Some inferences on the propagation mechanisms of dart leaders and return strokes

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Abstract. Inferences regarding the propagation mechanisms of dart leaders and return strokes are made from a comparison of the behavior of traveling waves on a lossy transmission line and the observed characteristics of these two lightning processes, in particular, their observed light profiles [*Jordan and Uman*, 1983; *Jordan et al.*, 1997]. The bottom kilometer or so of the subsequent-stroke lightning channel is modeled as an *R-L-C* transmission line whose resistance per unit length *R* is first reduced by the downward propagating dart leader and then further reduced by the upward propagating return stroke, while variations in the inductance per unit length *L* and the capacitance per unit length *C* during the dart-leader and return-stroke processes are neglected. The transmission line is assumed to be linear (R = const) but different ahead of and behind either the dart-leader or the return-stroke front with any nonlinear effects occurring at the front. The resistance per unit length is estimated to be as follows: (1) ahead of the dart-leader front about 18 kΩ/m or greater; (2) behind the dart-leader front (and ahead of the return-stroke front) about 3.5 Ω/m; and (3) behind the return-stroke front about 0.035 Ω/m. Comparison of the behavior of traveling waves on such a transmission line with the observed properties of luminosity waves associated with dart leaders and return strokes in conjunction with other observations suggests that there is a difference between these two lightning processes in terms of the dominant propagation mechanism. It appears that the return stroke is similar to a "classical" (linear) traveling wave that suffers appreciable attenuation and dispersion and whose advancement can be visualized as being due to the progressive discharging of elemental capacitors (previously charged by the leader process) of the equivalent *R-L-C* transmission line. Ionization does occur during the return-stroke process but has a relatively small effect on the wave propagation characteristics which are primarily determined by the transmission-line parameters ahead of the front, as opposed to being determined by the wave magnitude. On the other hand, the progression of the dart leader is apparently facilitated by sustained electrical breakdown at its front, so the downward propagation characteristics of the dart-leader wave are primarily determined by the wave magnitude, which largely determines the front electric field, as opposed to being determined by the transmission-line parameters of the channel ahead of the front. Thus the dart leader may be best described as a downward moving ionizing front which generates current waves that propagate upward along the dart-leader channel. Finally, it can be argued that the subsequent return stroke can be viewed as a ground "reflection" of the dart leader.

1. Introduction

The lightning channel is often represented by an equivalent *R-L-C* transmission line [e.g., *Strawe*, 1979; *Gorin*, 1985]. In the *R-L-C* transmission line approximation, voltage V and current I are the solutions of the telegrapher's equations:

$$-\frac{\partial V(z,t)}{\partial z} = L\frac{\partial I(z,t)}{\partial t} + RI(z,t) \quad (1)$$

$$-\frac{\partial I(z,t)}{\partial z} = C\frac{\partial V(z,t)}{\partial t} \quad (2)$$

where R, L, and C are, respectively, the series resistance, series inductance, and shunt capacitance, all per unit length, z is the vertical coordinate specifying position on the lightning channel, and t is the time. The second of the telegrapher's equations is equivalent to the continuity equation [e.g., *Sadiku*, 1994]. Equations (1) and (2) can be derived from Maxwell's equations assuming that the electromagnetic waves propagating on (guided by) the line exhibit a quasi-transverse electromagnetic field structure and that R, L, and C are constant [e.g., *Agrawal et al.*, 1980]. Note that the term "quasi-transverse electromagnetic field structure" implies that the transverse component of the total electric field is much greater than the z-directed component associated with a nonzero value of R [*Paul*, 1994]. For lightning subsequent strokes, all three transmission-line parameters are modified from the pre-dart-leader values, first by the dart leader and then by the return stroke. As a result, each of those transmission line parameters is a function of time and space; that is, in both cases

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the transmission-line representing a lightning channel is nonlinear and nonuniform. The channel inductance changes due to variation in the radius of the channel core that carries the z-directed channel current. The channel resistance changes due to variation in ionization level (electron density), heavy-particle densities, and radius of the channel core. The channel capacitance changes mostly due to the formation (dart leader) and neutralization (return stroke) of the corona sheath that surrounds the channel core and presumably contains the bulk of the channel charge. For the case of a nonlinear transmission line, equations (1) and (2) are still valid if L and C are understood to be the dynamic (as opposed to static) inductance and capacitance, respectively [e.g., *Gorin*, 1985].

This paper presents estimates of various characteristics of the lightning channel available for the propagation of either dart-leader or return-stroke waves, an approach somewhat similar to that of *Borovsky* [1995], for a subsequent comparison of the model-predicted behavior of these waves with experimental data. It is assumed that L and C are equal to some average values and constant throughout (a simplifying approximation justified later), while R is assumed to be constant but different ahead of and behind either the downward moving dart-leader front or the upward moving return-stroke front. The assumption of a constant value of R for the bottom kilometer or so of the pre-return-stroke channel implies that the channel decay, leading to an increase of R, within some tens of microseconds (see section 4) of the preceding leader is negligible. Thus the transmission line is assumed to be linear ahead of and behind either dart-leader or return-stroke front with any nonlinear effects occurring at the wavefront, where the transition between the two channel sections with different fixed values of R takes place. Inferences will be made on the relative degree of nonlinearity at the dart-leader front and at the return-stroke front from a comparison of the model predictions with the observed properties of these two lightning processes, in particular with their luminosity profiles [*Jordan and Uman*, 1983; *Jordan et al.*, 1997], assuming that within the bottom kilometer or so of the channel the current and luminosity vary in a roughly similar manner.

