AN ANTENNA THEORY MODEL FOR THE LIGHTNING RETURN STROKE

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Abstract: A new approach, based on antenna theory, is used to evaluate the lightning return-stroke current as a function of time and height. The lightning channel is modeled as a lossy, straight, and vertical monopole antenna above a perfectly conducting ground, and is fed by a source voltage. The source voltage is a function of the assumed current at ground level and the input impedance of the monopole antenna. An electric field integral equation (EFIE) is employed to describe the electromagnetic behavior of the antenna. The numerical solution of EFIE by the Method of Moments (MoM) in time domain provides the time-space distribution of the current along the lightning channel. This new antenna-theory model with specified current at the channel base requires only two adjustable parameters: the return-stroke propagation speed and the channel resistance per unit length. The new model is compared to the most commonly used lightning return-stroke models in terms of the temporal-spatial distribution of channel current and predicted electric fields.

1. Introduction

Return-stroke modeling can be viewed as the specification of the return-stroke channel current as a function of height and time. Most models of the return stroke specify an analytical relation between the current at each point of the channel and the channel base current (current at ground level). A suitable model should be consistent with the measured characteristics of the return-stroke, namely:

-Current at the base of the channel.

-Wave front propagation speed.

-Electric and magnetic fields at various distances from the channel at ground level.

-Variation of light intensity with height.

A brief discussion of some of the most used model [1-5] follows. In the transmission line model (TL) the current injected at the channel base propagates upward as it would on a losseless transmission line. The fields calculated from this model do not agree with the measurements, particularly at longer times and closer ranges.

In the modified transmission line model with exponential current decay with height (MTLE) the current wave suffers no distortion but its amplitude decays exponentially with height. The total charge density distribution is unrealistically skewed toward the bottom of the channel and consequently the model is not able to predict the very close electric field.

In the case of the modified transmission line model with linear current decay with height (MTLL), the calculated fields agree with the measurements, at all ranges.

In the modified transmission-line models (MTLL and MTLE) the current attenuation with height is specified arbitrarily, and current dispersion is ignored.

The Diendorfer and Uman (DU) model and its modifications consider the channel current as the linear sum of two components, one due to a fast discharge of the leader core and the other due to a slower discharge of the corona sheath surrounding the leader core. The DU model provides a good match between the model-predicted and measured electromagnetic fields and introduces current dispersion, but the specification of the dispersion is arbitrary.

In this paper, a new model based on antenna theory (AT) ,that is a complete solution to Maxwell's equations, is presented to describe the channel current profile. A monopole antenna of length H above a perfectly conducting ground is used to model the return-stroke channel (Fig. 1).The antenna is fed by a source whose voltage v(t) is given by the following equation:

$$v(t)=z(t) * i(0,t) \quad (1)$$

where $i(0,t)$ is the specified channel base current, $z(t)$ is the inverse Fourier transform of the input impedance of the monopole antenna and $*$ denotes the convolution. The input impedance of the monopole antenna, which is a function of its length, radius, and distributed resistance, is calculated applying the MoM to the EFIE.

In the AT model of the return-stroke, only two adjustable parameters are needed, the propagation speed and the resistance per unit length. The evolution of the current wave propagating along the channel is governed by antenna theory.

2. Theory

The response of a monopole antenna above a perfect ground to an incident electromagnetic wave, $\mathbf{e}^i$, can be found by considering the diffraction of a transient electromagnetic wave caused by a metallic obstacle [6]. Due to the presence of ground, the total incident wave at any point in space is:

$$\mathbf{e}^a=\mathbf{e}^i+\mathbf{e}^r \qquad (2)$$

Where $\mathbf{e}^r$ is the wave reflected from ground. The total incident electromagnetic field $\mathbf{e}^a$ induces current i(s,t) at any point s of antenna. This current according to Maxwell's equations produce the scattered electromagnetic field, $\mathbf{e}^d$. At any point P in space we have:

$$\mathbf{e}^d(P,t)=\mathbf{L}[i(s,t)] \qquad (3)$$

Where **L** is the integro-differential operator which is defined by Maxwell's equations [6,7]. The continuity of tangential component of the total electric field at any point on the antenna surface requires that:

$$\mathbf{s} \bullet \mathbf{e}^a(s,t)+\mathbf{L}[i(s,t)]=0 \qquad (4)$$

Using the definition of the **L** operator [5,6], we can write:

$$\mathbf{s}.\mathbf{e}^a(s,t)=\frac{\mu_0}{4\pi}\int_{antenna}\left[\frac{\mathbf{s}.\mathbf{s}_0}{R}\frac{\partial}{\partial t_0}i(s_0,t_0)+v\frac{\mathbf{s}.\mathbf{R}}{R^2}\frac{\partial}{\partial s_0}i(s_0,t_0)\right.$$
$$+v^2\frac{\mathbf{s}.\mathbf{R}}{R^3}\int_0^{t_0}\frac{\partial}{\partial s_0}i(s_0,t)dt-\frac{\mathbf{s}.\mathbf{s}_0^*}{R^*}\frac{\partial}{\partial t_0^*}i(s_0,t_0^*)$$
$$\left.-v\frac{\mathbf{s}.\mathbf{R}^*}{R^{*2}}\frac{\partial}{\partial s_0}i(s_0,t_0^*)-v^2\frac{\mathbf{s}.\mathbf{R}^*}{R^{*3}}\int_0^{t_0^*}\frac{\partial}{\partial s_0}i(s_0,t)dt\right]ds_0 \qquad (5)$$

Where $R=[(s-s_0)^2+a^2]^{1/2}$, $R^*=[(s-s_0^*)^2+a^2]^{1/2}$, $t_0=t-R/v$, $t_0^*=t-R^*/v$, and $v=(\varepsilon\mu_0)^{-1/2}$ is the propagation speed along the channel.

The last three terms in equation (5) represent the effect of perfect ground. s and s_0 are the observation and source points on the antenna respectively, s_0^* is the image point of a source point s_0. $\mathbf{s}$, $\mathbf{s}_0$, $\mathbf{s}_0^*$ are the corresponding unit tangential vectors. "a" is the radius of the antenna. The left hand-side of equation (5) represents the applied electric field. In this paper the excitation is realized by a voltage source, v(t), described in the introduction, which creates an electric field obtained as [7]:

$$\mathbf{e}^a(s,t) = -\nabla v(t) \qquad (6)$$

The numerical solution of equation (5), the electric field integral equation (EFIE), by the Method of Moments (MoM) in the time domain [6, 7] provides the time-space distribution of the current along the return stroke channel.

The propagation along the channel of the current injected at ground is governed by antenna theory. To slow the propagating current to a value consistent with observations, $v < 3\times10^8$, we use $\varepsilon > \varepsilon_0$ in calculating the current variation and then use that current to calculate the fields with $\varepsilon = \varepsilon_0$. The introduction of a distributed resistance of the antenna results in both attenuation and dispersion of the current, in agreement with time-resolved optical observations. The electromagnetic fields computed using this new model are in a good agreement with the measurements.

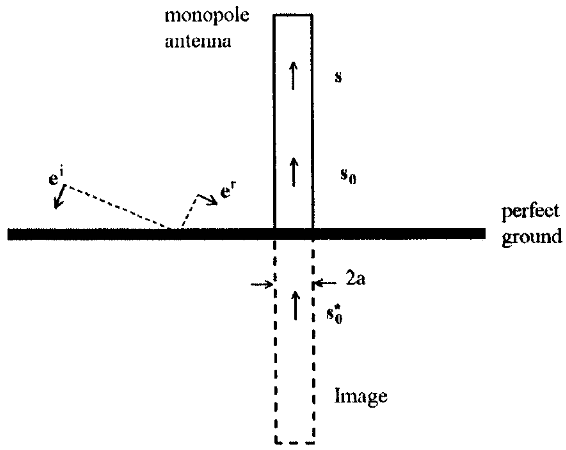
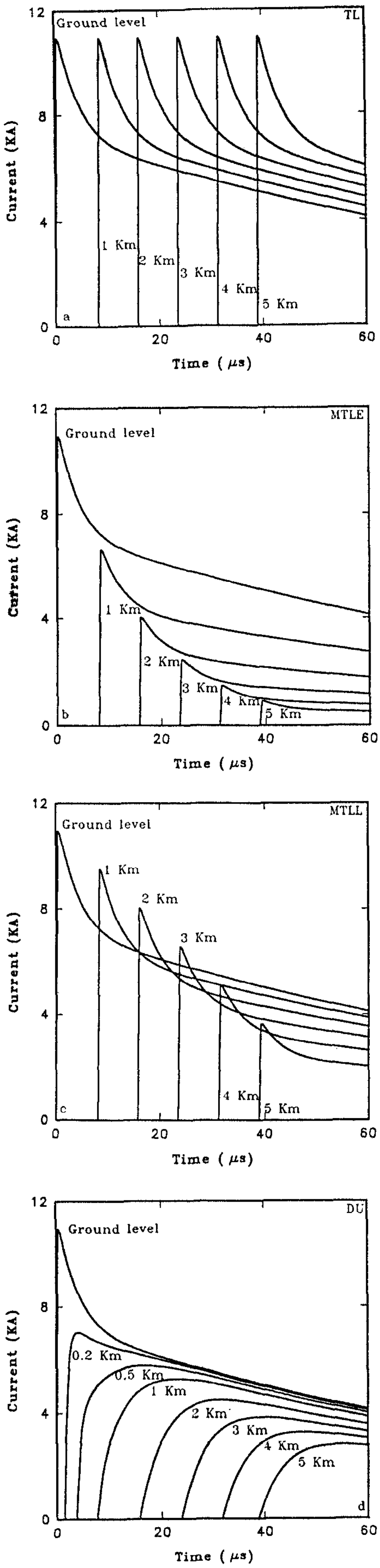


