COMMENTS ON THE SIGNIFICANCE OF RETARDATION EFFECTS IN CALCULATING THE RADIATED ELECTROMAGNETIC FIELDS FROM AN EXTENDING DISCHARGE

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Abstract: A cloud-to-ground lightning return-stroke channel is usually modeled as an extending line with one end fixed at ground. Return-stroke models describe how the current is varying with time along the line. The expressions for electric and magnetic fields due to retarded currents on a stationary straight wire antenna of fixed length have been derived in time domain previously (e.g., [1]). These general integral expressions can be used in lightning field calculations with suitable modifications to take into account retarded channel length and possible current discontinuity at the discharge front. The radiation fields far from the channel are generally expressed in terms of the time derivative of the retarded channel current. For the Transmission Line (TL) model which is characterized by a current wave moving vertically without distortion or attenuation at the same speed and in the same direction as the discharge front, the radiation fields observed at ground level very far from the channel are directly proportional to the channel current multiplied by the propagation speed v, as shown, for instance, by [2]. However, as demonstrated in [3] and [4], for the case of a channel that is arbitrarily oriented with respect to the observer's line of sight the field/current expressions noted above are in need of correction. A similar correction was also found to be needed in the field/current expressions for a travelling current discontinuity [5]. Both corrections are due to retardation effects and appear as a multiplier $[1-(v/c)\cos\theta]^{-1}$, where c is the speed of light, and θ is the angle between the direction of propagation of the current and the line joining the source and the field point, to the original equations. In this paper we (1) discuss the various situations in which the above factor can arise, (2) give physical interpretation of this factor, and (3) show that the retardation effects in calculating lightning radiation fields can be accounted for without the explicit use of the above factor.

1. Introduction

A lightning return stroke is modelled as a vertically extending electrical discharge from ground. Assume that the front of the discharge is moving with a speed v, which is usually of the order of one-third the speed of light. Ahead of the discharge front (or tip) the current is zero and behind the discharge front the current is changing with time and is different at various positions (heights) along the channel. Various return-stroke models describe how this current is varying and how the current at a given height z' is related to the current at ground z'=0 at any given instant t. To find the electric and magnetic fields at a distance due to the current distribution on the channel, first take a differential length of the channel dz' at a height z' as shown in Fig. 1. The element dz' is stationary even though the current in it is changing with time. The vector potential from this time varying current in dz' can be written as

$$d\bar{A} = \frac{\mu_0}{4\pi}\frac{i(z',t-R(z')/c)}{R(z')}dz'\hat{z} \qquad (1)$$

Note that in the above equation for the vector potential we used retarded currents, however we do not need any 'correction factor' $[1-(v/c)\cos\theta]^{-1}$, similar to that factor appearing in the Leonard-Wiechert potential of a <u>moving</u> charge, because we are finding the potentials from the time varying current in a <u>stationary</u> element of length dz' at a height z'. The discharge front is extending at a relativistic speed and hence the observer "sees" the retarded length L'(t), of the discharge at time t. The retarded length is derived in the next section. From the differential vector potential given by (1) we can find the scalar potential and then the electric fields and magnetic fields as outlined below (see [1]).

$$d\phi = -c^2\int_0^t \nabla.d\bar{A}d\tau \qquad (2)$$

$$d\bar{E} = -\nabla d\phi - \frac{\partial}{\partial t}d\bar{A} \qquad (3)$$

$$d\bar{B} = \nabla \times d\bar{A} \qquad (4)$$

$$\bar{E}(r,t) = \int_0^{L'(t)} d\bar{E} \qquad (5)$$

$$\bar{B}(r,t) = \int_0^{L'(t)} d\bar{B} \qquad (6)$$

Equations (5) and (6) in spherical co-ordinates can be written, following [1], as

$$\bar{E}(r,\alpha,t) = \frac{1}{4\pi\varepsilon_0}\int_0^{L'(t)}\left\{\cos(\theta)\times\left[\frac{2}{R^3(z')}\int_{\frac{z'}{v}+\frac{R(z')}{c}}^{t} i\left(z',\tau-\frac{R(z')}{c}\right)d\tau + \frac{2}{cR^2(z')}i\left(z',t-\frac{R(z')}{c}\right)\right]\hat{R} + \sin(\theta)\times\left[\frac{1}{R^3(z')}\int_{\frac{z'}{v}+\frac{R(z')}{c}}^{t} i\left(z',\tau-\frac{R(z')}{c}\right)d\tau + \frac{1}{cR^2(z')}i\left(z',t-\frac{R(z')}{c}\right) + \frac{1}{c^2R(z')}\frac{\partial i(z',t-R(z')/c)}{\partial t}\right]\hat{\theta}\right\}dz' \tag{7}$$