The application of the R-L-C transmission line model to lightning is an approximation, even if R, L, and C were truly constant, because the lightning electric and magnetic fields, particularly at the wave fronts, do not strictly satisfy the quasi-transverse electromagnetic field structure required for the derivation of the telegrapher's equations. Nevertheless, R-L-C transmission-line models of the lightning return stroke are commonly used and apparently are capable of the reproduction of a number of salient properties of the return-stroke process [e.g., *Strawe*, 1979; *Gorin*, 1985].

2. Lightning Channel as an *R-L-C* Transmission Line

If one represents a straight, vertical lightning channel as a lossy linear transmission line characterized by constant series resistance R, series inductance L, and shunt capacitance C, all per unit length, the propagation constant of such a channel in the frequency domain is [e.g., *Sadiku*, 1994]

$$\gamma = \sqrt{j\omega C(R + j\omega L)} \tag{3}$$

where ω is the angular frequency; and $\omega = 2\pi f$ with f the frequency in hertz. The attenuation constant α (the real part of γ) and the phase constant β (the imaginary part of γ) of the R-L-C line are

$$\alpha = Re\{\gamma\} = \omega\sqrt{LC}\sqrt{\frac{\sqrt{1 + \left(\frac{R}{\omega L}\right)^2} - 1}{2}} \tag{4}$$

$$\beta = Im\{\gamma\} = \omega\sqrt{LC}\sqrt{\frac{\sqrt{1 + \left(\frac{R}{\omega L}\right)^2} + 1}{2}} \tag{5}$$

The group velocity characterizing the line is usually defined as $v_g = d\omega/d\beta$ [*Cheng*, 1993], and the phase velocity v_p is

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}\left[\frac{\sqrt{1 + (R/\omega L)^2} + 1}{2}\right]^{-1/2} \tag{6}$$

The attenuation distance, defined as the distance over which the wave amplitude decreases to e^{-1}, or to approximately 37% of its original value, is $\delta = 1/\alpha$ [*Sadiku*, 1994]. Note that *Borovsky* [1995, equation (36)] used a different definition of attenuation distance (also called by him the "axial damping distance"): the distance over which the wave amplitude decays to $e^{-2\pi}$, or to less than 0.2% of its original value. (Equation (36) of *Borovsky* [1995] actually gives the wavelength in a "good conductor" and probably was used by mistake.) The phase velocity is the z-directed velocity of points of equal phase on periodic waves. The value of v_p is necessarily less than the velocity of light in free space even for the case $R = 0$ because the radius of the channel, including the radially formed corona sheath, which presumably contains the bulk of the channel charge, is considerably larger than the radius of the channel core that carries the longitudinal channel current. The value of v_p is additionally, and to a much greater degree, reduced with respect to the velocity of light due to energy losses represented by the series resistance R. The group velocity is the velocity of a point on the envelope of the wave packet representing a narrow region of the frequency spectrum [*Cheng*, 1993]. Measured dart-leader or return-stroke velocity represents that portion of the wave which actuates a measuring device, perhaps near the maximum of power losses in the front, and will differ from either v_p or v_g but is expected to be of the same order of magnitude. As will be shown later, for our model of either the dart leader or the return stroke, v_p and v_g do not differ more than a factor of about 2, so within the accuracy of the arguments made here, either velocity can be used. Note that *Borovsky* [1995] defines a complex group velocity that can be computed as $\partial\omega/\partial\gamma$ and states that its real part, $v_g^* = Re\{[\partial\gamma/\partial\omega]^{-1}\}$, is to be associated with the "velocity at which a wave pattern moves along the channel." We will see that this latter velocity is close to v_p for lower frequencies and is always bounded by v_p and v_g. The characteristic impedance of an R-L-C transmission line is

$$Z_0 = \sqrt{\frac{R + j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}\sqrt{1 + \frac{R}{j\omega L}} \tag{7}$$

with its magnitude $|Z_0|$ and phase angle θ being found as

$$|Z_0| = \sqrt{\frac{L}{C}\sqrt[4]{1+(R/\omega L)^2}} \quad \textbf{(8a)}$$

$$\theta = \frac{1}{2}\arctan\left(-\frac{R}{\omega L}\right) \quad \textbf{(8b)}$$

If R is equal to zero (a lossless or so-called ideal transmission line), $\alpha=0$, $v_p = v_g = v_g^* = 1/\sqrt{LC}$, and $Z_0 = \sqrt{L/C}$. Of the three channel parameters R, L, and C, the largest variation during either dart-leader or return-stroke process is expected in R, orders of magnitude as opposed to a factor of 2 or less for L and C, as discussed in section 3. As a simplifying approximation, noted earlier, variations in L and C will be neglected, and it will be assumed that variations in δ, v_p, v_g, v_g^*, and Z_0 are predominantly functions of frequency and of R. For a constant value of R, it follows from equations (4), (5), (6), and (8a) that an increase of a factor of 2 in C, the maximum change expected for subsequent strokes (see section 3 below), will result in an increase in α and β of about 40% and a reduction of v_p and $|Z_0|$ by about 30%.

