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Treatment of retardation effects in calculating the radiated electromagnetic fields from the lightning discharge

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Abstract. The analytical treatment of retardation effects in calculating lightning electromagnetic fields far from the source has often involved the use of a so-called *F* factor. The literature concerning the *F* factor in the lightning field equations is often confusing and sometimes in error. The aim of this paper is to clarify, to correct when needed, and to extend previous views of the *F* factor by considering and consolidating the various situations, both mathematically and physically, in which this factor can occur. The *F* factor arises because of the retardation effects occurring when the distance to the observer from a point on a propagating current is changing with time. In this paper we (1) discuss the various situations in which the *F* factor can arise, such as in the determination of the "radiating" channel length "seen" by the observer, in the calculation of fields due to a propagating current wave up or down the discharge channel, and in the calculation of fields due to a propagating current discontinuity extending the channel upward; (2) give a unifying physical interpretation for this factor; and (3) show that the retardation effects in calculating lightning fields can be accounted for without the explicit use of an *F* factor. Relative to item 2 above, we will show that for simple return-stroke models like the transmission line (TL) and traveling current source (TCS) models, in which the current at one point on the channel appears at another point at another time, the *F* factor associated with current behind the front can be interpreted physically as the ratio of the apparent propagation speed (upward or downward) of the current wave "seen" by a distant observer to its actual speed. In the TL model, a current wave moves upward at speed v, the same speed as that of the front, and the *F* factor is given by $[1-(v/c)\cos\theta]^{-1}$, where θ is the angle between the direction of propagation of the source and the line joining the source point and the field point (observer), and c is the speed of light in vacuum. In the TCS model, a current wave moves downward at a speed equal c, and the *F* factor is given by $[1+\cos\theta]^{-1}$. The *F* factor associated with an upward propagating current discontinuity can always (in any model) be interpreted physically as the ratio of the apparent propagation speed of the discontinuity to its actual speed and is given by the same expression as for the *F* factor for the upward propagating current wave in the TL model.

1. Introduction

The cloud-to-ground lightning return-stroke channel is generally idealized as a vertically extending line with one end fixed at ground. In most return-stroke models the current versus time is specified at each point along the extending line. General expressions for the remote electric and magnetic fields due to any assumed channel current distribution have been derived by *Uman et al.* [1975]. In order to use these time-domain integral equations in lightning electric and magnetic field calculations, appropriate account must be taken of the different retarded times $t-R/c$, for different points on the channel, where R is the distance from the source point to the observer. For the transmission line (TL) return-stroke model, characterized by a current wave moving vertically upward from ground without distortion or attenuation at the same constant speed as the discharge front, *Uman et al.* [1973] give an approximate analytical expression for the electric radiation field observed at ground level very far from the lightning channel. In this approximation the field is directly proportional to the channel-base current multiplied by the propagation speed v. We shall show (see also *LeVine and Willett* [1992] and *Krider* [1992]) that for the case of a channel that is not perpendicular to the observer's line of sight (e.g., an elevated observation point, an elevated vertical channel segment, or a nonvertical channel segment), the field/current equation (2) of *Uman et al.* [1973] relating the magnitude of the current at the channel (or channel segment) origin and the magnitude of the electric radiation field at the distant observation point on ground, rewritten here as $E(r,t) = vi(t-r/c)/(2\pi\epsilon_0 c^2 r)$, should be multiplied by a geometrical factor

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$\sin\alpha/[1-(v/c)\cos\alpha]$, where α is the angle between the direction of propagation of the discharge front and the line joining the channel origin and the field point (observer), and c is the speed of light. Part of this expression, $[1-(v/c)\cos\alpha]^{-1}$, is sometimes called the F factor and is due to the retardation effects occurring when the distance to the observer from a point on the traveling current wave is changing with time. We shall show that the F factor arises in analytical solutions of the type derived by *Uman and McLain* [1970a], *LeVine and Willett* [1992], and *Krider* [1992] whenever there is source motion that is not perpendicular to the observer's line of sight. We find the literature concerning the F factor to be often confusing and sometimes in error. The present paper can be viewed as clarification, generalization, and extension of the previous work on the F factor by *LeVine and Meneghini* [1978], *Meneghini* [1984], *Rubinstein and Uman* [1990, 1991], *LeVine and Willett* [1992], and *Krider* [1992].

2. Formulation of the Problem

We model the lightning return stroke as a vertically extending electrical discharge with its origin at ground. Assume that the front of the discharge is moving with a speed v, which is usually a significant fraction of the speed of light. Ahead of the discharge front (or tip) the current is zero. Behind the discharge front the current is changing with time and is different at different heights along the channel. Various return-stroke models describe how the current at a given height z' is related to the current at ground $z' = 0$, at any given instant of time t. To find the electric and magnetic fields due to the time- and height-varying current in the channel, consider first a differential-length segment of the channel dz' at a height z' as shown in Figure 1. The element dz' is stationary even though the current in it is changing with time. The vector potential (as a function of time) from the time-varying current in dz' can be found as

$$d\bar{A} = \frac{\mu_0}{4\pi}\frac{i(z',t-R(z')/c)}{R(z')}dz'\hat{z} \tag{1}$$

Note that in (1) we used retarded current, i.e., current evaluated at retarded times $t-R(z')/c$, but we do not need any factor similar to the factor $[1-(v/c)\cos\theta]^{-1}$ appearing in the expression for the Lienard-Wiechert potential of a moving charge, because we are finding the potential from the time-varying current in a stationary element of length dz' at a height z'. More discussion on this point can be found in section 3. From the differential vector potential given by (1) we can find the scalar potential $d\phi$

