A MODIFIED TRANSMISSION LINE MODEL FOR LIGHTNING RETURN STROKE FIELD CALCULATIONS

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Modifications to the transmission line model developed at the High Voltage Research Institute, Tomsk, USSR, are presented and compared between each other and with the modification proposed by the Italian-Swiss research team. The modifications considered differ from each other by the height dependent attenuation factor for the return stroke current pulse intensity. All of the attenuation factors, corresponding to various leader charge distributions along the channel, provide fairly good agreement of the calculated fields with the experimental data available. Some problems with modeling of the early and late stages of the return stroke process are discussed.

1. Introduction and literature review

First of all we note that the term "transmission line model" in title of the paper denotes a return stroke model described by Uman and McLain [1]. In this model a temporal and spatial behavior of the return stroke current is assumed, with the channel-base current and return stroke speed being specified in accordance with experimental data available. The term "transmission line model" is also used to label more sophisticated models which mathematically describe the return stroke channel as a R-L-C transmission line with circuit elements that may vary with height and time (e.g., [2-9]).

In the R-L-C models the temporal and spatial behavior of the current is determined by the telegrapher's equations. These models, being more physically oriented and potentially more informative, are in many respects not compatible with presently existing level of the understanding of the physics involved. For instance, the determining of the C circuit element requires a description of dynamics of the channel corona charge while this charge is collapsing into the channel core, the process for which even a time scale is a matter of great disagreement: nanoseconds [6], microseconds [10,11], milliseconds [12,13], hundreds of microseconds and seconds [14]. Due to the rudimentary knowledge on some pertinent lightning discharge processes one have to make too many arbitrary and speculative (not well grounded in observed lightning properties) assumptions. As a result, the R-L-C models are generally not able to provide an agreement between calculated and measured fields (compare, for instance, Fig. 3 in [5] and Fig. 1 in [15]). Although, this approach to modeling allows to carry out extensive numerical experiments whose results, being compared with observed lightning properties, can improve the understanding of lightning.

There has been considerable interest lately in developing and applying to field calculations of the lightning return stroke models with specified channel-base current, some of the models being, in fact, a modification of Uman and McLain's (1969) [1] model (e.g., Lin et al., 1980 [10], Dulzon and Rakov, 1980 [16], Master et al., 1981 [17], Rakov and Dulzon, 1987 [18], Nucci et al., 1988 [19,20], Nucci and Rachidi, 1989 [21], Rachidi and Nucci, 1990 [22]) while others representing a somewhat different approach (e.g., the so-called traveling current source model introduced by Heidler, 1985 [23], and Diendorfer and Uman's, 1990 [11] model). Probably these extensive efforts have been motivated by the need to have relatively straightforward techniques for (1) deriving the lightning current parameters from electromagnetic field measurements (the so-called inverse source problem), and (2) prediction of the coupling and resultant effects of the fields of nearby lightning on airborne vehicles and on ground based objects.

An initial version of the return stroke model we present here has been developed in the early seventies in the High Voltage Research Institute, Tomsk, USSR. The model description has been first published in 1974 in German [24] and in 1975 in Russian [25]. But, perhaps, due to the language barriers those papers appeared to be not available for the most of the lightning research community. The model was utilized for extensive electric and magnetic field calculations in both time and frequency domain. The results were used in developing estimating relations for deriving the return stroke peak current from electric and magnetic field measurements, and in substantiating the frequency response of the various lightning flash counters and other lightning field recording devices. The most complete set of calculated fields (both total and separate components: electrostatic, induction, electric radiation, magnetostatic, and magnetic radiation) in both time and frequency domain we organized in the Atlas (not published) which includes the calculated fields and their spectra for the typical first and typical subsequent stroke at two ranges, 10 and 100 km, for various channel heights and return stroke speeds. Waveforms in the Atlas show a reasonably good agreement with experimental data available in the literature. In particular, electric fields at 10 km show characteristic ramp and both electric and magnetic fields at 100 km show characteristic bipolar waveshape [15].