3. Characteristics of the Lightning Channel at Various Stages of the Lightning Discharge

Orville [1975] studied time-resolved (with 9-μs resolution) spectral emissions in the 390- to 510-nm region from a 13-m section of the lightning channel at some hundreds of meters above ground during five subsequent strokes in the same cloud-to-ground flash. He reported that the channel remained luminous between the time the dart-leader front crossed the field of view and the time of the ensuing return stroke. Further, for three dart leaders, *Orville* [1975], using the ratio of the intensity of singly ionized nitrogen (NII) emissions at two different wavelengths, estimated a typical "dart" temperature to be 20,000°K with an uncertainty of 10% and found that the channel retains this temperature until the return stroke occurs (for 20-30 μs), even though the light intensity initially decreases, implying a dart-leader radius decrease. The temperature of a lightning channel just before the passage of a dart leader could not be measured because channels were not sufficiently luminous at that time. The channel-cooling model of *Uman and Voshall* [1968] predicts that channels with radii of about 2 cm cool via thermal conduction to about 3000-6000°K during typical interstroke intervals of 10-100 ms. The study of *Uman and Voshall* [1968] was extended by *Picone et al.* [1981] to include convective energy loss, with essentially similar results. Since air makes the transition from a conductor to an insulator in the 2000 to 4000°K temperature range, and since essentially no current is detected at the channel base prior to the return stroke [e.g., *Fisher et al.*, 1993], a value near 3000°K seems to be a reasonable estimate of the temperature of the pre-dart-leader channel. Further, from spectroscopic measurements in a 10-m section of the lightning channel with 5-μs (eight strokes) and 2-μs (two strokes) resolution, the peak temperatures of return-stroke channels (no distinction is made between the first and the subsequent strokes) were reported by *Orville* [1968] to be of the order of 30,000°K. If the temperature varied more rapidly than could be resolved by the spectrometer, as may well be the case, these peak temperatures represent some sort of average over a few microseconds and hence are underestimates of the actual peak temperature [*Uman*, 1969]. A temperature rise has been detected in the first 10 μs for two (probably first) strokes recorded with 5-μs resolution, while two strokes recorded with 2-μs resolution and six strokes recorded with 5-μs resolution exhibited a monotonic decrease of temperature with time. The temperatures for all 10 strokes studied were below 30,000°K after 10 μs and were near or below 20,000°K after 20 μs. Because of the poorer time resolution for dart leaders as well as the different time resolution of the various measurements, possibly insufficient in each case, it is not clear if the temperature values reported by *Orville* for dart leaders and for return strokes can be meaningfully compared. Nevertheless, it appears that the dart leader heats the channel from about 3000°K to 20,000°K or higher after which it starts to cool but remains at 20,000°K or so for at least a few tens of microseconds, and the return stroke further heats it to 30,000°K or more. Calculation [*Yos*, 1963; *Plooster*, 1971] of the properties of heated air for temperatures expected ahead of and behind the dart-leader front indicates that channel electrical conductivity increases more than 5 orders of magnitude, from about 0.02 S/m to about 10^4 S/m, during the dart-leader process, whereas during the return-stroke process, it must remain more or less constant since the electrical conductivity of air at pressures ranging from 1 to 10 atm does not vary much with temperature for temperatures above 15,000°K or so. The conductivity of the post-dart-leader and the return-stroke channels of 10^4 S/m is consistent with an estimate of 1.8×10^4 S/m for a lightning channel temperature of 24,000°K reported by *Uman* [1964b] from similar calculations, as well as with values inferred from the long-spark experiments of about 4×10^4 S/m [*Norinder and Karsten*, 1952] and 1×10^4 to 3×10^4 S/m (*Gorin and Inkov* [1962] as presented by *Bazelyan et al.* [1978]). In order to estimate the resistance per unit length (which controls the attenuation distance δ, v_p, v_g, v_g^*, and Z_0 in our model) associated with various lightning processes, one needs, in addition to the electrical conductivities discussed above, estimates of the corresponding channel radii. Unfortunately, no experimental data are available on the pre-dart-leader and pre-return-stroke (i.e., dart-leader) channels. Measurements and theoretical estimates of the developed return-stroke channel radius usually yield values of the order of centimeters [e.g., *Uman*, 1987]. In particular, *Idone* [1992] reported that a direct measurement of the optical size of the channel, which was definitely not overexposed, yielded radii of 1-1.5 cm for six return strokes in two triggered lightning flashes.