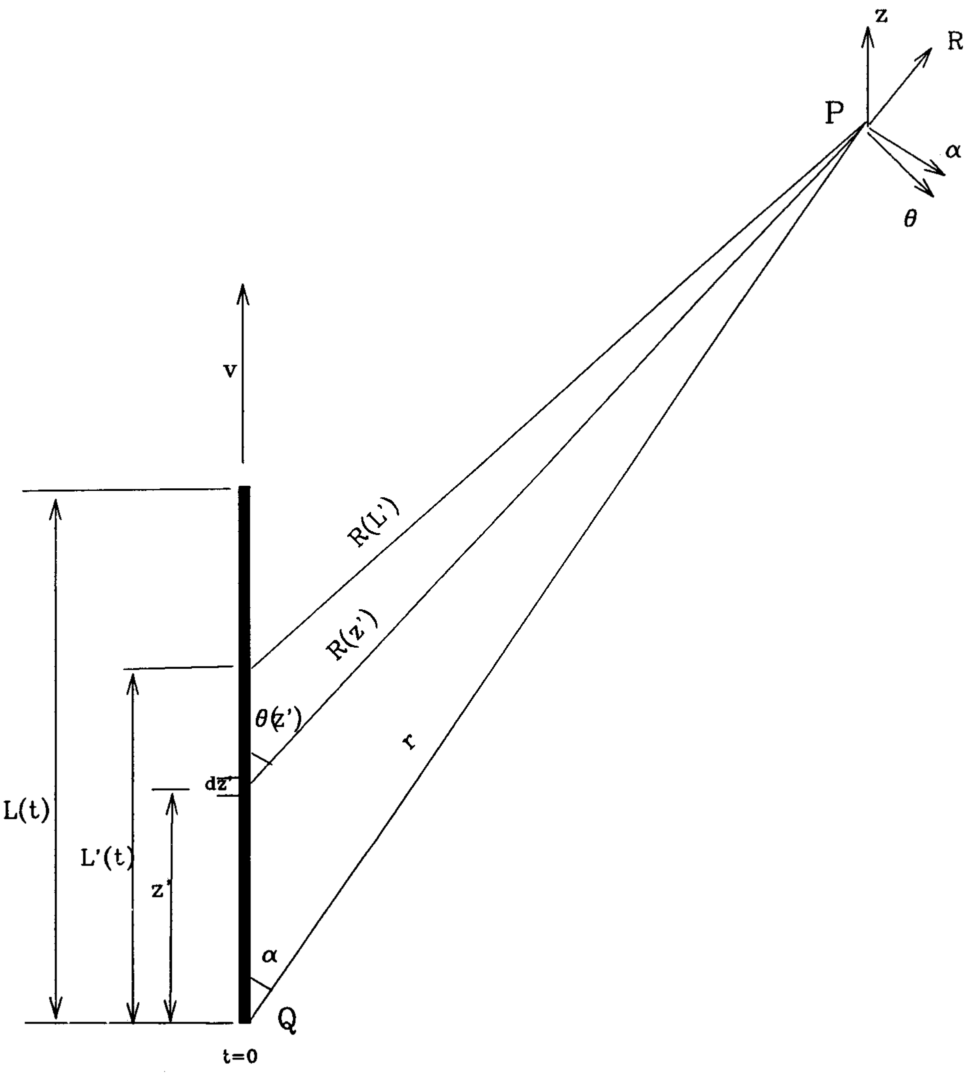

Figure 1. Lightning channel-observer geometry used to formulate the problem. See text for details.

and then the electric and magnetic fields as given by equations (2) to (6) below [see *Uman et al.*, 1975].

$$d\phi = -c^2 \int_{t_b}^{t} \nabla \cdot d\bar{A}\, d\tau \tag{2}$$

$$d\bar{E} = -\nabla d\phi - \frac{\partial}{\partial t} d\bar{A} \tag{3}$$

$$d\bar{B} = \nabla \times d\bar{A} \tag{4}$$

$$\bar{E}(r,t) = \int_0^{L'(t)} d\bar{E} \tag{5}$$

$$\bar{B}(r,t) = \int_0^{L'(t)} d\bar{B} \tag{6}$$

where t_b is the time at which the observer first "sees" the source, and $L'(t)$ is the radiating length of the discharge "seen" by the observer at time t, the so-called "retarded channel length" that is determined in section 3. Note that the scalar and vector potentials before the time t_b are both equal to zero. Equations (5) and (6) with the effect of ground neglected can be written in spherical coordinates, following *Uman et al.* [1975], as

$$\begin{aligned}\bar{E}(r,t) = {} & \frac{1}{4\pi\epsilon_0} \int_0^{L'(t)} \cos\theta \times \left[\frac{2}{R^3(z')} \int_{\frac{z'}{v}+\frac{R(z')}{c}}^{t} i\left(z',\tau - \frac{R(z')}{c}\right) d\tau \right. \\ & \left. + \frac{2}{cR^2(z')} i\left(z',t - \frac{R(z')}{c}\right) \right] dz'\hat{R} \\ & + \frac{1}{4\pi\epsilon_0} \int_0^{L'(t)} \sin\theta \times \left[\frac{1}{R^3(z')} \int_{\frac{z'}{v}+\frac{R(z')}{c}}^{t} i\left(z',\tau - \frac{R(z')}{c}\right) d\tau \right. \\ & + \frac{1}{cR^2(z')} i\left(z',t - \frac{R(z')}{c}\right) \\ & \left. + \frac{1}{c^2R(z')} \frac{\partial i(z',t - R(z')/c)}{\partial t} \right] dz'\hat{\theta}\end{aligned} \tag{7}$$

$$\begin{aligned}\bar{B}(r,t) = {} & \frac{\mu_0}{4\pi} \int_0^{L'(t)} \sin\theta \left[\frac{1}{R^2(z')} i\left(z',t - \frac{R(z')}{c}\right) \right. \\ & \left. + \frac{1}{cR(z')} \frac{\partial i(z',t - R(z')/c}{\partial t} dz'\hat{\phi} \right]\end{aligned} \tag{8}$$

Equations (7) and (8) above differ slightly from equations (9) and (7), respectively, of *Uman et al.* [1975] in the following way: Uman et al. considered an antenna of fixed length H, and hence their upper limit of integration on channel height was H instead of the retarded channel length $L'(t)$ used in this paper. Further, the lower limit of integration on τ in equation (9) of *Uman et al.* [1975] for the electric field is zero, whereas in equation (7) of this paper, the lower limit of integration on time τ is $z'/v + R(z')/c = t_b(z')$, the time taken by a return stroke to reach a height z' plus the time taken by an electromagnetic signal to travel from z' to the observer. The time at which the return stroke begins at $z' = 0$ is chosen as $t = 0$ (see Figure 1). The observer "sees" no current in the channel at z' before the time $t_b(z')$. Therefore the value of the integral remains the same whether the lower integration limit on τ is zero or $t_b(z')$. As an alternate but equivalent approach, one can find the vector potential $\bar{A}(r,t)$ for the whole retarded length $L'(t)$ as

$$\bar{A}(r,t) = \frac{\mu_0}{4\pi} \int_0^{L'(t)} \frac{i(z',t - R(z')/c)}{R(z')} dz'\hat{z} \tag{9}$$

and then derive field expressions identical to those given by (7) and (8). Equations (7) and (8) for calculating the fields from an extending discharge are applicable to any straight discharge, arbitrarily oriented with respect to the ground plane and attached or not attached to it, with one end of the channel fixed in space, provided that the z axis is chosen along the discharge and $\hat{z}$ is oriented in the direction of propagation. However, if the effect of the ground plane is to be considered or if the channel is composed of zigzagging straight segments, the appropriate coordinate transformations must be performed or a more suitable coordinate system must be chosen [see *LeVine and Willett*, 1992]. Equations (7) and (8) are exact for any return-stroke speed and for any position of the observer with respect to the channel, and they clearly do not contain any explicit factor similar to the factor $[1-(v/c)\cos\theta]^{-1}$ found in the Lienard-Wiechert potential. Far from the lightning channel the radiation fields are dominant and are given by the current derivative terms in (7) and (8). For example, the radiation electric field component is given by

$$\bar{E}_r(r,\theta,t) = \frac{1}{4\pi\epsilon_0} \int_0^{L'(t)} \frac{\sin(\theta)}{c^2R(z')} \frac{\partial i(z',t - R(z')/c)}{\partial t} dz'\hat{\theta} \tag{10a}$$

