LIGHTNING ELECTROMAGNETIC FIELDS: MODELING AND MEASUREMENTS

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Abstract: Modeling of lightning return strokes as sources of electromagnetic fields is reviewed. Validation of the models using measured fields due to natural and triggered lightning is discussed.

1. Introduction

All lightning processes, in both cloud and cloud-to-ground discharges, are associated with the motion of charges and, as a result, produce electromagnetic fields. Only one (presumably the most important from an EMC point of view) process, the return stroke in cloud-to-ground lightning, is considered in this review. There are basically four levels of sophistication in the mathematical modeling of the lightning return stroke. (1) The first, and most mathematically sophisticated level, typically involves the solution of three gas dynamic equations representing the conservation of mass, of momentum, and of energy, coupled to two equations of state, with the input parameter being channel current versus time, and the initial conditions imposed on the solution being the initial channel temperature, the initial channel radius, and the initial channel pressure or mass density. Such "physical" models are primarily concerned with the radial evolution of a short segment of lightning channel and its associated shock wave (primary model outputs include temperature, pressure, and mass density as a function of radial coordinate and time). (2) The second level of sophistication is represented by electromagnetic models that are based on a lossy, thin-wire antenna approximation to the lightning channel. These models involve a numerical solution of Maxwell's equations. (3) The third level of sophistication is represented by distributed-circuit models that describe the lightning discharge as a transient process on a vertical transmission line characterized by resistance (R), inductance (L), and capacitance (C) all per unit length. The line is usually assumed to be uniformly charged (by the preceding leader) to a specified potential and then closed at the ground end with a specified earth resistance to initiate the return stroke. There have been attempts to incorporate a "physical" model into a distributed-circuit model, the former being used to find R as a function of time. (4) The fourth level of sophistication is represented by "engineering" models that specify a spatial and temporal distribution of the channel current (or the channel charge density) based on such observed lightning return-stroke characteristics as current at the channel base, upward-propagating front speed, and luminosity profile.

2. Review of Models

The "physical" models do not address the physics of the current evolution along the channel and therefore cannot be directly used for electric and magnetic field calculations. Therefore these models are not further discussed here, unless they are part of a distributed-circuit model.

2.1 Lossy-antenna models

Return-stroke models based on a lossy, thin-wire antenna approximation have been proposed in [1, 2]. These models involve a numerical solution of Maxwell's equations using the method of moments (MOM), which yields the complete solution for current that includes both the antenna-mode current and the transmission-line-mode current [3]. In [1], the return-stroke front propagation speed is not specified but apparently it was equal to the speed of light. In [2], the permittivity of air surrounding the current-carrying channel was modified for the computation of the current distribution along the channel in order to force the front propagation speed to be less than the speed of light. The unmodified permittivity was used for the computation of electric fields due to that current distribution.

2.2. Distributed-circuit models

The derivation of any R-L-C transmission line model from Maxwell's equations requires that the electric field intensity and the magnetic field intensity satisfy the transverse electromagnetic (TEM) field structure, that is, they lie in a plane transverse or perpendicular to the axis of the line [3]. For a vertical lightning channel with the current "return path" being the channel image (assuming a perfectly conducting ground) the validity of the TEM assumption is questionable, in particular near the return-stroke tip where a relatively large longitudinal component of electric field is present. Usually, an R-L-C model of the lightning return stroke is postulated without any analysis of its applicability.

In R-L-C models, the channel voltage and channel current are found from solutions of the telegrapher's equations. An exact closed form solution of the telegrapher's equations can be obtained only in the case of the R, L, and C all being constants. Such an approximation was used, for instance, in [4-7]. For lightning, R, L, and C each is expected to be a function of time and space (nonlinear and nonuniform transmission line), and the solution of the appropriate telegrapher's equations (with L and C being dynamic as opposed to static inductance and capacitance, respectively) requires the use of a numerical technique, for instance, a finite-difference method. Attempts to take into account the lightning channel nonlinearities using various simplifying assumption have been made, for instance, in [8-14].

Combinations of an R-L-C model with a "physical" model, the latter one being used to determine R while L and C are kept constant, are presented in [15,16].