Fig. 1: A lossy monopole antenna above a perfectly conducting ground.

3. Results

For the current assumed at ground level (see Fig. 2) the propagation speed, v, is assumed to be 1.3×10^8 m/s, and the distributed resistance is taken to be about 0.1 Ω/m to provide an agreement between the computed and measured electric fields at 50 m (see Fig. 4.a). Fig. 2 illustrates the current versus time waveforms at different heights along the lightning channel for the TL, MTLE, MTLL, DU, and AT models. Fig. 3 shows current peak (CP), rise time (RT), and half peak width (HW) for these models as a function of height. In the TL model, the current propagates without either attenuation or distortion, so that the CP, RT and HW do not change with height. The MTLE model shows a very pronounced attenuation without dispersion. The DU and AT models exhibit attenuation and distortion, but in the DU model both the attenuation and distortion are more pronounced at the bottom of channel, and the dispersion vanishes after the first kilometer or so. In the AT model, the attenuation and dispersion increase with height gradually, consistent with the optical measurements of *Jordan et al.* [8]. The MTLL model shows an attenuation similar to the AT model but without dispersion.

Fig. 4 illustrates the electric fields predicted by various models at 50 m, 5 km and 100 km. Except for the TL and MTLE models, the 50 m electric field predicted by all the models are consistent with experimental data. At 5 km, the electric field exhibits a ramp for all the models except for the TL model, consistent with measurements. At 100 km, all the models predict the same peak and rise time. The MTLE and MTLL models exhibit a zero-crossing at some tens of microseconds in



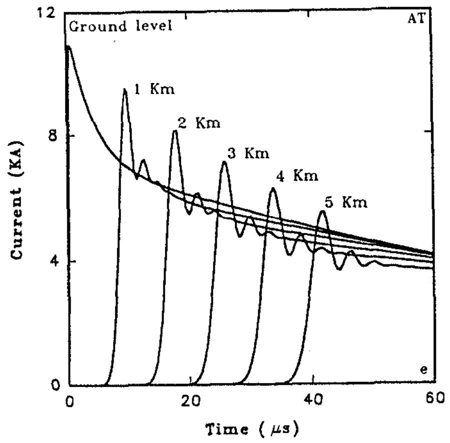


Fig. 2: Current as a function of time at different heights along the channel for the TL, MTLE, MTLL, DU and AT models, MTLE: λ=2000 m [3], MTLL: H=7.5 Km [2], DU: τ_{bd}=0.6 μs, τ_c = 5 μs [5] AT: R=0.07 Ω/m