In the following, an educated guess is made regarding the radius of the dart-leader channel. *Uman* [1964a] observed both millimeter-size and centimeter-size holes produced (melted) in fiberglass screen by lightning. The millimeter-size holes might be associated with upward unconnected leader-like discharges. Since these discharges are likely to propagate in virgin (cool) air, as opposed to dart leaders whose channels develop inside a column of warm (3000°K or so) air and, as a result, should expand to greater radii, their reported radii of 1 to 2.5 mm can be viewed as a lower limit to the radius of the dart-leader channel. If one assumes a uniform channel having a 2-mm radius r and an electrical conductivity σ of 10^4 S/m (consistent with a temperature in excess of 15,000°K), its resistance per unit length will be $R = (\pi r^2 \sigma)^{-1} = 8\ \Omega$/m. Further, the upper limit to the dart-leader channel radius (the lower limit to R) can be obtained from the consideration of the dart-leader channel as a stationary arc (*Orville* [1975] estimated that pressure equilibrium in the dart-leader channel is attained within 10 μs).

The longitudinal electric field intensity E in an arc channel carrying more than some tens of amperes current was observed to be about 1 kV/m [*King*, 1962]. One can estimate the dart-leader current by dividing the median value of charge transferred to ground in negative subsequent strokes (excluding continuing current), 0.95 C [*Berger et al.*, 1975], by the geometric mean duration of negative subsequent leaders that follow previously formed channel, 1.8 ms [*Rakov et al.*, 1994], with a resultant current I of approximately 500 A. Assuming an arc channel having an electrical conductivity σ of 10^4 S/m and a constant current density j, and using a point form of Ohm's law, $j = \sigma E$, one can estimate a channel radius $r = (I/\pi\sigma E)^{1/2}$ to be about 4 mm. The corresponding resistance per unit length is $R = E/I = 2\ \Omega$/m. On the basis of the above, the estimated limits to the dart-leader channel radius are 2 and 4 mm, and the corresponding values of R are 8 and 2 Ω/m. In the following, it is assumed that the radius is 3 mm, which leads to $R = 3.5\ \Omega$/m.

Further, if one assumes that the return-stroke channel is characterized by essentially the same electrical conductivity but by an order of magnitude greater radius than the leader channel, 3 cm (consistent with measurements of *Orville et al.* [1974]), the return-stroke resistance per unit length should be about 2 orders of magnitude lower, a value of roughly 0.035 Ω/m. Finally, if the return-stroke channel radius does not change too much during the cooling period (interstroke interval) [*Uman and Voshall*, 1968], the resistance per unit length for the pre-dart-leader channel should be about 18 kΩ/m, assuming $\sigma = 0.02$ S/m, corresponding to a temperature of 3000°K [*Yos*, 1963]. A smaller channel radius or a lower temperature at the end of the cooling period will result in a larger value of R. The value of R for the pre-dart-leader channel probably has little meaning since it implies a uniform, weakly conducting path between the cloud charge source and the ground, whereas at the end of the cooling interval, the lightning channel is likely to be highly nonuniform (fragmentation of the luminous channel is often observed as the channel decays [e.g., *Fisher et al.*, 1993]). Therefore, one should not attach too much significance to the value of $R = 18$ kΩ/m, perhaps viewing it just as very large compared to the values for the pre-return-stroke (dart-leader) and return-stroke channels, or as the lower limit to the actual value.

Characteristics of the lightning channel (except for L and C discussed next) associated with the various processes of the lightning discharge are summarized in Table 1. It is important to note that the values given should be viewed as order of magnitude estimates.

The inductance L and capacitance C, each per unit length, of the vertical lightning channel core were computed for a height $h = 500$ m above ground (approximately at the midpoint of the bottom kilometer or so of the channel being modeled here), using the following expressions [*Bazelyan et al.*, 1978]

$$C = \frac{2\pi\epsilon_0}{\ln(2h/r)} \tag{9}$$

$$L = \frac{\mu_0}{2\pi}\ln\frac{2h}{r} \tag{10}$$

where ϵ_0 and μ_0 are the permittivity and permeability of free space, respectively, and r is the radius of the channel core. In the following, it will be shown that L and C are unlikely to vary more than a factor of 2. For $r = 3$ cm and h varying from 10 m to 1 km (2 orders of magnitude), L and C change within about 60% with respect to the values for $h = 500$ m (even less for $r = 3$ mm). Further, for an equivalent subsequent-stroke channel radius of 2 m (channel core carrying the longitudinal current plus the expected corona sheath), $C = 8.9$ pF/m, about 2 times greater than for the 3-mm channel radius (4.4 pF/m) and less than 70% greater than the value for the 3-cm channel radius (5.3 pF/m). For the determination of L only the radius of the channel core r, which carries the longitudinal current, is needed. The values of L for $r = 3$ cm and $r = 3$ mm are 2.1 and 2.5 μH/m, respectively, a difference of less than 20%. In the following, it is assumed that $L = 2.3\ \mu$H/m and $C = 7$ pF/m. As seen in Tables 3 and 4, for very high frequencies, when $R \ll 2\pi fL$, Z_0, v_p, v_g, and v_g^* have values that are nearly equal to those for a lossless transmission line ($R = 0$): $Z_0 = \sqrt{L/C} = 0.57\ \mathrm{k\Omega}$, $v_p = v_g = v_g^* = 1/\sqrt{LC} = 2.5\times10^8\ \mathrm{m/s}$.