For the transmission line (TL) model applied to a vertical channel of a length that is very small compared to the distance to the observer from the channel and for a channel that is perpendicular to the line of sight of the observer, equation (10a) can be written in simplified form (see also *Uman et al.* [1973, Equation (2)])

$$\bar{E}_r(D,t) = -\frac{v}{4\pi\epsilon_0 c^2 D} i\left(0,t - \frac{D}{c}\right)\hat{z} \tag{10b}$$

where D is the horizontal distance between the observation point and the channel base, and $\hat{z}$ is an upward directed unit vector. Note that (10b) does not take into account the effect of ground and is valid only for times before the current reaches the channel top or for an extending discharge, as shown in later sections. Equation (10b) has been derived for the case that all channel source points are on a line essentially perpendicular to the observer's line of sight. That is, as the channel extends, all the source points are assumed to remain approximately equidistant from the observer. As will be shown later (see also *Rubinstein and Uman* [1990], *LeVine and Willett* [1992], and

Krider [1992]), if the channel is not essentially perpendicular to the observer's line of sight (for instance, for a vertical channel and a close observer, for an elevated vertical channel, or for a nonvertical channel), equation (10b) must be multiplied by the geometrical factor $\sin\alpha/[1-(v/c)\cos\alpha]$, partly to take account of retardation effects, where α is the angle between the direction of propagation and the line connecting the channel origin and the distant observation point (see Figure 1). *Uman and Mclain* [1970a], in deriving an equation for magnetic flux density due to an individual leader step and observer at ground (their equation (11)), failed to recognize this geometrical factor in its entirety. Only $\sin\alpha$ appears in their field/current equation. The portion of the geometrical factor $[1-(v/c)\cos\alpha]^{-1}$ missing in the work of *Uman and McLain* [1970a] is the F factor discussed in section 1. It is worth noting again that for channels whose lengths are very small compared to the distance and that are essentially perpendicular to observer's line of sight, $\alpha \approx 0$ so that $\sin\alpha \approx 1$, the F factor $[1-(v/c)\cos\alpha]^{-1}$ is equal to unity, and hence the use by Uman and coworkers of equation (10b) (with the right-hand side multiplied by 2 to take into account the effects of a ground, assumed to be perfectly conducting) for distant return strokes is correct. Considerable confusion regarding the use of F factor exists in the literature. *Krider* [1992] incorrectly interpreted the results of *Rubinstein and Uman* [1990] and *LeVine and Willett* [1992], who introduce the F factor into two special-case field equation, as suggesting that all previously published expressions for lightning electromagnetic fields that do not contain an explicit F factor are incorrect if v is a significant fraction of the speed of light. *Kumar et al.* [1995] erroneously claimed that the general expressions (7) and (8) above require corrections involving the F factor and used the erroneously "corrected" general expressions to calculate the fields at distances from 50 m to 5 km, both on ground and above ground.

3. Theory

One familiar example of the use of the F factor $[1-(v/c)\cos\alpha]^{-1}$ is in the expression for the scalar potential of a uniformly moving point charge, popularly known as the Lienard-Wiechert (LW) potential. Therefore it is instructive to begin our examination of the F factor by considering the origin of the F factor in the LW potential. The concepts developed are then applied to upward and downward propagating current waves and to extending discharge channels.

3.1. Lienard-Wiechert Scalar Potential

A point charge is an approximation for a finite-size charge distribution when the distance to the observer is very large compared to the dimensions of the distribution. The Lienard-Wiechert potential is an approximation to the potential of a uniformly moving finite charge distribution if the size of this distribution is very small compared to the distance to the point where the potential is evaluated [*Feynman et al.*, 1964; *Panofsky and Phillips*, 1962]. Because of the finite speed of electromagnetic waves, an observer at a given time "sees" the charge and its effects from an earlier time at an earlier position. If the distances to the observer from each point on the finite-size charge are essentially the same, then the scalar potential at the position of the observer at time t is given in terms of the retarded position and retarded value of the charge as

$$\phi = \frac{q(t-R/c)}{4\pi\epsilon_0 R} \tag{11}$$

where R is the distance from the observer to the charge at a time R/c earlier (retarded distance), and $q(t-R/c)$ is the value of the charge at a time R/c earlier. However if the different parts of the finite charge distribution are at different distances from the observer, the retarded time is different for those parts. Then the general expression for the scalar potential of a finite charge distribution is obtained by integrating the contributions to the potential from each elemental charge volume within the retarded charge volume and is

$$\phi(t) = \frac{1}{4\pi\epsilon_0}\int_{V'}\frac{\rho_v(t-R/c)}{R}dV' \tag{12}$$

where ρ_v is the volume charge density, dV' is the elemental retarded volume, R is the retarded distance, and V' is the retarded volume over which the integration is carried out. Let us first consider the application of (12) for the case of a line charge of length L, moving along its axis with uniform speed v (Figure 2). Later, we will consider the application of (12) to a point charge as a special case of L being vanishingly small compared to the retarded distance to the observer and thereby show the origin of the factor $[1-(v/c)\cos\alpha]^{-1}$. For simplicity, assume that the line has a uniform charge density ρ and total charge $q = \rho L$. Then the scalar potential is given by

$$\phi = \frac{1}{4\pi\epsilon_0}\int_0^{L'}\frac{\rho(t-R/c)}{R(z')}dz' \tag{13}$$

where dz' is a retarded elemental length of the line charge at position z' and at a retarded distance $R(z')$ from the observer, and L' is the retarded line length, the length of the line "seen" by the observer at time t. We can find L' as follows: In Figure 2a the moving line charge has a velocity component toward a stationary observer at P, and in Figure 2b the moving line charge has a velocity component away from the observer. Assume that at t=0, the lower end of the line charge is at point Q and its upper end at T. A line of length r connecting Q with P makes an angle α with the direction of motion of the charged line. The time required for an electromagnetic signal to propagate from Q to P is r/c, that is; the lower end of the charged line when it was at point Q is "seen" by the observer at $t=r/c$. In Figure 2a, where the velocity of the line has a component toward P, the distance from T to P is shorter than from Q to P, and hence the observer will "see" T first and successively lower sections until at $t=r/c$ the lower end Q, i.e., the full length of line L, is seen at $t=r/c$. Further, since the line is moving, the positions of its lower and upper ends change with time (both move upward). At $t=r/c$, the upper end of the moving line is "seen" by the observer in position T' at a distance R. It follows that at $t=r/c$, an observer at P simultaneously "sees" points Q (origin) and T' (and all points between Q and T'); that is, the length L' of the line "seen" by the observer is greater (for the geometry shown in Figure 2a) than the physical length L of the line. This apparent or retarded length L' depends on the velocity v and the geometry. From Figure 2a it follows that