2.3. "Engineering" models

An "engineering" return-stroke model is defined in this review as an equation relating the longitudinal channel current at any height z' and any time t to the current at the channel origin. An equivalent expression in terms of the line charge density on the channel can be obtained using the continuity equation [17]. The most used "engineering" models can be grouped in two categories: the transmission-line-type models and traveling-current-source-type models. The former include the transmission line (TL) model [18] and its two modifications: the modified transmission line model with linear current decay with height (MTLL) [19] and the modified transmission line model with exponential current decay with height (MTLE) [20]. The transmission-line-type models incorporate a current source at the channel base which injects a specified current wave into the channel, that wave propagating upward (1) without either distortion or attenuation (TL), or (2) without distortion but with specified attenuation (MTLL and MTLE).

The traveling-current-source-type models include the original traveling current source (TCS) model introduced in [21] and the Diendorfer-Uman (DU) model [22]. In the traveling-current-source-type models it is assumed that the return-stroke current is generated at the upward-moving return-stroke front and then propagates downward. The so-called Bruce-Golde (BG) model [23] is conveniently included in this category. In the TCS model, current at the front rises instantaneously and propagates downward at the speed of light. The TCS model reduces to the BG model if the downward current propagation speed is assumed equal to infinity. In the DU model, current at the front rises exponentially (actually two current components are considered in [22], each turning on with its own time constant, in order to match model-predicted fields with measured fields) and propagates downward at the speed of light.

Fig. 1 illustrates the relation between the three simplest and most used "engineering" models, namely the BG, TL, and TCS models. Current waveforms versus time at ground level ($z' = 0$) and at heights z'_1 and z'_2, are given for each of the three models. The channel current at any given height z' is specified as the product of the Heaviside function, u, and current at the channel base (at $z' = 0$), the current being appropriately shifted in time (zero shift for the BG model). The second multiplier is a continuous function of time that can exist (see Fig. 1) at a given height $z' > 0$ both before (blank portion of the current waveforms for the TCS and BG models) and after (dark portion of the waveforms) the return-stroke front arrives there, and the Heaviside function assures that current is turned on by the upward-moving front. It follows from Fig. 1 that if the channel-base current were a step function the TCS, BG, and TL models would be indistinguishable from each other.

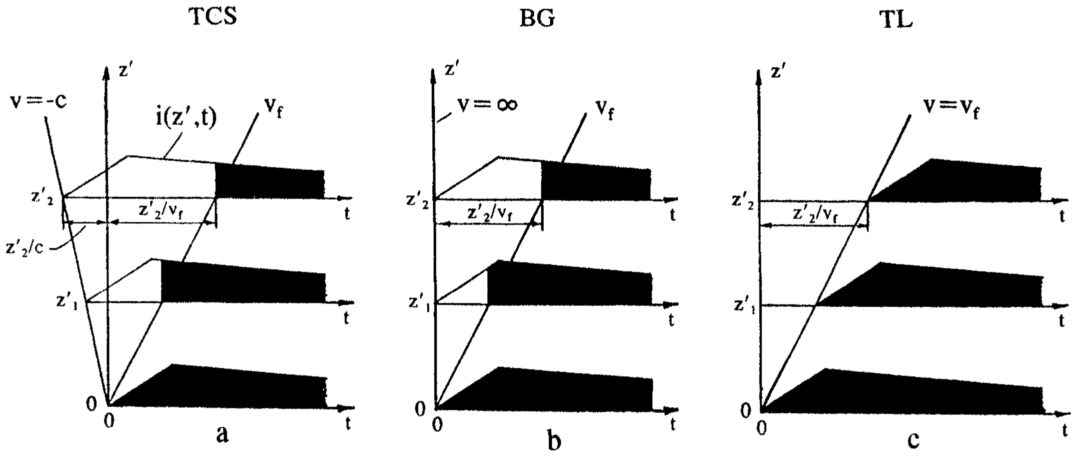
Let us first consider the TCS model (Fig. 1a) in which current at z' is equal to the current at ground a time z'/c later. This is equivalent to shifting the ground-level waveform by z'/c in the negative time direction. For instance, at z_2' the shift is z_2'/c. Further, the return-stroke front arrives at z_2' at time z_2'/v_f where v_f is the front propagation speed. Current is turned on at this instant, and the dark portion of the waveform indicates current that actually flows through the channel section at z_2' thereafter. The blank portion of the waveform (for $t < z'_2/v_f$) is shown for illustrative purpose only. As seen in Fig. 1a, the current turn-on time at each height is determined by the intersection of the straight line (labeled v_f), whose slope with respect to time axis is equal to the front propagation speed v_f, with the time axis at that height. The beginning points of the complete current waveforms at different heights lie on the line (labeled v) whose slope with respect to time axis is equal to the current-wave propagation speed, v, which for the TCS model is the negative of the speed of light.