Table 1. Estimated Characteristics of Lightning Channel Associated With Various Processes of Lightning Discharge

Channel Characteristics	Pre-Dart-Leader Channel (Ahead of Dart-Leader Front)	Pre-Return-Stroke Channel (Behind Dart-Leader Front and Ahead of Return-Stroke Front)	Return-Stroke Channel (Behind Return-Stroke Front)
Temperature, °K	~ 3000	≥ 20,000	≥ 30,000
Conductivity, S/m	~0.02	~ 10^4	~ 10^4
Radius, cm	~ 3	~ 0.3	~ 3
R, Ω/m	~18,000	~ 3.5	~ 0.035

For comparison the electrical conductivity of carbon is 3 x 10^4 S/m, of seawater is 4 S/m, and of copper is 5.8x10^7 S/m [*Sadiku*, 1994]; the temperature of solar interior is 10^7 °K, of solar surface is 6000°K, and the temperatures at which tungsten and lead melt are 3600°K and 600°K, respectively [*Halliday and Resnick*, 1974].

The values of δ, however, depend on the value of R.

Characteristics of R-L-C transmission lines representing the pre-dart-leader, pre-return-stroke, and return-stroke channels calculated as a function of frequency and using values of R from Table 1 are given in Tables 2, 3, and 4, respectively. Note that we neglect the electromagnetic skin effect that at relatively high frequencies causes a nonuniform cross-sectional distribution of current and, as a result, an increase of R with frequency. This effect can potentially affect values of Z_0, v_p, v_g, v_g^*, and δ and is more pronounced for higher σ and smaller radius [e.g., *Sadiku*, 1994]. The skin effect is negligible when the electromagnetic skin (penetration) depth, which is the same as the attenuation distance in the inward radial direction, for the longitudinal electric field is much greater than the channel radius. For the pre-dart-leader channel (Table 2) the skin effect is of no concern because of the low value of σ = 0.02 S/m. For both the pre-return-stroke channel (Table 3) and return-stroke channel (Table 4), $\sigma = 10^4$ S/m. In the former case, as frequency increases, the skin depth becomes comparable to the 3-mm channel radius at 1 MHz or so, and in the latter case, it becomes comparable to the 3-cm channel radius at 10 kHz or so. As seen from Tables 3 and 4, at frequencies when the skin effect comes into play, $|Z_0|$, v_p, v_g, and v_g^* attain the values corresponding to a lossless transmission line, $R = 0$ (see above), that is, they become independent of R and of f. Since R increases due to skin effect as $\sqrt{f}$ [e.g., *Sadiku*, 1994], a further increase of f will not change the above characteristics with respect to the lossless-line values (because the condition $R(\sqrt{f}) << 2\pi f L$ will remain true). Values of δ at higher frequencies are influenced by the change of R due to the skin effect, but the difference does not alter any of the inferences made from Tables 3 and 4.

4. Observed Characteristics of Dart-Leader and Return-Stroke Traveling Luminosity Waves

Jordan et al. [1997], using high-speed photography, have examined in detail three dart-leader/return-stroke sequences from two Florida natural-lightning flashes for relative light intensity as a function of time and height (within the bottom 1.4 km). Dart-leader light waveforms appear as sharp pulses with 20 to 80% risetimes of about 0.5-1 μs and widths of 2-6 μs followed by a more or less constant light level (plateau). The plateau continues (for up to some tens of microseconds) until it is overridden by the return-stroke light waveform, suggesting that a steady leader current flows through any channel section behind the downward moving leader tip before, and perhaps for some time after, the return-stroke front has passed that channel section. Similar dart-leader luminosity versus time waveforms were also reported, from photoelectric measurements, by *Mach and Rust* [1997] for both natural and triggered lightning. Return-stroke light waveforms near ground in *Jordan et al.*'s [1997] data exhibit 20 to 80% risetimes of about 1-2 μs. There is a significant difference between return strokes and dart leaders in terms of the variation of luminosity along the channel: The 20 to 80% risetime of the return-stroke light pulses increases from 1.5 to 4.0 μs (mean values), and the pulse peak decays as the return-stroke front propagates from ground to the cloud base at about 1.4 km, whereas the risetime of the dart-leader light pulses is essentially constant with height, and the pulse peak is either more or less constant or increases as the leader front approaches ground. *Mach and Rust* [1997] also find no significant difference between dart leader optical characteristics at the top and bottom of the channel. In two out of three of *Jordan et al.*'s [1997] events, the return-stroke luminosity peak at 480 m and 1400 m decayed to, respectively, 70-75% and 25-30% of its value at the bottom of the channel and in the third event to, respectively, 90-95% and about 70% of the channel-bottom value. *Jordan and Uman* [1983] reported an exponential decrease of the return-stroke luminosity peak with height with a decay constant of 0.6-0.8 km, resulting in a luminosity peak decrease at a height of 480 m to 45-55% of its value at ground and to 9.7-17% at 1400 m. Also, *Jordan et al.* [1995] observed a decrease to 33% at 600 m and to 19% at 1100 m for one subsequent return stroke which they examined in relation to their luminosity versus height analysis of M components [*Rakov et al.*, 1995]. On the basis of the observed risetimes of the luminosity pulses produced by dart