$$\bar{B}(r,\alpha,t) = \frac{\mu_0}{4\pi}\int_0^{L'(t)}\sin(\theta)\times\left[\frac{1}{R^2(z')}i\left(z',t-\frac{R(z')}{c}\right) + \frac{1}{cR(z')}\frac{\partial i(z',t-R(z')/c)}{\partial t}\right]\hat{\phi}\,dz' \tag{8}$$

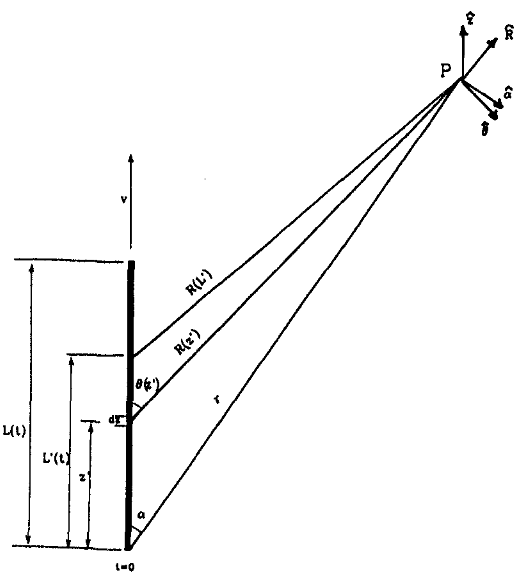

Fig.1 Geometry of the problem

Instead of the differential length dz', one could consider the whole retarded length L'(t) and find the vector potential **A(r,t)** as

$$\bar{A}(r,t) = \frac{\mu_0}{4\pi}\int_0^{L'(t)}\frac{i(z',t-R(z')/c)}{R(z')}dz'\hat{z} \tag{9}$$

and then proceed to find the fields. This method will give expressions identical to those given by (7) and (8). Equations (7) and (8) are exact for any return-stroke speed, for any orientation of channel with respect to the observer, and do not contain any explicit correction factor. Far from the channel, the radiation fields are dominant and are given by the current derivative terms in (7) and (8). For example, the radiation electric field component is given by

$$\bar{E}_r(r,\theta,t) = \frac{1}{4\pi\varepsilon_0}\int_0^{L'(t)}\frac{\sin(\theta)}{c^2R(z')}\frac{\partial i(z',t-R(z')/c)}{\partial t}dz'\hat{\theta} \tag{10a}$$

For some return-stroke models and at very large distances from the lightning channel, simplified field equations for radiation field (without the integral) can be derived from (10a). For the transmission line (TL) model, the simplified equation is [2]

$$\bar{E}_r(D,t) = -\frac{v}{4\pi\varepsilon_0 c^2 D}i\left(0,t-\frac{D}{c}\right)\hat{z} \tag{10b}$$

where D is the horizontal distance between the observation point and the channel base, and $\hat{z}$ is a vertically directed unit vector (equation (10b) do not take into account the effect of ground). It is important to note that equation (10b) has been derived for the channel essentially perpendicular to the observer's line of sight. If the channel is not perpendicular to the observer's line of sight (for instance, an elevated observation point), equation (10b) should be multiplied by a geometrical factor $\sin\theta/[1-(v/c)\cos\theta]$ where θ is the angle between the direction of propagation and the line connecting the channel segment of interest and the observation point. In [2] an incomplete expression with sinθ for the geometrical factor is used, as pointed out in [3] and [5]. Part of the geometrical factor, $[1-(v/c)\cos\theta]^{-1}$, missing in [2] is sometimes called a correction factor or F factor. Some confusion regarding the use of F factor can also be found in literature. Some authors [4] incorrectly interpreted the results of [3] and [5] as suggesting that all previously published expressions for lightning electromagnetic fields are incorrect if v is a significant fraction of the speed of light. Some others [6] erroneously claimed that the general expressions (7) and (8) require corrections involving F factor and used the "corrected" general expressions to calculate the fields. In the following sections we will show how the correction factors suggested by [3-5] arise from the general expression (10a) for radiation electric field under various approximations.