Next we will show with reference to Fig. 1 how the TCS model can be transformed to the BG model and further to the TL model by simply changing the position of the v line.

If we rotate the v line, with all the current waveforms still beginning on it, clockwise with respect to the origin of coordinates until the v line coincides with the vertical axis, we obtain the BG model (Fig. 1b). The vertical position of the v line corresponds to v equal to infinity.

If we further rotate the v line clockwise until it coincides with the v_f line, we obtain the TL model (Fig. 1c), in which both the return-stroke front and the return-stroke current wave propagate in the same direction and at the same speed. The relation between the TL and TCS models is further illustrated in Fig. 2 which shows that the front of the spatial current wave moves in the positive z' direction for the TL model and in the negative z' direction for the TCS model.

All the return-stroke models introduced above except for the DU model can be expressed by the generalized equation given in Table 1. In this equation, $P(z')$ is the height-dependent current attenuation factor introduced in [24], u is the Heaviside function equal to unity for $t \geq z'/v_f$ and zero otherwise, v_f is the front propagation speed, and v is the current-wave propagation speed. Table 1 also summarizes $P(z')$ and v for the five "engineering" models. In the Table, H is the total

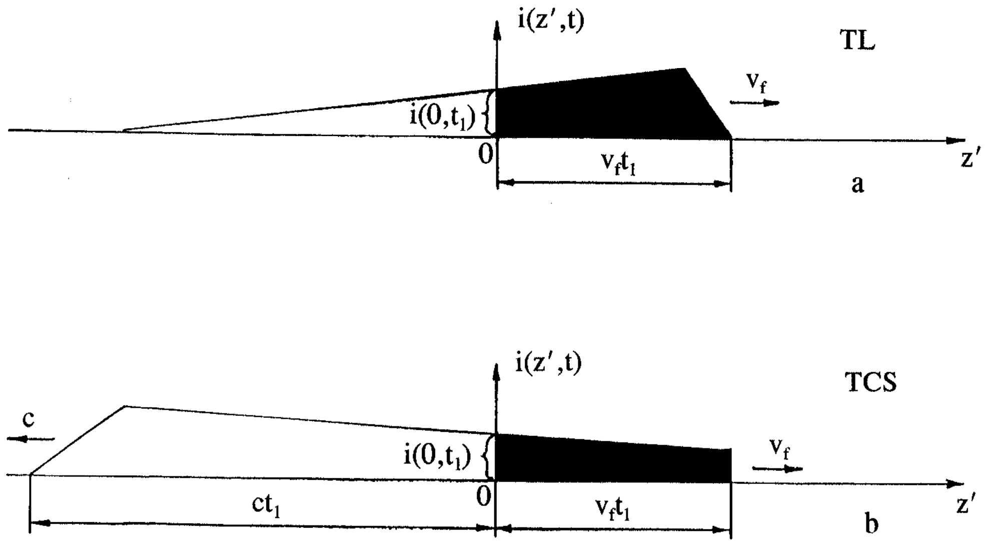


$i(z',t)=u(t-z'/v_f)i(0,t+z'/c)$ $\quad$ $i(z',t)=u(t-z'/v_f)i(0,t)$ $\quad$ $i(z',t)=u(t-z'/v_f)i(0,t-z'/v_f)$

$$u\left(t-\frac{z'}{v_f}\right)=\begin{cases}0 \text{ if } t<z'/v_f\\ 1 \text{ if } t\geq z'/v_f\end{cases}$$

v_f = front speed
v = current wave speed
c = speed of light

Fig. 1: Comparison of the (a) TCS, (b) BG, and (c) TL return-stroke models. Current versus time waveforms at ground (z'=0) and at two heights z_1' and z_2' above ground.



$v_f=c/2$ $\qquad$ $t=t_1$

Fig. 2: Comparison of the (a) TL and (b) TCS return-stroke models. Current versus height waveforms at fixed instant of time $t = t_1$.

channel length and λ is the current decay constant (assumed in [20] to be 2000 m).