Table 2. Characteristics of Pre-Dart-Leader Channel Having R = 18 kΩ/m, L = 2.3 μH/m, and C = 7 pF/m

f, kHz	Z_0, kΩ	v_p, m/s	v_g, m/s	v_g^*, m/s	δ, m
0.01	$6\times10^3\ \angle\text{-}45°$	3.2×10^4	6.3×10^4	3.2×10^4	503
0.1	$2\times10^3\ \angle\text{-}45°$	1.0×10^5	2.0×10^5	1.0×10^5	159
1	$640\ \angle\text{-}45°$	3.2×10^5	6.3×10^5	3.2×10^5	50
10	$202\ \angle\text{-}45°$	1.0×10^6	2.0×10^6	1.0×10^6	16
100	$64\ \angle\text{-}45°$	3.2×10^6	6.3×10^6	3.2×10^6	5
10^3	$20\ \angle\text{-}45°$	1.0×10^7	2.0×10^7	1.0×10^7	1.6
10^4	$6.4\ \angle\text{-}45°$	3.1×10^7	6.2×10^7	3.2×10^7	0.51

$Z_0 = \sqrt{L/C}\ \sqrt{1+R/(j\omega L)}$; $\quad v_p = \omega/\beta$; $\quad v_g = \partial\omega/\partial\beta$;

$v_g^* = Re\{[\partial\gamma/\partial\omega]^{-1}\}$; $\quad \delta = 1/\alpha$; $\quad \gamma = \alpha + j\beta$

Table 3. Characteristics of Pre-Return-Stroke Channel Having $R = 3.5\ \Omega$/m, $L = 2.3\ \mu$H/m, and $C = 7$ pF/m

f, kHz	Z_0, kΩ	v_p, m/s	v_g, m/s	v_g^*, m/s	δ, km
0.01	$89 \angle{-45°}$	2.3×10^6	4.5×10^6	2.3×10^6	36
0.1	$28 \angle{-45°}$	7.2×10^6	1.4×10^7	7.2×10^6	11
1	$8.9 \angle{-45°}$	2.3×10^7	4.5×10^7	2.3×10^7	3.6
10	$2.8 \angle{-44°}$	7.0×10^7	1.3×10^8	7.6×10^7	1.2
100	$0.93 \angle{-34°}$	1.9×10^8	2.7×10^8	2.5×10^8	0.44
10^3	$0.58 \angle{-6.8°}$	2.5×10^8	2.5×10^8	2.5×10^8	0.33
10^4	$0.57 \angle{-0.69°}$	2.5×10^8	2.5×10^8	2.5×10^8	0.33

leaders and return strokes and on their propagation speeds, *Jordan et al.* [1997] speculated that the average electric field intensity across the dart-leader front is at least an order of magnitude higher than that across the return-stroke front and that it should exceed the critical value for electron-impact ionization.

5. Analysis and Discussion

5.1. Return stroke

For the pre-return-stroke channel, the attenuation distance δ for frequencies of 100 kHz-1 MHz, expected in the return-stroke front near the ground, is some hundreds of meters (see Table 3) and increases for lower frequencies. As a result, the maximum of the frequency spectrum of the return-stroke wave should shift toward lower frequencies (loss of the higher-frequency content) as the wave propagates upward. The dominant frequency of the wave front decreases, with the lower frequencies being characterized by lower velocities (see Table 3). In the time domain these effects correspond to the degradation of the wave front with increasing height (the front duration becomes longer and the wave amplitude decreases) and probably to a reduction of the effective propagation speed of the wave, qualitatively consistent with optical observations [*Jordan and Uman*, 1983; *Jordan et al.*, 1997; *Idone and Orville*, 1982; *Mach and Rust*, 1989]. For a pre-return-stroke channel having $R = 3.5\ \Omega$/m, $L = 2.3\ \mu$H/m, $C = 7$ pF/m, and a typical return-stroke current wave injected at the channel base, a computer simulation using the ATP version of the Electromagnetic Transients Program (EMTP) [e.g., *Almeida and Correia de Barros*, 1994] shows that the current peak at 480 m and 1400 m is about 40-45% and about 20-25% of its value at ground, respectively. Within the accuracy of the arguments made here, the agreement between the amplitude decay of traveling waves on the *R-L-C* transmission line representing the pre-return-stroke channel and observed luminosity decay with height for the return stroke is fairly good. Thus it appears that the return stroke is largely similar to a "classical" (linear) traveling wave whose advancement can be visualized as being due to the progressive discharging of elemental shunt capacitors (previously charged by leader) of the equivalent *R-L-C* transmission line. It is worth noting that a realistic return-stroke model (see, for example, *Rakov* [1997] for a recent review) should take into account both attenuation and dispersion of the return-stroke wave, as per the discussion above.