2. Analysis

2.1 Retarded channel length to be used in calculating radiated fields

For a lightning return stroke modeled as a current-carrying line extending vertically upward at constant speed v with its lower end fixed at ground, the physical length of the discharge channel at a given time t, where t=0 is the discharge start time, is the product vt. If r is the distance from the channel bottom at ground to an observation point P (Fig. 1) and c is the speed of light, the channel bottom is "seen" by the observer at t=r/c. Hence no information on the channel length is available at the observation point for t<r/c. For the channel-observer geometry shown in Fig. 1, the length (or height) of the channel "seen" by the observer at time t, L'(t), is different from vt because of the finite propagation times of the electromagnetic signals from the end points of the channel to the observer. The height L'(t) can be found as follows. If we define the time t such that it is the sum of the time required by the return stroke wavefront to reach a height L'(t) and the time required for a signal to travel from the wavefront at L'(t) to the observer at P, t can be written as

$$t = \frac{L'(t)}{v} + \frac{R(L'(t))}{c} \tag{11a}$$

and from Fig. 1,

$$R(L'(t)) = \sqrt{r^2 + L'^2(t) - 2L'(t)r\cos(\alpha)} \qquad (11b)$$

Note that when L'=0, R=r from (11b) and t=r/c from (11a). Equation (11) is a second degree equation in L'(t) which can be solved to obtain

$$L'(t) = \left(\frac{1}{v^2} - \frac{1}{c^2}\right)^{-1} \times \left[-\frac{r\cos(\alpha)}{c^2} + \frac{t}{v} - \frac{1}{c}\sqrt{r^2\left(\frac{1}{v^2}-\frac{1}{c^2}\right) + t^2 + r\cos(\alpha)\left(\frac{r\cos(\alpha)}{c^2} - \frac{2t}{v}\right)} \right] \qquad (12)$$

If the ground is treated as perfectly conducting, (12) can also be used, with α replaced by 180°-α, to find the apparent length of the image channel "seen" by the observer. Even if the velocity of the discharge front is varying with height, (11) is valid if v is replaced by an "average" velocity and can be solved for L'(t) (see [7,8] for details).

2.2 Approximation to the retarded length

If the channel length is very small compared to the distance to the observer, i.e., if L'<<r, then R(L'(t)) can be approximated as (see Fig. 1)

$$R(L'(t)) = r - L'(t)\cos\alpha \qquad (13)$$

$$L'(t) = \frac{v.(t - r/c)}{1 - \frac{v}{c}\cos\alpha} = v.F.(t - r/c) \qquad (14)$$

where v.(t-r/c) is the actual length of the discharge at time t-r/c. Now we have seen how the factor, $F=[1-(v/c)\cos\alpha]^{-1}$ can appear in the approximation to retarded length when the observer is far away. Let us go on to consider other situations in which the correction factor, or F factor, appears.

2.3 Transmission line model

In the Transmission Line (TL) model of the return stroke a current wave originate at ground and propagate up the channel without attenuation and distortion (e.g., [2,3]). The retarded current distribution along the channel can be expressed in terms of the current at the base of the channel as

$$i(z', t - \frac{R(z')}{c}) = i(0, t - \frac{z'}{v} - \frac{R(z')}{c}) \qquad (15)$$

From Fig. 1, R(z') can be written as,

$$R(z') = \sqrt{z'^2 + r^2 - 2z'r\cos(\alpha)} \qquad (16)$$

which is same as (11b) except it is for any z', not only for z'=L'(t). For a given model and a given channel base current, the expression for radiation electric field (10) can be evaluated numerically. For each value of t, equation (12) has to be solved to find L'(t) and the integration in (10) has to be carried out through the upper limit L'(t). As noted in the previous subsection, when the electric radiation field is calculated as indicated above, there is no need for any explicit use of correction factor, F. However, calculations using (10) are time consuming and do not provide an analytical relation (without the integral) between current and remote electric fields, as is sometimes desirable. Next we will show how, for the case of the electric radiation field far from the channel, (10) can be approximated by a simpler expression, without the integral but involving an F or correction factor.