2. Model description

Major features of the model being presented here are the following. As in original transmission line model (TLM) a specified channel-base current pulse is assumed propagating without distortion vertically upwards from the surface of a perfectly conducting ground plane. Neither channel branching, nor attachment process are taken into consideration. In contrast with the original TLM the current pulse intensity is allowed to decrease with height above ground level. Thus, the current behavior as a function of time (t) and height (z) is expressed as

$$i(z,t) = P(z)\cdot i(0,t - tz), \qquad (1)$$

where P(z) is the current attenuation factor which is a function of only z (is a constant at fixed z); tz is the time for the return stroke front to reach the height z.

We assumed that entire charge to be neutralized by the return stroke is deposited onto the channel (mostly in the corona sheath surrounding the highly conducting channel core) of the effective height H. This implies that only charge stored above the given channel section (no more, no less) is to be transferred through this section to ground, i.e., total channel charge will flow through the channel base (z = 0), and no charge will flow through the effective channel top (z = H). Since the current waveshape is assumed to be independent on height (no distortion, only attenuation) this decrease in charge transfer with height corresponds to a decrease in return stroke current pulse intensity with the rate of decrease being the same in both the cases. Hence, the current attenuation factor at height z simply is a ratio of the leader charge distributed along the channel above the channel section at height z and total leader charge deposited onto the channel of effective height H:

$$P(z) = \frac{\int_z^H q(z)\cdot dz}{\int_0^H q(z)\cdot dz}, \qquad (2)$$

where q(z) is the charge per unit length at height z. It is clear from (2) that P(z) varies from 1 at z = 0 to 0 at z = H. Note that in the models with exponential current decay [19-22] this attenuation factor is non-zero at z = H.

The behavior of P(z) within the boundary values indicated (P(0) = 1 and P(H) = 0) is determined by the distribution of charge along the channel. First [24,25,16] we used the uniform charge distribution, and then we explored influence of the distribution type on the calculated fields considering both linear [18] and exponential decrease of the charge with height.

If the leader charge is uniformly distributed along the channel (q(z) = = const), then it follows from (2) that

$$P(z) = 1 - z/H. \qquad (3)$$

For the linear decrease of the charge

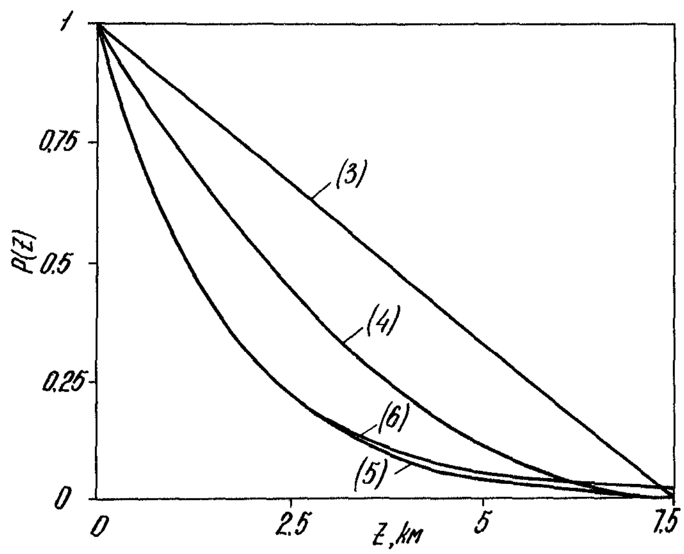

Fig. 1. Return-stroke current attenuation factors defined by expressions (3) through (6) as a function of height z (H = 7.5 km)

with height

$$P(z) = (1 - z/H)^2 \quad (4)$$

If the leader charge per unit length decays exponentially with height ($q(z) = q_0 \cdot \exp(-z/L_q)$, where subscript 0 refers to the channel base, z = 0, and L_q is the decay rate of the charge with height), then

$$P(z) = \frac{\exp(-H/L_q) - \exp(-z/L_q)}{\exp(-H/L_q) - 1}. \quad (5)$$

Expressions (3) through (5) are displayed in Fig. 1. Also shown in Fig. 1 is the current attenuation factor proposed in [19]:

$$P(z) = \exp(-z/L_i), \quad (6)$$

where L_i = 1.7 km [21] represents the decay rate of the lightning current pulse intensity. Due to lack of the relevant experimental data any value of L_q is to a great extent arbitrary. We chose it equal to the foregoing value of L_i, which is close to the commonly used values L_q (e.g., [26]).