$$\frac{L'-L}{v} + \frac{R}{c} = \frac{r}{c} \tag{14}$$

where

$$R = \sqrt{r^2+L'^2-2r'L\cos\alpha} \tag{15}$$

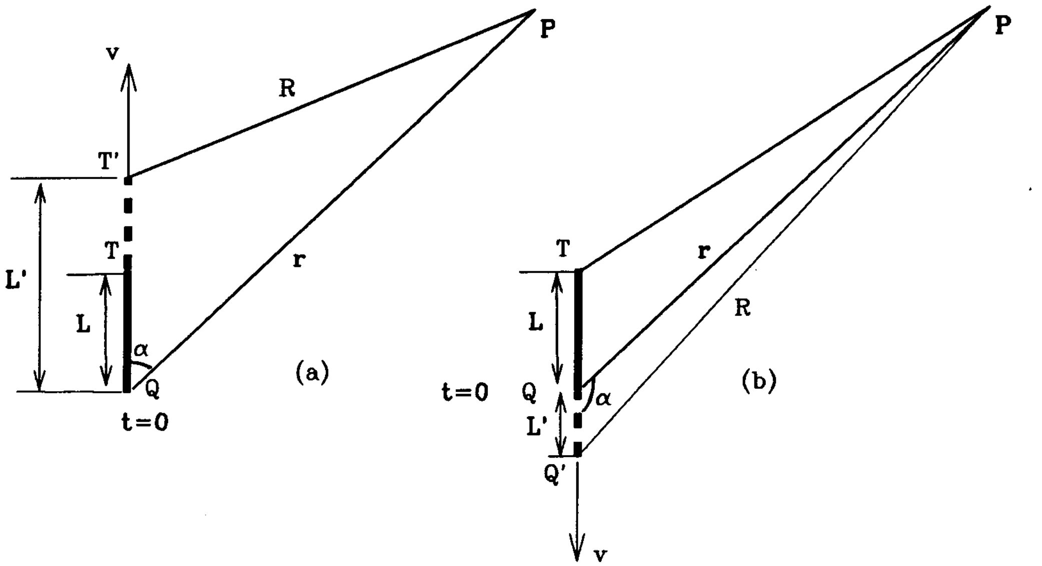

Figure 2. Charged line (a) moving closer to observer at P and (b) receding from observer at P. See text for details.

In Figure 2b the observer "sees" the top end T of the line passing through Q in a time $L/v+r/c$. In that time the bottom end has moved and is "seen" by the observer at position Q' at a distance R. The apparent length L' of the line is smaller than the physical length L of the line. From Figure 2b it follows that

$$\frac{L'}{v} + \frac{R}{c} = \frac{L}{v} + \frac{r}{c} \tag{16}$$

which is in the same form as (14). Now, referring to Figure 2a, we will examine solutions of (14) for L' for different positions of P with respect to the line. For $\alpha = 0°$, i.e., when the observer is along the axis of the moving line with the line approaching the observer, $R=r-L'$, and (14) yields the following expression for L':

$$L' = \frac{L}{1-\frac{v}{c}} \tag{17}$$

From (17) it is clear that when the line is approaching the observer $L'>L$; that is, the apparent length is greater than the actual. For $\alpha = 180°$, i.e., when the observer is along the axis of the moving line with the line receding from the observer, (14) yields

$$L' = \frac{L}{1+\frac{v}{c}} \tag{18}$$

From (18) it is clear that when the line is receding from the observer $L'<L$; that is, the apparent length is smaller than the actual length. For an arbitrary α, if $L<<r$, then $R \approx r-L'\cos\alpha$, and therefore (14) leads to (see also *Griffiths* [1981, p. 367])

$$L' = \frac{L}{1-\frac{v}{c}\cos\alpha} \tag{19}$$

Equation (14) (and (16)) can be written as a second degree polynomial in L', and a general solution for L', for any arbitrary angle α and ratio L/r, can be found as

$$L' = \frac{r}{1-\frac{v^2}{c^2}}\left[\frac{v}{c}\left(1-\frac{v}{c}\cos\alpha\right) + \frac{L}{r} - \frac{v}{c}\sqrt{\left(1-\frac{v}{c}\cos\alpha\right)^2 - 2\frac{L}{r}\left(\cos\alpha-\frac{v}{c}\right)+\left(\frac{L}{r}\right)^2}\,\right] \tag{20}$$

Note that the general solution for L' given in (20) does contain the expression $[1 - (v/c)\cos\alpha]$, found in the F factor, but it does not appear as a simple "correction" to L as in the case of far-distance approximation given in (19). If $L<<r$, the point charge approximation (19) applies and equation (13) reduces to

$$\begin{aligned}\phi &= \frac{\rho(t-R/c)}{4\pi\epsilon_0 R}\,\frac{L}{1-\frac{v}{c}\cos\alpha} \\ &= \frac{q(t-R/c)}{4\pi\epsilon_0 R}\,\frac{1}{1-\frac{v}{c}\cos\alpha}\end{aligned} \tag{21}$$

where R is the retarded position of the charge. Equation (21) is the retarded scalar potential or Lienard-Wiechert scalar potential of a uniformly moving point charge in which the F factor appears as a correction to the potential of a point charge at rest. If the far-distance approximation does not apply, (13) has to be used for finding the potential with the upper limit of integration, L', given by (20). In the integral equation (13) each point along the moving line charge is at a different distance from the observer, and it may appear reasonable that to take care of retardation effects each differential length dz' of the channel should be multiplied by the F factor. However, this reasoning is not correct because the differential length dz' in (13) is the retarded differential length. The incremental time required, as seen by the observer, for a point on the line charge to move an incremental distance $\Delta z'$ is given by

$$\Delta t = \frac{\Delta z'}{v} + \frac{R(z'+\Delta z')}{c} - \frac{R(z')}{c} \quad (22)$$

In the limit $\Delta z' \to 0$, (22) can be rearranged and written as

$$\frac{dz'}{dt} = \frac{v}{1 - \frac{v}{c}\cos\theta(z')} = v \cdot F(z') \quad (23)$$

where $\theta(z')$ is the angle between the direction of motion at z' and the observer's line of sight. In (23) the F factor appears as a modifying factor to the propagation speed v. It is clear from (23) that for different positions along the line charge the apparent speed given by (23) is different, with apparent speed equal to v only when $\theta(z')=90°$, i.e., when the motion is perpendicular to the observer's line of sight. By the same reasoning used to formulate (22), we can show that the apparent speed of a uniformly moving point charge is given by (23), containing the F factor.

In section 3.2 the results obtained for a moving line charge of fixed length and constant charge density are adapted to an extending discharge with one end fixed in space, a typical model for the lightning return stroke. In general, the charge density and current in the lightning channel vary as a function of height and time.