The variables t and z' are independent since z' is arbitrarily chosen, and thus we can write

$$\frac{\partial i(0,t - z'/v - R(z')/c)}{\partial z'} = \frac{\partial i(0,t - z'/v - R(z')/c)}{\partial(t - z'/v - R(z')/c)} \cdot \frac{\partial(t - z'/v - R(z')/c)}{\partial z'}$$
$$= -\frac{\partial i(0,t - z'/v - R(z')/c)}{\partial t} \cdot \frac{1}{v} \cdot \left(1 + \frac{v}{c}\frac{z' - r\cos\alpha}{\sqrt{z'^2 + r^2 - 2z'r\cos\alpha}}\right) \qquad (17)$$

Rearranging (17), we obtain the relationship between the time and spatial derivatives of the retarded current, involving the F factor, as dictated by the TL model

$$\frac{\partial i(0, t - \frac{z'}{v} - \frac{R(z')}{c})}{\partial t} = -\frac{\partial i(0, t - \frac{z'}{v} - \frac{R(z')}{c})}{\partial z'} . v . F_{TL}(z') \qquad (18)$$

where the factor $F_{TL}(z')$ is a function of z' and given by

$$F_{TL}(z') = \left[1 + \frac{v}{c}\frac{z' - r\cos\alpha}{\sqrt{z'^2 + r^2 - 2z'r\cos\alpha}}\right]^{-1} = \left[1 - \frac{v}{c}\cos\theta(z')\right]^{-1} \qquad (19)$$

and θ(z') is the angle the channel makes with a line joining the point P with the point on the channel at height z' (see Fig. 1). Here we see how the factor can arise while converting the time derivative of the retarded current into its spatial derivative. There is a physical meaning to the F factor in this context, which is given below. If an observer at P "sees" a certain current at the ground at time t, the observer will "see" the same current at z at a time t" later. Then z' is a function of t" and is given by the solution of

$$t'' = \frac{z'(t'')}{v} + \frac{R(z'(t''))}{c} - \frac{r}{c} \qquad (20)$$

where R(z') is given by (16). The speed of the current wave at z' as "seen" by the observer (apparent speed), dz'/dt, is obtained by differentiating both sides of (20) with respect to time and rearranging as

$$\frac{dz'}{dt} = \frac{dz'}{dt''} = v.(1 - \frac{v}{c}\cos\theta)^{-1} = v.F_{TL} \qquad (21)$$

since t equals t" plus a constant. It follows from (21) that the F factor for the TL model can be defined as the ratio of the apparent speed to the actual speed of the current wave in the channel. Using (15) and (18) we can write (10) for TL model as

$$\bar{E}_r(r,\theta,t) = -\frac{1}{4\pi\varepsilon_0} \times \int_0^{L'(t)} \frac{\sin\theta}{c^2 R(z')} . v . F_{TL}(z') \frac{\partial i(0, t - z'/v - R(z')/c)}{\partial z'} dz' \hat{\theta} \qquad (22)$$

If the observer is distant as is necessary for the radiation field to be dominant, L'(t)<<r, $\theta \cong \alpha$, R(z') $\cong$r, and thus E is given by

$$\bar{E}_r(r,\alpha,t) = -\frac{1}{4\pi\varepsilon_0}\frac{\sin\alpha}{c^2 r}\frac{v}{1 - \frac{v}{c}\cos\alpha}\int_0^{L'(t)} di(0, t - \frac{z'}{v} - \frac{R(z')}{c})\,\hat{\alpha}$$
$$= -\frac{1}{4\pi\varepsilon_0}\frac{\sin\alpha}{c^2 r}\frac{v}{1 - \frac{v}{c}\cos\alpha}\left[i(0,0) - i(0, t - \frac{r}{c})\right]\hat{\alpha} \qquad (23)$$

noting that i(0,t - L'(t)/v - R(L'(t))/c) = i(0,0), as follows from equation (11). The factor $[1-(v/c)\cos(\alpha)]^{-1}$ in (23) appears as a multiplier (or "correction") to the speed v of the upward propagating current wave in the channel, which in the TL model is the same as the speed of the discharge front. The factor is equal to one when $\alpha=90°$ (when the line connecting the channel bottom and point P is perpendicular to the channel), the situation considered by [2].