Returning to expression (1), the time tz for the return stroke front to reach the height z is a function of the return stroke speed. This speed is probably correlated with the return stroke current since the higher current the faster transformation of the leader channel to the return stroke channel. If this be so, then current decay should be accompanied by a decrease in return stroke speed, even in a branchless channel. We arbitrarily assumed an exponential decrease of the speed with time:

$$v(t) = v_0 \cdot \exp(-G \cdot t), \quad (7)$$

where v_0 is the speed at the ground level, and G is the speed decay rate. Although, in view of the subsequent return stroke speed being commonly claimed uniform (e.g.,[10,19]) with reference to Schonland et al. (1935) [27], we also considered a case of G = 0. Thus, it follows from (7) that

$$tz = \begin{cases} -[\ln(1-z \cdot G/v_0)]/G & \text{for } G \neq 0; \quad (8) \\ z/v_0 & \text{for } G=0. \quad (9) \end{cases}$$

Distributions of the return stroke current along the channel at different times and current waveshapes at different heights of the channel for the attenuation factors defined by expressions (3)-(6) are presented in Fig. 2 and 3 respectively, assuming H = = 7.5 km, $v_0 = 1.5 \cdot 10^8$ m/s, G = 0, and $L_q = L_i$ = 1.7 km. To describe a channel-base current we used here an analytical approximation of the typical current waveshape for subsequent return stroke [28] proposed in [21, Fig. 2].

3. Calculated electric fields and discussion

To calculate vertical electric fields at various ranges we used well-known expressions (e.g., [29]) based on the solution of Maxwell's equations in terms of retarded scalar and vector potentials. Upper limit to height integrals in the expressions for electrostatic, induction and radiation field components [29] was assigned to be z as derived from following expression:

$$t - tz - \frac{\sqrt{D^2 + z^2}}{c} = 0, \quad (10)$$

where D is a horizontal distance between observation and ground strike points, and c is speed of light, until the return stroke front reaches the effective channel top, and to be H otherwise.

Calculated vertical electric fields E(t) at three different ranges (2, 10, and 200 km) are plotted by the attenuation factor expression in Fig. 4 for H = 7.5 km, $v_0 = 1.5 \cdot 10^8$ m/s, G = = 0, and $L_q = L_i$ = 1.7 km. The time origin (t = 0) indicated in Fig. 4 for the observation point is by D/c later with respect to the time origin at the ground strike point. Change of the value of G from zero to $0.9 \cdot v_0/H$, which is close to the upper limit for G as it follows from (8), does not make the calculated fields much different from those shown in Fig. 4.

In general, the calculated fields presented in Fig. 4 are quite similar at the same range for different P(z) and all are in fairly good agreement with the experimental data available (e.g., [15]). Although none of the current attenuation factors considered provides field zero crossing at 50 km which is reported to be typical in [15, Fig. 1].

Another discrepancy between the calculated fields and experimental data [15] is the absence in the former of the electric field hump appeared to be typical at some ranges: 1, 5, perhaps 10, 50, and 200 km, although it is not