Table 1: Generalized equation for five "engineering" return-stroke models.

$$i(z',t) = u\left(t-\frac{z'}{v_f}\right) P(z')\ i\left(0,t-\frac{z'}{v}\right)$$

Model	P(z')	v
TL (Uman and McLain, 1969)	1	v_f
MTLL (Rakov and Dulzon, 1987)	1 - z'/H	v_f
MTLE (Nucci et al., 1988)	exp(-z'/λ)	v_f
BG (Bruce and Golde, 1941)	1	∞
TCS (Heidler, 1985)	1	-c

A characteristic feature of the "engineering" models is the small number of adjustable parameters, usually one or two besides the channel-base current. In these models, the physics of the lightning return stroke is deliberately downplayed, and the emphasis is put on achieving an agreement between model-predicted and observed lightning electromagnetic fields.

3. Validation of Return-Stroke Models Using Measured Electric and Magnetic Fields

3.1. Lossy-antenna models

No model predicted fields are given in [1]. If, as discussed in section 2.1, the return-stroke front propagation speed is assumed in [1] to be equal to the speed of light, the model- predicted fields are unlikely to be consistent with measurements. Fairly good agreement between the model-predicted and typical measured fields at distances ranging from tens of meters to tens of kilometers has been demonstrated in [2]. However, the model in [2] predicts no zero-crossing at some tens of microseconds for electric fields at 100 km, inconsistent with the published experimental data (e.g., [25]).

3.2. Distributed-circuit models

Electromagnetic fields calculated in [13] and [16] using nonlinear R-L-C models are inconsistent with typical measured fields (e.g., [25]). Other authors do not present model-predicted electromagnetic fields leaving their models unverified by the most readily available experimental data.

3.3. "Engineering" models

Two primary approaches to model validation have been used: The first approach involves the use of a typical channel-base current waveform and a typical average propagation speed as inputs to the model, and a comparison of the model-predicted electromagnetic fields with typically observed fields. In the second approach, the channel-base current waveform and propagation speed, both measured for the same individual event, are used to compute fields that are compared to the measured fields for that same event. The second approach appears to be able to provide a more definitive answer regarding model validity, but it is feasible only in the case of triggered-lightning return strokes or natural lightning strikes to tall towers where channel-base current can be measured. In the field calculations, the channel is assumed to be straight and vertical with its origin at ground (z' = 0), conditions expected to be valid for subsequent strokes, but potentially not for first strokes. The first approach has been adopted in [26], [24], and [17], and the second one in [27]. The results of model validation can be summarized as follows.

(a) The relation between the initial (predominantly radiation) field peak and the initial current peak is reasonably well predicted by the TL, MTLL, MTLE, and DU models. The TL model can be recommended for engineering use in predicting peak currents from measured peak fields as the mathematically simplest one.

(b) Electric (essentially electrostatic) fields at tens of meters from the channel after the first 10-15 μs are reasonably reproduced by the BG, MTLL, TCS and DU models, but not by the TL and MTLE models. Additionally, the MTLE model is inconsistent with the observed net ratio of leader-to-return-stroke electric field change at far ranges: The measured ratio is near unity (in support of the BG, MTLL, TCS, and DU models), whereas the MTLE model predicts a value near 3.

(c) Based on the overall field waveforms at 5 km (the only distance at which the "individual-return-stroke" model validation approach has been used) none of the considered models can be preferred.

4. Discussion

In this section we briefly discuss a few important aspects of the return-stroke modeling that are either ignored to keep the models easy-to-use or simply not recognized.

4.1. Treatment of the upper, in-cloud part of the channel

It is the common view that subsequent return strokes are easier to model than the first strokes. First strokes are commonly branched, may involve an upward connecting discharge from ground of appreciable length, and typically exhibit a significant variation of propagation speed along the channel. This view is correct as long as the lightning channel is predominantly vertical, a condition that is less likely to be satisfied for subsequent strokes than for first strokes when the return stroke reaches cloud charge height, typically after 25-75 μs assuming that the return-stroke front propagation speed in the cloud is approximately the same as that below the cloud base. Subsequent strokes are expected to follow predominantly horizontal paths in the cloud. Additionally, all the "engineering" models except for the MTLL model do not specify boundary conditions at the channel top. In general, a reflection should be produced when the return-stroke front arrives at the channel top. Various boundary conditions at the channel top have been used in the distributed-circuit models including an open circuit [15, 16], a capacitor or an L-C transmission line [7], and an R-C network [12,13].

4.2. Boundary conditions at ground

In the transmission-line-type "engineering" models, the boundary conditions at ground are determined by the specified channel-base current (current source at the channel bottom). In the TCS and DU models, those models which assume that the return-stroke current is generated at the upward-moving front and propagates toward ground, it is implied that the channel is terminated at ground in its characteristic impedance so that the current reflection coefficient at ground is equal to zero. This implication is invalid for the case of a lightning strike to a well-grounded object where an appreciable reflection from ground is expected. In the distributed-circuit models, the boundary conditions at ground are specified explicitly, with a terminating resistor simulating earth resistance in the range of tens to hundreds of ohms being typical.