2.4 Traveling current source model

In the previous section we found how the F factor can arise in the relation between the time and spatial derivatives of the retarded current for the TL model. If the channel current is described by another model we may get a different F factor. In order to illustrate the model dependency of the F factor, we now derive an analytical expression similar to (23) for the radiation field far from a return stroke channel using the Traveling Current Source (TCS) model proposed by [9]. The relation between the channel current at z' and the channel base current for the TCS model is given by

$$i(z',t-\frac{R(z')}{c}) = i(0,t+\frac{z'}{c}-\frac{R(z')}{c}) \qquad (24)$$

where R(z') is given by (16). Equation (24) represents a model in which the upward propagating return stroke wavefront, with speed v, instantaneously turns on current sources along the lightning channel as it passes them. The resultant current is assumed to travel downward at the speed of light c, without distortion and attenuation. First consider the current behind the return-stroke front. The variables t and z' are independent since z' is chosen arbitrarily, and hence using the same procedure as in the case of TL model, we can derive the relationship between the time and spatial derivatives of the retarded current as dictated by the TCS model,

$$\frac{\partial i(0,t+\frac{z'}{c}-\frac{R(z')}{c})}{\partial t} = \frac{\partial i(0,t+\frac{z'}{c}-\frac{R(z')}{c})}{\partial z'}.c.F_{TCS}(z') \qquad (25)$$

where the factor $F_{TCS}(z')$, different from $F_{TL}(z')$, is given by

$$F_{TCS}(z') = \frac{1}{\left(1-\frac{z'-r\cos(\alpha)}{\sqrt{z'^2+r^2-2z'r\cos(\alpha)}}\right)} = \frac{1}{1+\cos\theta(z')} \qquad (26)$$

and $\theta(z')$ is as shown in Fig.1. We can provide a physical interpretation of the factor F_{TCS}, similar to that given for F_{TL}. A current that is "seen" at ground by the observer at P would have been "seen" by the observer at height z' at an earlier time t" where t" is given by

$$t'' = -\frac{z'(t'')}{c}+\frac{R(z'(t''))}{c}-\frac{r}{c} \qquad (27)$$

and R(z') is given by (16). The speed of the current wave at z', as "seen" by the observer (apparent speed), dz'/dt, is obtained by taking the time derivative of (27) and rearranging

$$\frac{dz'}{dt} = \frac{dz'}{dt''} = -c.\frac{1}{1+\cos\theta} = -c.F_{TCS} \qquad (28)$$

since t equals t" plus a constant. Therefore, from (28), the F factor for the TCS model can be defined as the ratio of the apparent speed to the actual speed of the current wave in the channel (negative sign due to direction of travel being downward, that is, opposite to the direction of propagation of the discharge front). Substituting (25) in (10), using the far field approximations; that is, letting L'(t) << r, $\theta\cong\alpha$, and $R(z')\cong r$, performing the integration, and using (11a), we obtain

$$\bar{E}_r(r,\alpha,t) = \frac{1}{4\pi\varepsilon_0}\frac{\sin\alpha}{c^2 r}\frac{c}{1+\cos\alpha} \times\left[i\left(0,L'(t)(\frac{1}{c}+\frac{1}{v})\right)-i(0,t-\frac{r}{c})\right]\hat{\alpha} \qquad (29)$$

Equation (29) gives the radiation electric field from the channel current behind the discharge front only; that is, it does not take account of the radiation from the discontinuous discharge front inherent in the TCS model. The factor $[1+\cos\alpha]^{-1}$ in (29) appears as a correction to the speed c of the downward propagating current wave. Comparing (29) with (23) we can see that the F-factor for the TCS model, not taking into account the discontinuity at the front, is completely different from that for the TL model.

2.5 Current discontinuity at the discharge front

The TCS model involves a current discontinuity at the front even when the current at ground level at t=0 is zero. As a result, equation (10) has to be applied separately to the current behind the front and to the current discontinuity at the front. The TL and other models can, in general, also have a current discontinuity at the return-stroke front. We will see how the F factor arises in the general radiation field expression for a current discontinuity at the front.