3.2. Extending Discharge With One End Fixed

Consider a lightning return-stroke channel with one end fixed at Q as shown in Figure 1. It takes a time r/c for the information from Q to reach the observer at P and hence the observer "sees" the channel emerging from Q at time r/c. The length $L(t)$ of the channel at a time $t\text{-}r/c$ is given by

$$L(t) = v \cdot \left(t - \frac{r}{c}\right) \quad (24)$$

which corresponds to length L in Figures 2a and 2b. If the line is assumed to be perpendicular to a perfectly conducting ground plane passing through Q, then L' in Figures 2a and 2b corresponds to the apparent length $L'(t)$ of the channel and its image, respectively. Replacing L in (20) with $L(t)$ given by (24), we obtain the general expression for $L'(t)$ applicable to the lightning return-stroke,

$$L'(t) = \frac{r}{1 - \frac{v^2}{c^2}} \left[-\frac{v^2}{c^2}\cos\alpha + \frac{vt}{r} - \frac{v}{c}\sqrt{\left(1 - \frac{v^2}{c^2}\right) + \frac{v^2t^2}{r^2} + \frac{v^2}{c^2}\cos^2\alpha - \frac{2vt}{r}\cos\alpha} \right] \quad (25)$$

If we define the time t such that it is the sum of the time required for the return-stroke wavefront to reach a height $L'(t)$ and the time required for a signal to travel from the wavefront at $L'(t)$ to the observer at P, t can be written as

$$t = \frac{L'(t)}{v} + \frac{R(L'(t))}{c} \quad (26a)$$

and from Figure 1,

$$R(L'(t)) = \sqrt{r^2 + L'^2(t) - 2L'(t) r \cos\alpha} \quad (26b)$$

The retarded length $L'(t)$ given in (25) can also be obtained by directly solving (26) for $L'(t)$. If all channel sections were equidistant from the observer, i.e., if the discharge were to extend in a circular arc of radius r with the observer at the center, the length of the discharge seen by the observer would be $L'(t) = v \cdot (t\text{-}r/c)$. If electromagnetic propagation were with infinite speed, the observer would see the actual channel length; that is, if c in (25) is replaced by ∞, $L'(t) = vt$. If the ground is treated as perfectly conducting, (25) can also be used, with α replaced by $(180° - \alpha)$, to find the apparent length of the channel image "seen" by the observer. Even if the velocity of the discharge front is varying with height, (26) is valid if v is replaced by an "average" velocity (see *Thottappillil et al.* [1991] and *Thottappillil and Uman* [1994] for details) and can be solved for $L'(t)$ iteratively.

If the channel is straight and vertical and its length is very small compared to the distance to the observer, i.e., if $L'(t) \ll r$, then $R(L'(t))$ can be approximated as (see Figure 1)

$$R(L'(t)) = r - L'(t)\cos\alpha \quad (27)$$

Substituting (27) in (26a) we obtain

$$L'(t) = \frac{v \cdot (t - r/c)}{1 - \frac{v}{c}\cos\alpha} = F \cdot v \cdot (t - r/c) \quad (28)$$

where $v \cdot (t\text{-}r/c)$ is the actual length of the discharge at time t. Thus the F factor $F=[1-(v/c)\cos\alpha]^{-1}$ appears in the far-distance approximation to the retarded channel length that can be used in equations (7), (8), and (10a).

For any given model and any given channel-base current, (10a) can be evaluated numerically. For each value of t the integration in (10a) has to be carried out through the upper limit $L'(t)$, where $L'(t)$ is either given by the analytical expression (25) or by the numerical solution of (26). When the electric radiation field is calculated numerically as indicated above, there is no need for the explicit use of the F factor.

So far we have not made any specific assumptions regarding the charge and current distributions along the return-stroke channel. Many return-stroke models specify the relationship between the time-varying current at ground and current at a height on the channel [e.g., *Rakov*, 1997]. In some return-stroke models currents are assumed to travel up and in others to travel down the return-stroke channel behind the upward extending discharge front (e.g., transmission line model and traveling current source model, respectively). To be general, let us assume that the speed of travel of the current wave is u, different from the speed of travel of the discharge front v. The incremental time required, as seen by the observer, for a point on the current wave to move an incremental distance $\Delta z'$ is given by

$$\Delta t = \frac{\Delta z'}{u} + \frac{R(z'+\Delta z')}{c} - \frac{R(z')}{c} \quad (29)$$

In the limit $\Delta z' \to 0$, (29) can be rearranged and written as

$$\frac{dz'/dt}{u} = \frac{1}{1 - \frac{u}{c}\cos\theta(z')} = F(z') \quad (30)$$

where dz'/dt is the speed of the wave as "seen" by the observer, and $\theta(z')$ is same as in (23). Note that in (29) and (30) the speed u can have either sign, positive if u is in the same direction as v, and negative if u is in the direction opposite to v. From (30) it is evident that the F factor is the ratio of the apparent speed to the actual speed of traveling waves in a return stroke channel.

Differentiating both sides of equation (26a) and rearranging, we find that the apparent speed of the discharge front is

$$\frac{dL'(t)}{dt} = v\cdot\frac{1}{1 + \frac{v}{c}\cdot\frac{L'(t) - r\cos\alpha}{\sqrt{L'^2(t) + r^2 - 2L'(t)r\cos\alpha}}} = v\frac{1}{1 - \frac{v}{c}\cos\theta(L')} \quad (31)$$

Note that the factor $[1-(v/c)\cos\theta(L')]^{-1}$ in (31) is obtained for a traveling step function in *Rubinstein and Uman* [1990] using an alternate derivation involving Heaviside and delta functions. As follows from (31), this factor is the ratio of the apparent speed to the actual speed of the propagating discharge front, depends only on the front propagation speed and geometry, and is applicable to the discharge front discontinuity in any return-stroke model. The apparent speed $dL'(t)/dt$ of the discharge front can also be calculated numerically as $\{L'(t+\Delta t)-L'(t)\}/\Delta t$, where Δt is the incremental time, and $L'(t+\Delta t)$ and $L'(t)$ are numerically determined from (26a).

In section 4 we consider first the transmission line (TL) model for the return stroke used by *Uman et al.* [1973], *LeVine and Willett* [1992], and *Krider* [1992] and inspect how the factor $[1-(v/c)\cos\alpha]^{-1}$ arises in the approximate expression relating the channel-base current and the remote radiation field. Then we consider the traveling current source (TCS) model and show that the approximate expression relating the channel-base current and the remote radiation field (excluding the radiation from the front) involves an F factor equal to $[1+\cos\alpha]^{-1}$, clearly different from that for the TL model. Finally, we generalize the field/current equations to include radiation from the front, inherent in the TCS model and possible in the TL model if there is a current discontinuity at the front.

4. Application to Return-Stroke Models

4.1. Radiation Field/Current Relation for the Transmission Line Model

In the transmission line (TL) model of the return stroke, a current wave originates at ground and propagates up the channel without attenuation and distortion [e.g., *Uman and McLain*, 1969]. The retarded current distribution along the channel can be expressed in terms of the current at the base of the channel as

$$i\left(z', t - \frac{R(z')}{c}\right) = i\left(0, t - \frac{z'}{v} - \frac{R(z')}{c}\right) \quad (32)$$

From Figure 1, $R(z')$ can be written as

$$R(z') = \sqrt{z'^2 + r^2 - 2z'r\cos\alpha} \quad (33)$$

which is same as (26b) except it is for any z', not only for $z'=L'(t)$.