Some confirmation of the lack of dependence of the return-stroke wave propagation characteristics on the wave magnitude, as expected for a "classical" traveling wave, comes from the observed absence of a significant correlation between the return-stroke propagation speed and the return-stroke peak current [*Willett et al.*, 1989; *Mach and Rust*, 1989]. It is worth noting that *Idone et al.* [1984], who analyzed in detail three New Mexico triggered-lightning flashes, did report "a nonlinear relationship" between the return-stroke propagation speed and the return-stroke peak current. However, this observation does not appear to be inconsistent with our inferences if other findings of *Idone et al.* [1984], namely, "a strong linear correlation" between the dart-leader propagation speed and the return-stroke peak current and "a nonlinear correlation" between the return-stroke propagation speed and the dart-leader propagation speed, are taken into account. The former correlation, confirmed by *Jordan et al.* [1992], is consistent with the treatment of the dart leader as an ionizing (nonlinear) wave, as discussed below. The latter correlation can be interpreted as indicative of the dependence (expected for the "classical" traveling wave) of the return-stroke speed on the channel cooling time since the preceding leader, this time largely determining the conditions along the channel in front of the return-stroke wave. Since the leader speed is essentially proportional to the return-stroke peak current, the relationship between the return-stroke and dart-leader speeds translates to the apparent relation between the return-stroke speed and the return-stroke peak current. Note that if one excludes the relatively small events that are characterized by return-stroke peak currents less than 6-7 kA from the sample of *Idone et al.* [1984], the relationship between the return-stroke speed and the return-stroke peak current disappears, as does the relationship between the return-stroke and the dart-leader speeds, bringing *Idone et al.*'s [1984] data in line with other published data on both triggered and natural lightning.

Ionization that does occur during the return-stroke process (predominantly due to a temperature rise caused by Joule heating) and serves to counteract attenuation apparently does not have a very marked effect on the wave propagation

Table 4. Characteristics of Return-Stroke Channel Having $R = 0.035$ Ω/m, $L = 2.3$ μH/m, and $C =$ 7 pF/m

f, kHz	Z_0, kΩ	v_p, m/s	v_g, m/s	v_g^*, m/s	δ, km
0.01	8.9 ∠-45°	2.3×10^7	4.5×10^7	2.3×10^7	361
0.1	2.8 ∠-44°	7.0×10^7	1.3×10^8	7.6×10^7	116
1	0.93 ∠-34°	1.9×10^8	2.7×10^8	2.5×10^8	44
10	0.58 ∠-6.8°	2.5×10^8	2.5×10^8	2.5×10^8	33
100	0.57 ∠-0.69°	2.5×10^8	2.5×10^8	2.5×10^8	33
10^3	0.57 ∠-0.069°	2.5×10^8	2.5×10^8	2.5×10^8	33
10^4	0.57 ∠-0.007°	2.5×10^8	2.5×10^8	2.5×10^8	33

characteristics which are primarily determined by the transmission line parameters ahead of the wave front. The characteristic impedance of the channel created by the dart leader, which is “seen” by frequency components between 100 kHz and 1 MHz of the return-stroke wave, is about 0.5-1 kΩ (see Table 3), comparable to the estimates reported by *Gorin et al.* [1977], 1-2 kΩ (21 out of 24 values), and *Gorin and Shkilev* [1984], 0.6-2.5 kΩ, from their measurements of lightning current at different points along a 530-m tower.