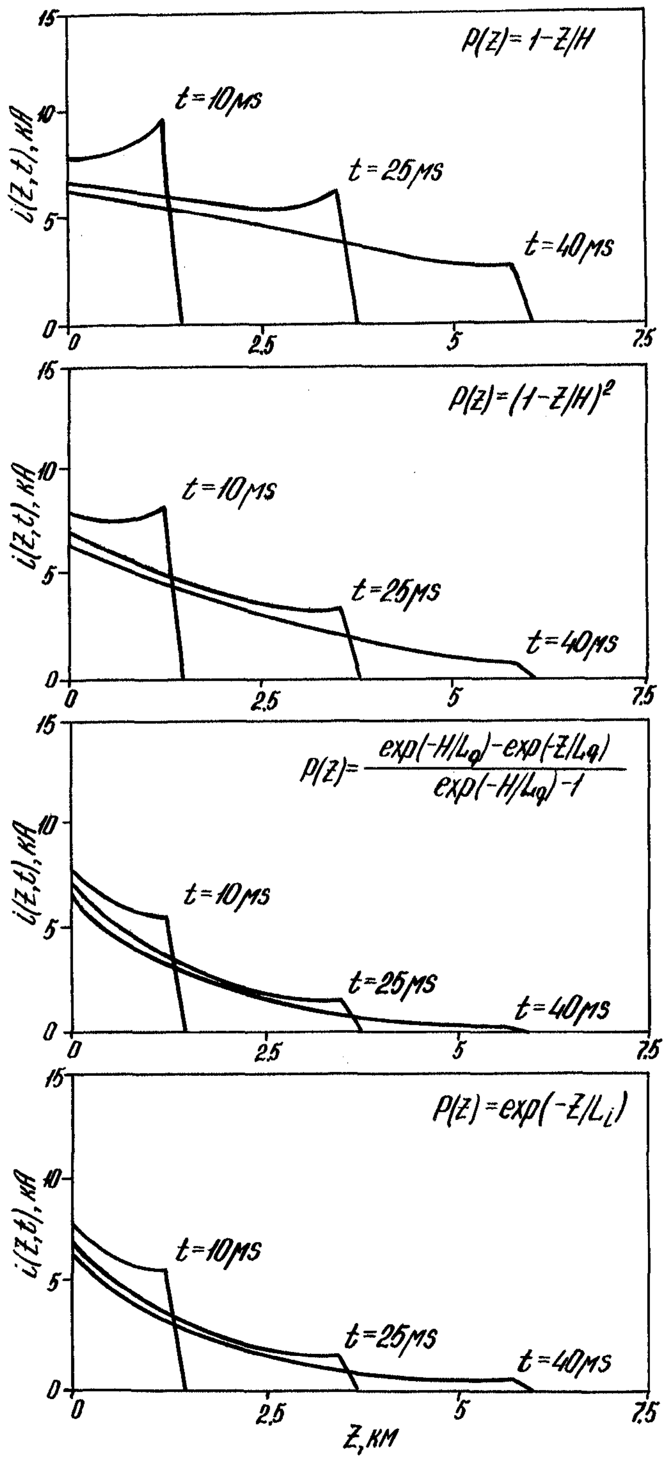

Fig. 2. Distributions of the return stroke current along the channel at different times for the current attenuation factors defined by expressions (3) through (6)