4.3. Return-stroke front speed at early times

It is argued in [11] that at the instant of return-stroke initiation the geometry of the bottom some tens of meters of the leader channel is an inverted circular cone because the corona has not had enough time for its full development. Propagation speeds of radial corona streamers from conductors subjected to negative high voltage in laboratory were reported to be about 10^5 m/s (0.1 m/μs) [28], so that some microseconds are required for the development of corona sheath with a radius of the order of meters. For stepped leaders, the downward propagation speed is also of the order of 10^5 m/s, so that there is a relatively short delay in the corona-sheath formation as a stepped leader moves toward ground. For dart leaders, the downward propagation speeds (10^7 m/s) are about two order of magnitude higher than the radial-streamer speeds, so that the delay may be appreciable. The conical model of the bottom part of the channel predicts an initial return-stroke speed of nearly c, the speed of light, because both the longitudinal channel current and channel charge near ground are confined in a volume of approximately the same radial dimension. The speed is predicted in [11] to decrease when the return-stroke front reaches the height (of the order of tens of meters) of the fully developed corona sheath when the channel geometry is cylindrical, and the radii of the current-carrying channel core and the charge-containing corona sheath appreciably differ from each other. It is claimed in [11] that support of the conical-model prediction regarding the initial return-stroke speed being nearly c comes from the results presented in [29] where (a) measured peak time-derivatives of the channel-base current, (b) measured peak time-derivatives of the electric field at 50 m and (c) the relation between the peak time-derivatives of the current and radiation electric field based on the TL model are used to estimate the return-stroke speed. The estimated return-stroke speed is reported in [29] to be on average near c, with 14 out of 40 values being greater than the speed of light. On the other hand, similar estimates of speed but using peak electric field derivatives measured at about 5 km give mean value of approximately two-thirds of c [30]. Further, the use of (a) measured channel-base current peak, (b) measured electric field peak at about 5 km, and (c) the relation between the peak current and peak radiation electric field based on the TL model leads to a mean return-stroke speed of about one-half of c, consistent with corresponding optical speed measurements over the bottom 400-600 m of the channel [30]. It is possible that, since the peak derivative precedes the peak of electric field or current, speed estimates using the peak time- derivatives of electric field and current, are representative of somewhat lower channel section than those based on peak electric field and peak current, this conjecture implying a very rapid speed decay within the bottom 100 m or so. One possible explanation [29, 31] for the discrepancy between the speeds inferred using the 50-m and 5-km field derivative data is the contribution to the electric field derivative peak from the induction and electrostatic field components at 50 m, this contribution being not accounted for in the field derivative/current derivative relation derived for radiation field component only. The discrepancy between the three return-stroke speed estimates, near c, 2c/3, and c/2, discussed above, remains a subject of controversy.

A different view of the initial return-stroke speed is suggested in [9]. According to the distributed-circuit model in [9], the speed initially increases to its maximum (appreciably less than c). The initial speed increase in [9] is associated with the so-called break-through (or switch-closing) phase thought to be responsible for the formation of the initial rising portion of the return-stroke current pulse (see also [24] and [32]). In [33], based on the experimental data published in [34], a bi-exponential expression for the return-stroke speed as a function of time is proposed, according to which the speed rises from zero to its peak and falls off afterwards. More experimental data on the attachment process and on the early stages of the return-stroke process are needed to deduce the typical return-stroke speed profile near ground.

5. Concluding Remarks

The continuing interest in lightning modeling has been motivated by the desire to have relatively straightforward techniques (a) for deriving lightning current parameters from remotely measured electromagnetic fields (the so-called inverse source problem) and (b) for predicting the coupling and resultant effects of lightning fields on airborne vehicles and ground based objects and systems. The most recent application of lightning models is associated with studying the interaction of lightning with the mesosphere and the lower ionosphere (red sprites, blue jets, elves). Whatever the application, the lightning model is a crucial element that usually involves more uncertainties than any other element of that application. There has been significant progress lately in developing lightning models. However further studies are needed to validate the existing models by measuring fields at different distances from the lightning channel and to extend the modeling to lightning processes other than the return stroke in cloud-to-ground discharge.

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