The variables t and z' are independent since z' is arbitrarily chosen, and thus we can write

$$\frac{\partial i(0,t - z'/v - R(z')/c)}{\partial z'} = \frac{\partial i(0,t-z'/v-R(z')/c)}{\partial(t-z'/v-R(z')/c)}\cdot\frac{\partial(t-z'/v-R(z')/c)}{\partial z'}$$
$$= -\frac{\partial i(0,t-z'/v-R(z')/c)}{\partial t}\cdot\frac{1}{v}\cdot\left(1 + \frac{v}{c}\frac{z'-r\cos\alpha}{\sqrt{z'^2+r^2-2z'r\cos\alpha}}\right) \quad (34)$$

Rearranging (34), we obtain the relationship between the time and the spatial derivatives of the retarded current, involving the F factor,

$$\frac{\partial i\left(0, t - \frac{z'}{v} - \frac{R(z')}{c}\right)}{\partial t} = -\frac{\partial i\left(0, t - \frac{z'}{v} - \frac{R(z')}{c}\right)}{\partial z'}\cdot v\cdot F_{TL}(z') \quad (35)$$

where the factor $F_{TL}(z')$ is a function of z' and is given by

$$F_{TL}(z') = \left[1 + \frac{v}{c}\frac{z'-r\cos\alpha}{\sqrt{z'^2+r^2-2z'r\cos\alpha}}\right]^{-1} = \left[1 - \frac{v}{c}\cos\theta(z')\right]^{-1} \quad (36)$$

and $\theta(z')$ is the angle between the direction of propagation and a line joining the point P with the point on the channel at height z' (see Figure 1). Here we see how the F factor arises in analytically converting the time derivative of the retarded current into its spatial derivative. The physical meaning of the F factor in this context of a traveling wave is the same as in (30). Replacing u with v, we observe that (30) and (36) become identical. Using (32) and (36), we can rewrite the equation for the radiation field (10a) for the TL model as

$$\bar{E}_r(r,\theta,t) = -\frac{1}{4\pi\epsilon_0}\int_0^{L'(t)}\frac{\sin\theta}{c^2R(z')}\cdot v\cdot F_{TL}(z')\frac{\partial i(0,t-z'/v-R(z')/c)}{\partial z'}dz'\hat{\theta} \quad (37)$$

The integration in (37) can be performed analytically if the observer is distant, as is also necessary for the radiation field to be dominant, so that $L'(t) \ll r$, $\theta \approx \alpha$, $R(z') \approx r$,

$$\bar{E}_r(r,\alpha,t) = -\frac{1}{4\pi\epsilon_0}\frac{\sin\alpha}{c^2r}\frac{v}{1-\frac{v}{c}\cos\alpha}\int_0^{L'(t)} di\left(0, t - \frac{z'}{v} - \frac{R(z')}{c}\right)\hat{\alpha}$$
$$= -\frac{1}{4\pi\epsilon_0}\frac{\sin\alpha}{c^2r}\frac{v}{1-\frac{v}{c}\cos\alpha}\left[i(0,0) - i\left(0, t - \frac{r}{c}\right)\right]\hat{\alpha} \quad (38)$$

noting that $i(0,t - L'(t)/v - R(L'(t))/c) = i(0,0)$, as follows from equation (26a). The factor $[1-(v/c)\cos\alpha]^{-1}$ in (38) appears as a multiplier to the speed v of the upward propagating current wave in the channel, which in the TL model is the same as the speed of the discharge front. The factor is equal to unity when $\alpha=90°$ (when the channel is perpendicular to the line of sight of the observer), the situation considered by *Uman et al.* [1973] (see equation (10b) above). In the Appendix we further discuss equation (38) and compare it with equation (9a) of *LeVine and Willett* [1992], also based on the TL model. It is worth noting that the term containing i(0,0) in (38) will be canceled by an equal and opposite-sign term representing the radiation from the front, a result derived in a later section.

4.2. Radiation Field/Current Relation for the Traveling Current Source Model

In the previous section we found how the F factor can arise in the relation between the time and spatial derivatives of the retarded current for the TL model and interpreted the factor as the ratio of the apparent speed to the actual speed of the current wave. In the traveling current source (TCS) model proposed by *Heidler* [1985] the relation between the channel current at z' and the channel-base (z'=0) current is given by

$$i\left(z',t-\frac{R(z')}{c}\right) = i\left(0,t+\frac{z'}{c}-\frac{R(z')}{c}\right) \tag{39}$$

where $R(z')$ is given by (33). Equation (39) represents a model in which the upward propagating, with speed v, return-stroke wavefront instantaneously activates current sources distributed along the lightning channel as it passes them. The resultant current is assumed to travel downward at the speed of light c, without distortion and attenuation. We first consider the current behind the return-stroke front. In the TCS model there always exists a current discontinuity at the return-stroke front, the retardation effects of that discontinuity being considered separately in a later section. The variables t and z' are independent since z' is chosen arbitrarily, and hence using the same procedure as in the case of TL model, we can derive for the TCS model the relationship between the time and spatial derivatives of the retarded current,

$$\frac{\partial i\left(0,t+\frac{z'}{c}-\frac{R(z')}{c}\right)}{\partial z'} =$$

$$= \frac{\partial i\left(0,t+\frac{z'}{c}-\frac{R(z')}{c}\right)}{\partial\left(t+\frac{z'}{c}-\frac{R(z')}{c}\right)}\cdot\frac{\partial\left(t+\frac{z'}{c}-\frac{R(z')}{c}\right)}{\partial z'}$$

$$= \frac{\partial i\left(0,t+\frac{z'}{c}-\frac{R(z')}{c}\right)}{\partial t}\cdot\frac{1}{c}\left(1-\frac{z'-r\cos\alpha}{\sqrt{z'^2+r^2-2z'r\cos\alpha}}\right) \tag{40a}$$

Rearranging the terms in (40a), we get

$$\frac{\partial i\left(0,t+\frac{z'}{c}-\frac{R(z')}{c}\right)}{\partial t} = \frac{\partial i\left(0,t+\frac{z'}{c}-\frac{R(z')}{c}\right)}{\partial z'}\cdot c\cdot F_{TCS}(z') \tag{40b}$$

where the factor $F_{TCS}(z')$, clearly different from $F_{TL}(z')$, is

$$F_{TCS}(z') = \frac{1}{\left(1-\frac{z'-r\cos(\alpha)}{\sqrt{z'^2+r^2-2z'r\cos\alpha}}\right)} = \frac{1}{1+\cos\theta(z')} \tag{41}$$

and $\theta(z')$ is as shown in Figure 1. Physical interpretation of the factor F_{TCS} is similar to that given earlier for F_{TL}. If we replace u with $-c$ in (30), we obtain (41).. Substituting (40b) in (10a), using the far-field approximations, i.e., letting $L'(t) \ll r$, $\theta\approx\alpha$, and $R(z')\approx r$, performing the integration, and using (26a), we obtain

$$\bar{E}_r(r,\alpha,t) = \frac{1}{4\pi\epsilon_0}\frac{\sin\alpha}{c^2 r}\frac{c}{1+\cos\alpha}\left[i\left(0,L'(t)\left(\frac{1}{c}+\frac{1}{v}\right)\right) - i\left(0,t-\frac{r}{c}\right)\right]\hat{\alpha} \tag{42}$$

Equation (42) gives the radiation electric field from the channel current behind the discharge front only; that is, it does not take account of the radiation from the current discontinuity at the discharge front, inherent in the TCS model, which is considered below.

4.3. Current Discontinuity at the Discharge Front (General)

The TL model may or may not have a discontinuity in current at the front. If current at ground level is zero at $t = 0$, there is no discontinuity in the TL model, as we shall see in section 4.4. The TCS model involves a current discontinuity at the front whether or not the current at ground level at t=0 is zero. As a result, (10a) has to be applied separately to the current behind the front and to the current discontinuity at the front. We show now how the F factor arises in the general radiation field expression for a current discontinuity at the discharge front.