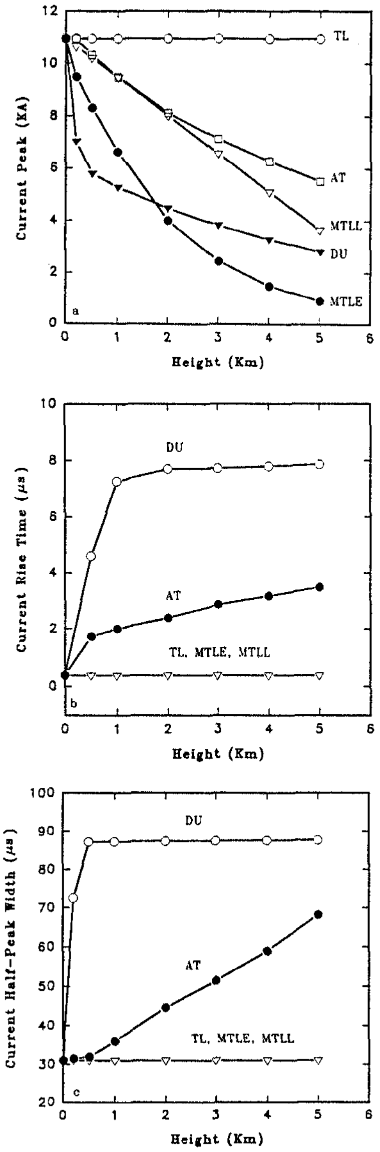


Fig. 3: Characteristics of the spatial current distribution for the TL, MTLE, MTLL ,DU, and AT models.

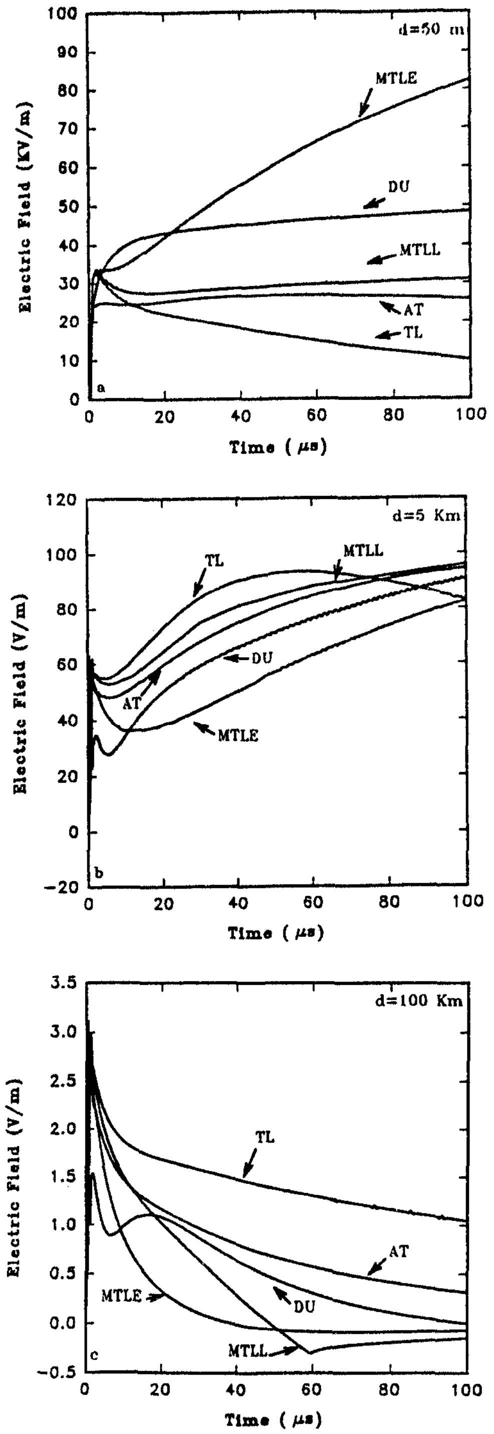


Fig. 4: Electric field at 50 m, 5 km, and 100 km, predicted by the TL, MTLE, MTLL, DU, and AT models.

agreement with generally accepted typical field signature at this range. The zero-crossing for AT and DU models does not occur within the first 100 microseconds considered in Fig. 4, however this feature can be achieved choosing a faster decaying current waveform at ground level.

Ringing after the initial peak in Fig. 2e and the abrupt change in field slope at 60 microseconds for the MTLL model (Fig. 4c) are due to numerical instability.

4. Conclusion

A new model for the lightning return-stroke based on antenna theory is introduced and compared with other models. The specification of appropriate speed of propagation and distributed resistance of the channel in addition to the channel base current is sufficient to make the model-predicted electric fields consistent with measurements. The AT model results are intermediate in characteristics between models that have previously been most used.

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