# Lightning electric field intensity at high altitudes: Inferences for production of elves 

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Received 20 March 2003; revised 21 July 2003; accepted 7 August 2003; published 23 October 2003.
[1] Distant electric fields predicted by the transmission line (TL) model and by the modified transmission line model with exponential current decay with height (MTLE) are examined as a function of polar angle (elevation) and return stroke propagation speed. The lightning return stroke current waveform was approximated by a step function. The resultant electric field waveform for the TL model is also a step function, while for the MTLE model the field instantaneously rises to the same value as for the TL model and then decays exponentially. The exponential current attenuation with height in the MTLE model results in a considerable reduction in the electric field intensity within $1 \mu \mathrm{~s}$ after the initial peak, particularly for smaller polar angles (larger elevations) and higher propagation speeds. Combinations of current and speed (as a function of polar angle) that are conducive to the production of transient optical emissions (elves) in the lower ionosphere are examined. According to the TL model, for a typical negative first-stroke current of 30 kA , elves would be produced only if the return stroke speed were greater than about $2.5 \times 10^{8} \mathrm{~m} / \mathrm{s}$. For the MTLE model, considerably larger currents are needed for the production of elves than for the TL model. INDEX TERMS: 0619 Electromagnetics: Electromagnetic theory; 0684 Electromagnetics: Transient and time domain; 3304 Meteorology and Atmospheric Dynamics: Atmospheric electricity; 3324 Meteorology and Atmospheric Dynamics: Lightning; KEYWORDS: lightning, electric field, elves

Citation: Rakov, V. A., and W. G. Tuni, Lightning electric field intensity at high altitudes: Inferences for production of elves, J. Geophys. Res., 108(D20), 4639, doi:10.1029/2003JD003618, 2003.

## 1. Introduction

[2] Lightning electromagnetic fields are often computed using the so-called transmission line type models [e.g., Uman and McLain, 1969; Rakov and Dulzon, 1987; Nucci et al., 1990; Krider, 1992, 1994]. In these models, lightning return stroke current at a height $z^{\prime}$ above perfectly conducting ground at time $t$ is given by [e.g., Rakov, 1997]

$$
\begin{equation*}
i\left(z^{\prime}, t\right)=P\left(z^{\prime}\right) i\left(0, t-z^{\prime} / v\right) \tag{1}
\end{equation*}
$$

where $P\left(z^{\prime}\right)$ is the current attenuation function, $v$ is the return stroke speed, and the lightning channel is usually assumed to be straight and vertical. For the original transmission line (TL) model [Uman and McLain, 1969], $P\left(z^{\prime}\right)=1$, while for the modified TL model with exponential current decay with height (MTLE) [Nucci et al., 1988] $P\left(z^{\prime}\right)=\exp \left(-z^{\prime} \lambda\right)$ where $\lambda$ is the current attenuation distance.
[3] Krider [1992, 1994], using the TL model, studied the lightning electric field intensity (and the Poynting vector) as a function of polar angle, return stroke current, and return stroke propagation speed. He was apparently the first to predict the enhancement of the radiation field at smaller polar angles (measured with respect to vertical) when the
return stroke speed is a significant fraction of the speed of light. Thottappillil et al. [2001], using the TL model with the return stroke speed equal to the speed of light, also discussed the focusing of electromagnetic radiation from a vertical lightning channel in the upward direction. Wait [1998] extended the study of Krider [1992, 1994] to the case of the MTLE model, using a step-function current, $I(0, t)=$ $I_{0} u(t)$ where $u(t)$ is the Heaviside function equal to unity for $t \geq 0$ and zero otherwise. Note that the TL model can be viewed as a special case of the MTLE model when $\lambda=\infty$. Wait [1998] noted that for the MTLE model the dependence of the lightning electric or magnetic field on polar angle is more complex than for the TL model, although he did not quantify the differences.
[4] In this paper, we compute the lightning electric field intensity as a function of time for different polar angles (elevations) and return stroke speeds, as predicted by the MTLE model. We also compute the electric field intensity at three retarded times, $0,0.1$, and $1 \mu \mathrm{~s}$, for the MTLE model and compare the results with those for the TL model, the latter results being previously published by Krider [1992, 1994]. We show that current attenuation with height results in a considerable reduction of fields within as little as $1 \mu \mathrm{~s}$ after the initial peak for relatively high propagation speeds and relatively small polar angles (high elevations). Further, we examine the influence on the production of transient optical emissions (elves) in the lower ionosphere of the
causative lightning return stroke current, lightning return stroke propagation speed, and current attenuation with height. The influence of current was previously considered by Rowland et al. [1996] and Fernsler and Rowland [1996] and of both current and propagation speed by Krider [1994]. We show that in the presence of current attenuation with height a higher current at the channel base would be required for the production of elves, this effect being more pronounced for higher propagation speeds.