Let i(z',t-R(z')/c) describe the retarded current in the return stroke channel. Then the current and current derivative at the return-stroke front "seen" by the observer are given by

$$i\left(L'(t),t-\frac{R(L'(t))}{c}\right) = i\left(L'(t),\frac{L'(t)}{v}\right) \qquad (30a)$$

$$\frac{\partial i(L'(t),t-R(L'(t))/c)}{\partial t} = \frac{di(L'(t),L'(t)/v)}{dt} \qquad (30b)$$

using t as given in (11a). Let $L'_-(t)$ and $L'_+(t)$ be the positions just below and just above the wavefront at L'(t), respectively. The integral of the current derivative across the wavefront is equal to the product of the current at the wavefront and the velocity of the wavefront as seen by the observer at P [10]. That is,

$$\int_{L'_-(t)}^{L'_+(t)}\frac{di(L'(t),t-R(L'(t))/c)}{dt}dz' = i(L'(t),\frac{L'(t)}{v})\cdot\frac{dL'(t)}{dt} \qquad (31)$$

Differentiating both sides of (1) with respect to t and rearranging the terms, we find

$$\frac{dL'(t)}{dt} = v.\frac{1}{1+\frac{v}{c}.\frac{L'(t)-r\cos(\alpha)}{\sqrt{L'(t)^2+r^2-2L'(t)r\cos(\alpha)}}} = v\frac{1}{1-\frac{v}{c}\cos(\theta(L'))} \qquad (32)$$

Note that the factor $[1-(v/c)\cos\theta]^{-1}$ in (32) is obtained for a traveling step discontinuity in [5] using Heaviside and delta functions. As follows from (32), this factor is the ratio of the apparent speed to the actual speed of the propagating discharge front, depends only on the front propagation speed and geometry, and is applicable to a discharge front discontinuity of any return stroke model. The apparent speed, dL'(t)/dt, of the discharge front can also be calculated numerically as $\{L'(t+\Delta t)-L'(t)\}/\Delta t$, where

Δt is the incremental time, and L'(t+Δt) and L'(t) are determined from (12). Substituting (32) in (31), using the resulting equation in (10), we find the radiation electric field from a traveling current discontinuity (the "turn-on" field)

$$\bar{E}_t(r,\alpha,t)_t = \frac{1}{4\pi\varepsilon_0}\frac{\sin\theta(L')}{c^2R(L')}\, i\left(L'(t),\frac{L'(t)}{v}\right).v.\frac{1}{1-\frac{v}{c}\cos\theta(L')}\hat{\theta} \qquad (33a)$$

Using the approximation for far-fields, that is, letting L'(t)<<r, θ≅α, and R(L')≅r, we get the "turn-on" field in the far-field as

$$\bar{E}_t(r,\alpha,t) = \frac{1}{4\pi\varepsilon_0}\frac{\sin\alpha}{c^2r}\, i\left(L'(t),\frac{L'(t)}{v}\right).v.\frac{1}{1-\frac{v}{c}\cos\alpha}\hat{\alpha} \qquad (33b)$$

The factor $[1-(v/c)\cos\alpha]^{-1}$ in (33b) appears as a "correction" to the return stroke discharge front speed v and is applicable to any return stroke model with a current discontinuity at the wavefront. The correction factor associated with the current discontinuity at the return-stroke front derived here and for a traveling step discontinuity of [5] and [10] is the same as the expression obtained for the continuous current waveform in the channel for the TL model of [3]. Note that the F factor associated with the channel current behind the discharge front is different for models other than the TL, as illustrated previously.

2.6 Far-distance radiation field taking into account current discontinuity at the discharge front

2.6.1 TL model

The current at the return-stroke front for the TL model is i(L'(t),L'(t)/v) = i(0,0). Therefore, if the current at ground starts from zero at t = 0, i(0,0) = 0, there is no current discontinuity at the propagating discharge front. If the current at the ground starts from a non-zero value at t=0, the current will have the same non-zero value at the propagating front. The "turn-on" far radiation field is obtained from (33b) as

$$\bar{E}(r,\alpha,t) = \frac{1}{4\pi\varepsilon_0}\frac{\sin\alpha}{c^2r}.\frac{v}{1-\frac{v}{c}\cos\alpha}\, i(0,0)\hat{\alpha} \qquad (34)$$

which is equal in magnitude and opposite in sign to the term containing i(0,0) in (23). Adding (23) and (34) we get

$$\bar{E}(r,\alpha,t) = \frac{1}{4\pi\varepsilon_0}\frac{\sin(\alpha)}{c^2r}\frac{v}{1-\frac{v}{c}\cos(\alpha)}\, i(0,t-r/c)\hat{\alpha} \qquad (35)$$

which is the total radiation electric field far from the channel for the TL model, whether or not there is a discontinuity at the discharge front.