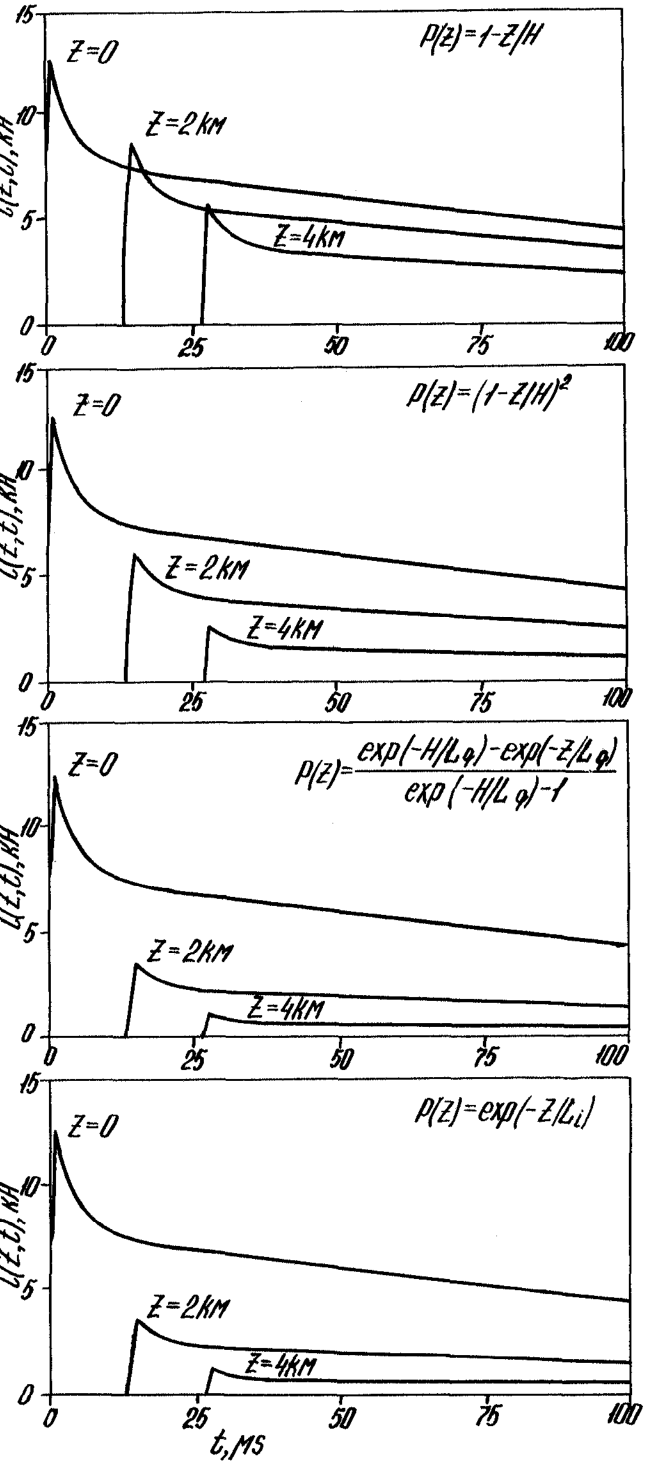

Fig. 3. Return stroke current waveshapes at different heights for the current attenuation factors defined by expressions (3) through (6)

evident at 2 and 15 km (see Fig. 1 in [15]). Similar electric field hump at 10 to 20 microseconds is produced by the corona current in Diendorfer and Uman's model [11] which is probably most physically oriented among all presently existing return stroke models with specified channel-base current. Although, in [11, Fig. 15b] the hump occurs at the same time at any range (10 to 200 km) while in the experimental data the time of the hump occurrence seems to be (at least sometimes) distance dependent (see Fig. 5 in [30] and Fig. 4.11, waveforms labeled E2, in [31]).

It follows from Fig. 4 that apparently no special uniform return stroke current component [10] is needed to provide the characteristic field ramp at close ranges [15].

Now we briefly discuss a couple of problems (sometimes not recognized) with return stroke modeling in general, not in any particular approach mentioned above. These are related to the very beginning and to the very end of the return stroke process, and should be kept in mind to avoid applying of a model beyond its limitations.

Probably a return stroke process may be viewed starting just after the so-called streamer zone [4] of the downward-moving leader contacts ground or streamer zone of the upward-moving connecting leader. At this moment potential of the downward-going leader tip is still close to the cloud potential and, hence, the first stage of the return stroke should provide nearly ground potential to that point on the leader channel, the process called the break-through phase [4]. This phase was inferred to exist in lightning discharge by analogy with long spark. Physically, the break-through phase is the transformation of the relatively high longitudinal potential gradient streamer zone (a volume of some tens of meters in longitudinal dimension occupied by numerous filaments) to relatively low-gradient plasma channel. This phase is characterized by reducing the streamer zone length due to simultaneous propagation toward each other of the two plasma channels from the upper and lower extremities of the streamer zone. The process has very steep negative voltage-current characteristic and is thought to last from 1 microsecond to even some tens of microseconds [4].

It seems to be not unreasonable to hypothesize [4] that the initial rising portion of the return stroke current pulse, including the peak value, at the channel base (i.e., first few microseconds or so of the return stroke process) is associated with the break-through phase, the process physically different from those modeled using any of the transmission line and other abovementioned approaches (see Introduction and literature review section) except the break-through phase model developed by Gorin [4].

Thus, any attempt to improve the lightning return stroke model for early times of the process (e.g., [9]) may be not productive without taking account of the break-through phase.