Let $i(z',t-R(z')/c)$ describe the retarded current in the return-stroke channel. Then the current and current derivative at the return-stroke front "seen" by the observer can be expressed, using (26a), as

$$i\left(L'(t),t-\frac{R(L'(t))}{c}\right) = i\left(L'(t),\frac{L'(t)}{v}\right) \tag{43a}$$

$$\frac{\partial i(L'(t),t-R(L'(t))/c)}{\partial t} = \frac{di(L'(t),L'(t)/v)}{dt} \tag{43b}$$

Let $L'_-(t)$ and $L'_+(t)$ be the positions just below and just above the wavefront at $L'(t)$, respectively. The integral of the current derivative across the wavefront is equal to the product of the current at the wavefront and the velocity of the wavefront as seen by the observer at P. That is,

$$\int_{L'_-(t)}^{L'_+(t)} \frac{di(L'(t),t-R(L'(t))/c)}{dt}dz' = i\left(L'(t),\frac{L'(t)}{v}\right)\cdot\frac{dL'(t)}{dt} \quad (44)$$

We have seen earlier that the apparent speed of the discharge front $dL'(t)/dt$ is given by (31), containing the F factor. Substituting (31) in (44) and using the resulting equation in (10a), we find the radiation electric field from a traveling current discontinuity (the so-called "turn-on" field [*Uman and McLain*, 1970a])

$$\bar{E}(r,\alpha,t) = \frac{1}{4\pi\epsilon_0}\frac{\sin\theta(L')}{c^2R(L')}i\left(L'(t),\frac{L'(t)}{v}\right)\frac{v}{1-\frac{v}{c}\cos\theta(L')}\hat{\theta} \quad (45a)$$

Using the far-field approximation, i.e., letting $L'(t)<<r$, $\theta\approx\alpha$, and $R(L')\approx r$, we can express the "turn-on" field as

$$\bar{E}(r,\alpha,t) = \frac{1}{4\pi\epsilon_0}\frac{\sin\alpha}{c^2r}i\left(L'(t),\frac{L'(t)}{v}\right)\frac{v}{1-\frac{v}{c}\cos\alpha}\hat{\alpha} \quad (45b)$$

Note that in the expression for the "turn-on" field, *Uman and McLain* [1970a] did not include the factor $[1-(v/c)\cos\alpha]^{-1}$ appearing in (45b), an oversight that was later corrected by *Rubinstein and Uman* [1990]. The factor $[1-(v/c)\cos\alpha]^{-1}$ in (45b) appears as a "correction" to the return-stroke discharge front speed v and is applicable to any return-stroke model with a current discontinuity at the wavefront. The F factor associated with the current discontinuity at the return-stroke front derived here for any return-stroke model and for a traveling step function in the work of *Rubinstein and Uman* [1990, 1991] is the same as the F factor obtained for the upward propagating continuous current wave in the TL model, because all these radiation sources move in the same manner with respect to the observer on ground (upward at speed v). On the other hand, the F factor associated with the channel current behind the discharge front is different for the TL and TCS models, because the radiation sources move at different speeds and in different directions: upward at speed v in the TL model and downward at speed c in the TCS model, as illustrated in Figures 1 and 2 of *Rakov* [1997].

4.4. Far-Distance Radiation Field/Current Relations Taking Into Account Current Discontinuity at the Discharge Front

4.4.1. TL model. The current at the return-stroke front for the TL model is $i(L'(t),L'(t)/v) = i(0,0)$. Therefore if the current at ground starts from zero at $t = 0$, $i(0,0) = 0$, and hence there is no current discontinuity at the propagating discharge front. If the current at the ground starts from a nonzero value at t=0, the current will have the same nonzero value at the propagating front. The "turn-on" far radiation field is obtained from (45b) as

$$\bar{E}(r,\alpha,t) = \frac{1}{4\pi\epsilon_0}\frac{\sin\alpha}{c^2r}\cdot\frac{v}{1-\frac{v}{c}\cos\alpha}i(0,0)\hat{\alpha} \quad (46)$$

which is equal in magnitude and opposite in sign to the term containing i(0,0) in (38). Adding (38) and (46), we get

$$\bar{E}(r,\alpha,t) = \frac{1}{4\pi\epsilon_0}\frac{\sin\alpha}{c^2r}\frac{v}{1-\frac{v}{c}\cos\alpha}i(0,t-r/c)\hat{\alpha} \quad (47)$$

which is the total radiation (subscript "r" used in (38) is dropped here) electric field far from the channel for the TL model, whether or not there is a current discontinuity at the discharge front.

For the special case of an observer at ground far from the channel, α=90°, $\hat{\alpha} = -\hat{z}$ (see Figure 1), and r = D, equation (47) in a Cartesian coordinate system becomes the same as equation (10b), the relation used (with the right-hand side multiplied by 2 to account for the effects of the ground) by *Uman et al.* [1973] for the calculation of the far electric radiation fields at ground level from return strokes assumed to obey the TL model.

4.4.2. TCS model. The current at the return-stroke front for the TCS model is $i(L'(t),L'(t)/v) = i(0,L'(t)/v+L'(t)/c)$. Therefore unlike the TL model, even if the current at ground starts from zero, the current at the return-stroke front above ground will have a non-zero value. The "turn-on" far radiation field is obtained from (45b) as

$$\bar{E}(r,\alpha,t) = \frac{1}{4\pi\epsilon_0}\frac{\sin\alpha}{c^2r}\cdot\frac{v}{1-\frac{v}{c}\cos\alpha}i\left(0,L'(t)(\frac{1}{v}+\frac{1}{c})\right)\hat{\alpha} \quad (48)$$

The total radiation electric field for the TCS model is obtained by summing (42) and (48), which yields

$$\bar{E}(r,\alpha,t) = \frac{1}{4\pi\epsilon_0}\frac{\sin\alpha}{c^2r}\left[\frac{c}{1+\cos\alpha}\left(i(0,L'(t)(\frac{1}{c}+\frac{1}{v}))-i(0,t-\frac{r}{c})\right) + \frac{v}{1-\frac{v}{c}\cos\alpha}i\left(0,L'(t)(\frac{1}{c}+\frac{1}{v})\right)\right]\hat{\alpha} \quad (49)$$

For the special case of an observer at ground, α=90°, equation (49) in a Cartesian coordinate system becomes

$$\bar{E}(r,t) = -\frac{1}{4\pi\epsilon_0}\frac{1}{cr}\cdot\left[i\left(0,L'(t)(\frac{1}{c}+\frac{1}{v})\right) - i(0,t-\frac{r}{c}) + \frac{v}{c}i\left(0,L'(t)(\frac{1}{c}+\frac{1}{v})\right)\right]\hat{z} \quad (50)$$

where $\hat{z}$ is an upward directed unit vector.