For the special case of measurements at ground far from the channel, α=90°, equation (35) in a Cartesian coordinate system where $\hat{z}$ is an upward directed unit vector, becomes

$$\bar{E}(r,t) = -\frac{1}{4\pi\varepsilon_0}\frac{v}{c^2r}\, i(0,t-r/c)\hat{z} \qquad (36)$$

which is identical to the relation obtained by [2] for the far radiation fields at ground level from return strokes assumed to obey the TL model. If we additionally include the effect of the perfectly conducting ground plane, we must multiply the right hand side of (36) by 2.

2.6.2 TCS model

The current at the return-stroke front for the TCS model is i(L'(t),L'(t)/v) = i(0,L'(t)/v+L'(t)/c). Therefore, unlike the TL model, even if the current at ground starts from zero, the current at the return-stroke front above ground will have a non-zero value. The "turn-on" far radiation field is obtained from (33b) as

$$\bar{E}(r,\alpha,t)_t = \frac{1}{4\pi\varepsilon_0}\frac{\sin\alpha}{c^2r}.\frac{v}{1-\frac{v}{c}\cos\alpha}\, i\left(0,L'(t)(\frac{1}{v}+\frac{1}{c})\right)\hat{\alpha} \qquad (37)$$

The total radiation electric field for the TCS model is obtained by summing (29) and (37) to give

$$\bar{E}(r,\alpha,t) = \frac{1}{4\pi\varepsilon_0}\frac{\sin\alpha}{c^2r}\Big[\frac{c}{1+\cos\alpha}\left(i(0,L'(t)(\frac{1}{c}+\frac{1}{v})) - i(0,t-\frac{r}{c})\right) + \frac{v}{1-\frac{v}{c}\cos\alpha}\, i(0,L'(t)(\frac{1}{c}+\frac{1}{v}))\Big]\hat{\alpha} \qquad (38)$$

For the special case of measurements at ground, α=90°, equation (38) in a Cartesian coordinate system becomes

$$\bar{E}(r,t) = \frac{1}{4\pi\varepsilon_0}\frac{1}{cr}.\left[i(0,L'(t)(\frac{1}{c}+\frac{1}{v})) - i(0,t-\frac{r}{c}) + \frac{v}{c}\, i(0,L'(t)(\frac{1}{c}+\frac{1}{v}))\right]\hat{z} \qquad (39)$$

Noting that for α=90°, L'(t)<<r, L'(t)≅L(t)=v(t-r/c), and defining

$$k = 1+\frac{v}{c} \qquad (40)$$

equation (39) becomes

$$\bar{E}(r,t) = \frac{1}{4\pi\varepsilon_0}\frac{1}{cr}.[k\,i(0,k(t-r/c)) - i(0,t-r/c)]\hat{z} \qquad (41)$$

which is the same relation obtained by [11], for far radiation fields at ground level from return strokes assumed to obey the TCS model. If we additionally include the effect of perfectly conducting ground plane, we must multiply right hand side of (41) by 2.