One may argue that the break-through phase is important only for first strokes, not for subsequents. We think that there is no great phenomenology difference between first and subsequent strokes. Even upward-moving connecting leader thought to be attributable exclusively to first strokes was recently found occurring in subsequents as well (see [32,33] for natural lightning; and [34,35] for triggered lightning). Probably, the streamer zone for subsequent strokes is shorter than for first strokes

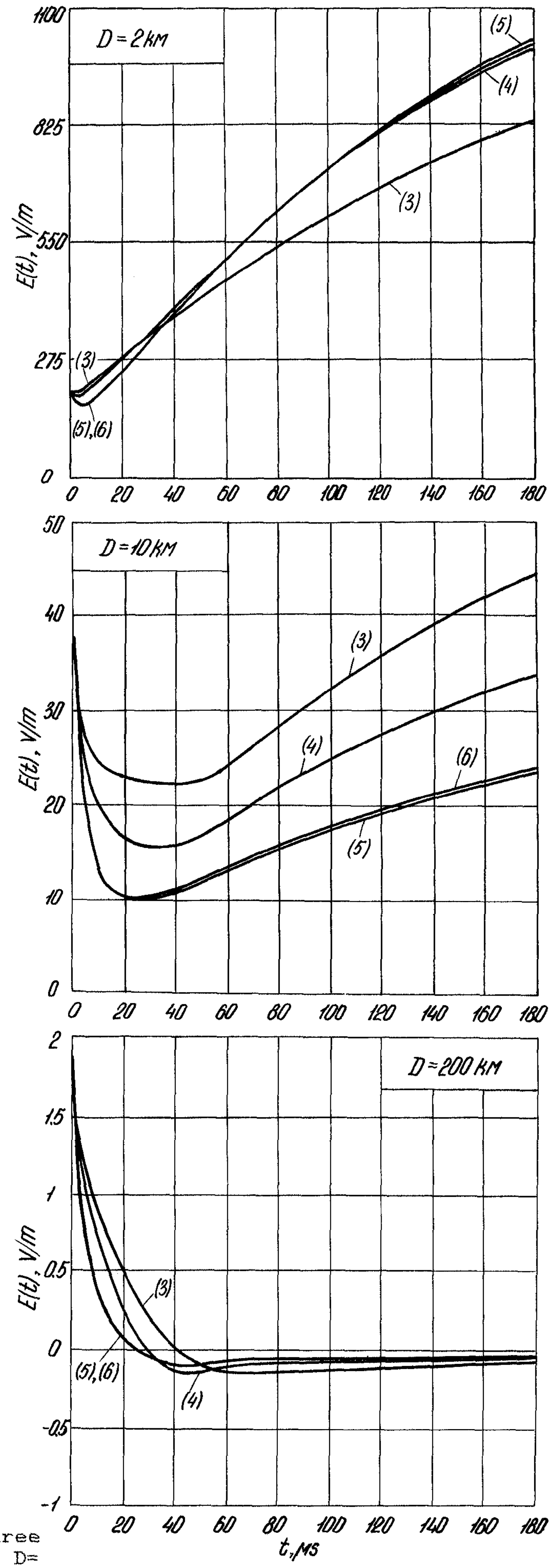

Fig. 4. Calculated lightning return stroke electric fields at three different ranges (D = 2 km, D= = 10 km, and D = 200 km) for the current attenuation factors defined by expressions (3) through (6)

resulting in shorter break-through phase duration and, hence, shorter return stroke current rise time duration, the latter being in accordance with the experimental data available.

It is common (e.g., [10,19]) view that subsequent return strokes are easier to model as compared to first strokes. It is certainly so but until the process enters the cloud and nears the region previously discharged by the first stroke since then the subsequent stroke channel usually changes its orientation from predominantly vertical to predominantly horizontal [36-38] with the horizontal extension being up to 8 km [36] and more. Hence, the calculated fields may be not comparable with experimental data on the higher order strokes for late times (later than about 50 microseconds for fields presented in Fig. 4) if the change in the channel orientation is not taken into account.

Further, the leader process depositing charges onto the channel for following neutralizing by the return stroke should collect those charges from the cloud hydrometeors initially isolated from each other. This funneling process is probably associated with numerous heavily branched ionized channels pervading a relatively large cloud volume and serving to supply charges for the highly organized movement along the channel. We suspect that the return stroke process after its front has reached this funneling cloud region is not adequately reflected by any of the presently existing return stroke models.

Thus, any return stroke model assuming propagation of the current wave along the previously charged single channel is justifiable only for middle portion of the channel. Below this portion at the early times the channel should be viewed as a sort of closing switch, and above this portion (at late times) the leader funneling region should be taken into account. For subsequent strokes of the higher order the change in the channel orientation from predominantly vertical to predominantly horizontal for the late times may be also important.

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