5.2 Dart leader

For frequencies of 100 kHz-1 MHz expected in the dart-leader wave front, the velocities are of the order of 10^6-10^7 m/s, consistent with observations [e.g., *Jordan et al.*, 1992], but the attenuation distance δ is only several meters or less (see Table 2). Even much lower frequencies in the range 100 Hz - 1 kHz would be severely attenuated after propagating only a few hundreds of meters on poorly conducting channels typically available for dart leaders, according to Table 2. Therefore the "classical" traveling wave mechanism of propagation does not appear to be possible for the dart leader. On the other hand, dart leaders do propagate on such channels and, furthermore, they do not exhibit any degradation of the light wave front as the leader approaches ground [*Jordan et al.*, 1997]. This result suggests that there must be significant ionization at the leader front, which counteracts (more successfully than in the case of return stroke) its degradation, a nonlinear effect perhaps equivalent to the dart-leader current's being generated at the leader front rather than being originated at the cloud charge source and traveling toward the ground. The generation of current at the dart-leader front also has been considered by *Bazelyan* [1995], *Cooray* [1996], and *Thottappillil et al.* [1997]. The downward progression of the dart leader is apparently facilitated by sustained breakdown at the leader front and therefore is controlled by the front electric field (inferred by *Jordan et al.* [1997] to be higher than the critical breakdown value and at least an order of magnitude higher than the electric field across the return-stroke front) as opposed to the transmission-line parameters of the channel ahead of the front. This inference is consistent with the observation [*Jordan et al.*, 1992] that (1) the dart-leader speed is positively correlated with the following return-stroke current peak, the latter inferred [*Rubinstein et al.*, 1995] to be positively correlated with the average leader charge density at the bottom of the channel, which largely determines the front electric field, and (2) the dart-leader speed appears to be relatively insensitive to the age of the channel (only the upper speed limit appears to be dependent on previous interstroke interval), which probably determines the equivalent transmission line parameters, particularly R, ahead of the dart-leader tip. In other words, the experimental data suggest that the dart-leader speed primarily depends on the magnitude of the dart-leader wave (in contrast with the case of return stroke discussed above), as opposed to the conditions along the transmission line representing pre-dart-leader channel. If indeed the dart leader is an ionizing wave whose propagation is primarily governed by the breakdown processes at its front, as opposed to the channel parameters ahead of the front, the representation by *Borovsky* [1995] of the dart leader as a guided "classical" electromagnetic wave and his conclusion that the "dart leader and the return stroke are caused by the same type of guided electromagnetic waves" are incorrect.

If the dart-leader current is viewed as generated at the downward moving leader front, this current should predominantly propagate upward, toward the cloud. The upward progression of the dart-leader current is controlled by the properties of the pre-return-stroke channel formed behind the dart-leader front (see Table 3). For that upward progression the phase velocity for the frequency components between 100 kHz and 1 MHz is about 200 m/μs (similar to measured return-stroke velocities) and the attenuation distance is of the order of some hundreds of meters (see Table 3), while the frequency components between 100 Hz and 1 kHz move at characteristic dart-leader velocities, apparently all the way to the cloud charge source. It is possible that the microsecond-scale dart-leader light pulse (see section 4 above and *Jordan et al.* [1997]) and the corresponding downward-moving luminous channel section, or dart, some tens of meters in length [e.g., *Jordan et al.*, 1997] are associated predominantly with the relatively short-lived upward-propagating higher-frequency components and that the light plateau following the dart-leader light pulse is primarily associated with the relatively long-lived lower-frequency components.

5.3. Relation Between Dart Leader and Return Stroke

The characteristic impedance of the channel behind the ionizing dart-leader front for frequencies of 100 kHz-1 MHz is about 0.5-1 kΩ (see Table 3), and some tens of kilo-ohms (see Table 2) ahead of the dart-leader front. Therefore when a contact between the dart-leader wave and ground having an impedance less than 50 Ω or so is established (details of the attachment process are not considered here), the dart-leader current wave must be "reflected" from ground as if the ground were a short circuit. This "reflection" is the return-stroke wave propagating upward along the post-dart-leader channel. Because the resistance of the channel encountered by the return-stroke wave is 3 to 4 orders of magnitude lower than that of the pre-dart-leader channel (see Table 1), the return-stroke wave is capable of propagation at a speed about an order of magnitude higher than the typically observed [e.g., *Jordan et al.,* 1992] dart-leader speed. Since both the dart-leader and the return-stroke processes operate on the same charge density per unit length (the former deposits, the latter removes), an order of magnitude higher speed should be associated with an order of magnitude higher current, as is apparently the case [e.g., *Idone and Orville*, 1985; *Uman*, 1987]. The light pulses due to dart leaders and return strokes in the luminosity profiles presented by *Jordan et al.* [1997] first clearly separate at heights of 100 to 300 m. Even at such relatively small heights, the light pulses produced by these two lightning processes exhibit somewhat different shapes, in particular, risetimes for the return-stroke pulses are about a factor of 2 greater (see section 4 above and *Jordan et al.* [1997]). If we assume that the return stroke can be viewed as a ground "reflection" of the dart leader, the latter observation suggests that appreciable attenuation of the higher-frequency components in the return-stroke wave occurs within the first few hundreds of meters, qualitatively consistent with the attenuation distances for frequencies of 100 kHz and higher given in Table 3. Additionally, the lightning ground-attachment process can influence the return-stroke current waveshape making its risetime longer than that of the dart-leader current waveshape.

6. Summary

The behavior of traveling waves representing dart leaders and return strokes on a linear *R-L-C* transmission line is compared with the observed properties of the waves of luminosity associated with those two processes [*Jordan and Uman*, 1983; *Jordan et al.*, 1997] and with other observed characteristics of dart leaders [*Jordan et al.*, 1992] and return strokes [*Willett et al.*, 1989; *Mach and Rust*, 1989]. It is inferred from this comparison that the return stroke is similar to a "classical" (linear) traveling wave that suffers appreciable attenuation and dispersion during its propagation along the channel, while the dart leader can be viewed as a downward moving ionizing front generating current waves that propagate in the upward direction. It appears that the subsequent return-stroke wave can be viewed as a ground "reflection" of the dart-leader wave.

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