Noting that for α=90°, $L'(t)<<r$, $L'(t)\approx L(t)=v(t-r/c)$, and defining

$$k = 1+\frac{v}{c} \quad (51)$$

we can rewrite equation (50) as

$$\bar{E}(r,t) = -\frac{1}{4\pi\epsilon_0}\frac{1}{cr}\cdot[ki(0,k(t-r/c)) - i(0,t-r/c)]\hat{z} \quad (52)$$

which is similar to the expression obtained by *Heidler* [1986] for the far radiation field at ground level from return strokes assumed to obey the TCS model. If we additionally wish to include the effect of perfectly conducting ground plane, we must multiply the right-hand side of (52) by 2.

5. Discussion and Summary

We have shown that in calculating the electromagnetic fields from lightning return strokes, the F factor arises as a manifestation of retardation effects because the distance R from the observer to each source point changes as the source moves with respect to observer. The F factor becomes significant in calculations only when the speed v is a significant fraction of the speed of light c. The F factor can appear explicitly as a multiplier (1) in the approximate expression for the retarded length of the channel when the observer is very far away (or when the length of channel is vanishingly small), (2) in the expression for the apparent speed of the propagating discharge front, and (3) in the expression for the apparent speed of the traveling waves in the channel (or in the expression relating the time derivative of the traveling current wave and its spatial derivative). Situations 1 and 2 are applicable to any return-stroke model, while situation 3 is applicable only to return-stroke models that involve a traveling wave such as the transmission line (TL) and the traveling current source (TCS) models (the F factor expression (41) for the TCS model can be obtained if we substitute $v = -c$ in the F-factor expression (36) for the TL model). The presence of the F factor in the expressions for the Lienard-Wiechert (LW) potentials of a charge in uniform motion at relativistic speeds and the fact that return-stroke discharge front is moving at relativistic speeds (typically one third of c), as well as work of *Rubinstein and Uman* [1990] and *LeVine and Willett* [1992] on the F factor for specific models, has led some authors to erroneously believe that general field expressions similar to (7) and (8) above require corrections involving the F factor (e.g., the first paragraph of Krider [1992] and Kumar et al. [1995]). The vertical and horizontal electric field integral expressions (3) and (4) of *Kumar et al.* [1995] contain multipliers $[1 - (v/c) \cos\theta]^{-3}$ for the current integral (static) terms, $[1 - (v/c) \cos\theta]^{-2}$ for the current (induction) terms, and $[1 - (v/c) \cos\theta]^{-1}$ for the current derivative (radiation) terms. *Kumar et al.* [1995] do not show the details of their mathematical derivation; however, they have used the F-factor-corrected vector potential (LW vector potential) for an elemental length of the channel dz', and this may be the source of the additional multipliers F^{-3}, F^{-2}, and F^{-1} in their general field equation (3) and (4). In section 3 of this paper we have shown why the derivation of general integral equations (7) and (8) does not require explicit application of the F factor and have also shown various situation in which the F factor does arise. The structure of equations (3) and (4) of *Kumar et al.* [1995] is similar to that of the general field expression (7) of this paper, and it appears that equations (3) and (4) of *Kumar et al.* [1995] are incorrect on account of the multipliers F^{-3}, F^{-2}, and F^{-1}. Therefore the calculated electric fields at various altitudes and ranges, that *Kumar et al.* [1995] present in their Figures 2, 3, 4, and 5, are likely in error. The general expressions (7) and (8) do not need corrections involving the F factor even when the speed of the return stroke is a significant fraction of the speed of light. Equations (7) and (8) can be used to find the electric and magnetic fields in free space at any position with respect to the lightning return stroke, with proper account taken of the effects of the ground. Lightning return stroke field calculations using the general expressions (7) and (8) applied to specific return-stroke models and observer at ground are presented, for example, by *Rakov and Dulzon* [1987, 1991], *Diendorfer and Uman* [1990], *Nucci et al.* [1990], *Thottappillil et al.* [1991], and *Thottappillil and Uman* [1993, 1994].

It is shown in the Appendix that the implicit "angle-dependent" element (inherent in the time delays) of the radiation field expressions of *LeVine and Willett* [1992] for the TL model is due to the fixed length of the current-carrying line. For the case of the TL model, the radiation field from an extending discharge does not have any implicit angle dependency as opposed to the radiation field from the channel of fixed length discussed by *LeVine and Willett* [1992] and in the Appendix.

Appendix

In this appendix we discuss some important differences and similarities of equation (38) of this study and equation (9a) of *LeVine and Willett* [1992] reproduced as (A1) below, both equations representing the electric radiation field very far from the lightning channel for the transmission line (TL) model. According to *LeVine and Willett* [1992],

$$\bar{E}(r,t) = -\frac{1}{4\pi\epsilon_0}\frac{\sin\theta}{c^2 R_0}\frac{v}{1-\frac{v}{c}\cos\theta}\cdot[I(t-t_a)-I(t-t_b)]\,\theta \quad \text{(A1)}$$

where R_0 is the distance from the center of a line segment of length L to the field point, and θ is the angle between the direction of propagation and the line connecting the midpoint of the segment and the field point. In (A1), t_a and t_b represent the times at which the current I reaches the end points a and b, respectively, of the segment plus the time required for the radiation to propagate from the corresponding end point to the observer. Equation (A1) contains the factor $[1-(v/c)\cos\theta]^{-1}$ explicitly as a multiplier, similar to that in (38). In addition, equation (A1) contains the factor $[1-(v/c)\cos\theta]^{-1}$ implicitly in the time delays $t_a=R_0/c-(L/2v)[1-(v/c)\cos\theta]$ and $t_b=R_0/c+(L/2v)[1-(v/c)\cos\theta]$, given by equations (10a) and (10b), respectively, of *LeVine and Willett* [1992], whereas the time delays r/c and 0 in equation (38) of this paper do not contain such a factor. The reason for the above difference is the following. Equation (9a) of *LeVine and Willett* [1992] gives the radiation field of a channel of fixed length stationary in space which is traversed by a propagating current wave, whereas equation (38) gives the radiation field of a channel whose length is being extended by the propagating discharge front. We can see that if the upper limit of the integral in equation (38) is changed from $L'(t)$ to L, a constant length, $R(L)$ in the expression $I(0,L/v - R(L)/c)$ is expanded in a power series, and only the first-order terms of L/r are taken, as done by *LeVine and Willett* [1992], we get an angle-dependent term in one of the time delays, $r/c+(L/v)[1-(v/c)\cos\alpha]$, as given in (A2) below.

$$\bar{E}_r(r,\alpha,t) = -\frac{1}{4\pi\epsilon_0}\frac{\sin\alpha}{c^2 r}\frac{v}{1-\frac{v}{c}\cos\alpha}\left[i\left(0,t-\frac{r}{c}-\frac{L}{v}\left(1-\frac{v}{c}\cos\alpha\right)\right) - i\left(0,t-\frac{r}{c}\right)\right]\alpha \quad \text{(A2)}$$

It is worth noting that the time reference used in the present paper (the source is at z=0 at t=0) is different from that adopted by *LeVine and Willett* [1992] who assumed that at t=0, the source is at the center of the line segment, as seen in their Figure 1b. The remaining apparent difference between (A1), *LeVine and Willett's* equation, and (A2) is due to this difference in time origins.

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