3. Discussion and conclusion

It follows from the analysis in the previous section that in calculating lightning radiation fields one can use either the general formula (10a) without any explicit "correction factor" (F factor) or its approximations involving F factors, such as (35) for the TL model. We have shown that the correction factor appears as various manifestations of retardation effects: in the far-distance approximation to the retarded channel length and while converting the time derivative of the retarded current into spatial derivative. For simple models like the TL and TCS models, in which the current at one point on the channel appears at another point at another time, the F factor associated with current behind the front can be interpreted physically as the ratio of the apparent propagation speed of the current wave "seen" by an observer some distance from the discharge to its actual speed. The F factor associated with a propagating discontinuity can always be interpreted as the ratio of the apparent speed of the front to its actual speed. The expressions for calculating the fields given in [1] and [12, section 7.3], are general expressions derived from Maxwell's equations and require no correction factors regardless of discharge extension speed (even near the speed of light), provided the retarded channel length is used in the calculations. In this respect, the allegation of the first sentence of [4] that the field equations in [1] and[12] describing the electromagnetic fields that are radiated by current pulses similar to lightning need correction when the speed of the pulse is a significant fraction of the speed of light is misleading. In [6] the F factors are applied to the vector and scalar potentials of the current carrying element dz' and general integral electric field expression similar to (7), but having additional multipliers $[1-(v/c)\cos\theta]^{-3}$ for current-integral (static) terms, $[1-(v/c)\cos\theta]^{-2}$ for current (induction) terms and $[1-(v/c)\cos\theta]^{-1}$ for current-derivative (radiation) terms, are obtained. The general expressions for electric fields in [6] are incorrect and therefore the calculated field waveshapes in [6] at various elevation angles and distances in their figures 2-5 may be wrong. In [6] the Lienard-Wiechert potentials of a uniformily moving point charge, which contain the F factor $[1-(v/c)\cos\theta]^{-1}$, is used to find the potentials from the current in an elemental channel segment dz′. This approach is incorrect for the following reasons. Point charge is an approximation to finite size charge distribution when the observation distance is very large compared to the size (volume) of the distribution. The Lienard-Wiechert potential is an approximation to the potential of a uniformily moving finite charge distribution if the size of the charge is very small compared to the distance to the point where the potential is found. The exact expression for the potential would be obtained, without any explicit involvement of the F factor $[1-(v/c)\cos\theta]^{-1}$, by integrating the contribution to the potential from each elemental charge volume within the retarded charge volume at each instant. If ρ is the uniform charge density of the line the total charge q = ρ.L and the charge in a short segment on the line is ρ.dz′. The potential at a point is given by

$$\phi = \int_0^{L'} \frac{\rho}{4\pi\varepsilon_0 R(z')} dz' \qquad (42)$$

where R(z′) is the distance from the observer to the retarded position of each segment dz′ of the line at a distance z′ from the bottom end of the line. If L<<r, as for point charge approximation, and using the far-distance approximation to retarded length, (42) can be reduced to

$$\phi = \frac{q}{4\pi\varepsilon_0 R'} \frac{1}{1-\frac{v}{c}\cos\theta} \qquad (43)$$

where R′ is the retarded position of the charge with respect to the observer. Equation (43) is the retarded potential (Lienard-Wiechert potential) of a uniformly moving point charge. For finite size charges (42) is the exact expression for potential and do not have the F factor, while (43) is a far-distance approximation and contain the F factor.

References

[1] Uman, M. A., D. K. McLain, and E. P. Krider, The electromagnetic radiation from a finite antenna, Am. J. Phys., 43 (1), 33-38, 1975.

[2] Uman, M. A., and D. K. McLain, Lightning return-stroke current from magnetic and radiation field measurements, Geophys. Res., 75, 5143-5147, 1970b.

[3] LeVine, D. M., and J. C. Willett, Comment on the transmission-line model for computing radiation from lightning, J. Geophys. Res., 97, 2601-2610, 1992.

[4] Krider, E. P., On the electromagnetic fields, Poynting vector, and peak power radiated by lightning return strokes, J. Geophys. Res., 97, 15913-15917, 1992.

[5] Rubinstein, M., and M. Uman, On the radiation field turn-on term associated with traveling current discontinuities in lightning, J. Geophys. Res., 95, 3711-3713, 1990.

[6] Kumar, R., J. Rai, and V. Singh, Lightning return stroke electric fields above ground, J. Atmos. Terr. Phys., 57, 1247-1254, 1995.

[7] Thottappillil, R., D. K. Mclain, M. A. Uman, and G. Diendorfer, Extension of Diendorfer-Uman lightningreturn stroke model to the case of a variable upward return stroke speed and a variable downward discharge current speed, J. Geophys. Res., 96, 17143-17150, 1991.

[8] Thottappillil, R., and M. A. Uman, Lightning return stroke model with height-variable discharge time constant, J. Geophys. Res., 99, 22773-22780, 1994.

[9] Heidler, F., Traveling current source model for LEMP calculation, Proc. 6th symp. Electromagn. Compat. paper 29F2, (Zurich, Switzerland), 157-162, 1985.

[10] Rubinstein, M., and M. Uman, Transient electric and magnetic fields associated with establishing a finite electrostatic dipole, revisited, IEEE Trans. on Electromag. Compatibility, 33, 312-320, 1991.

[11] Heidler, F., Some deductions from the travelling current source model, Proc. Int. Wroclaw Symp. on Electromag. Compatibility, 245-252, 1986.

[12] Uman, M. A., Lightning Discharge, Academic Press, NewYork, 1989.