## 2. Theory

[5] The lightning channel was assumed to be straight and vertical, and ground was assumed to be an infinitely large horizontal, perfectly-conducting plane. Lightning current was approximated by a step function. A general current versus time waveform may be considered as a succession of step functions, and the associated field can be found using Duhamel's integral. Only the $\theta$ component of far electric field is analyzed in this paper, similar to the studies of Krider [1992, 1994] and of Wait [1998]. For distances and times considered in this paper, the radial component of electric field that varies as $R^{-2}$ [Wait, 1998] is expected to be negligible compared to the $\theta$ component that varies as $R^{-1}$ where $R$ is the radial distance from the channel base. Optically detected elves are usually associated with larger return strokes of either polarity [Barrington-Leigh and Inan, 1999; Barrington-Leigh et al., 2001]. Only negative return strokes in cloud-to-ground lightning are considered here. It is possible that other impulsive processes in negative lightning discharges, such as the initial breakdown [e.g., Rakov, 2001] can produce elves.
[6] An expression for the $\theta$ component of far electric field for the MTLE model and $I(0, t)=I_{0} u(t)$ has been derived by Wait [1998]. We re-write his equation in a form that allows a more straightforward interpretation:

$$
\begin{array}{cc}
E_{\theta}(t)=\frac{Z_{0} \sin \theta}{4 \pi R c}\left(v_{u} I_{u}+v_{d} I_{d}\right) u(t-R / c) \\
v_{u}=v /(1-\beta \cos \theta) & v_{d}=v /(1+\beta \cos \theta) \\
h_{u}=v_{u}(t-R / c) & h_{d}=v_{d}(t-R / c) \\
I_{u}=I_{0} \exp \left(-h_{u} / \lambda\right) & I_{d}=I_{0} \exp \left(-h_{d} / \lambda\right)
\end{array}
$$

where $Z_{0}=120 \pi$ is the intrinsic impedance of free space, $c$ is the speed of light, $\theta$ and $R$ are the polar angle measured with respect to vertical (lightning channel) and radial distance from the origin of coordinates (lightning channel base), respectively, $v$ is the return stroke propagation speed, $\beta=v / c, \lambda$ is the current attenuation distance, $v_{u}, h_{u}$, and $I_{u}$ are the apparent channel extension speed, the apparent channel length, and apparent current at the return stroke front, respectively, all as "seen" from the field point, for the actual lightning channel extending upward from the origin on the ground surface, and $v_{d}, h_{d}$, and $I_{d}$ are similar quantities for the image channel extending downward from the origin. For a field point on the ground surface, $\theta=90^{\circ}$,
$v_{u}=v_{d}=v$ and $h_{u}=h_{d}=v(t-R / c)$, that is, the actual channel and its image contribute equally to the electric field. For a field point above ground, $\theta<90^{\circ}, h_{u}>v(t-R / c)$ and $h_{d}<v(t-R / c)$, with the difference between $h_{u}$ and $h_{d}$ increasing with increasing $\beta$ and decreasing $\theta$ (see equations (2') and ( $\left.2^{\prime \prime}\right)$ ). When $\theta \neq 90^{\circ}$, as $\beta$ increases, $v_{u}$ increases and $I_{u}$ decreases, while $v_{d}$ decreases and $I_{d}$ increases. When $\lambda \gg h_{u}$ and $\lambda \gg h_{d}$, equation (2) still applies, but since in this case $I_{u}=I_{d}=I_{0}$, it can be reduced to the equation derived by Krider [1992, 1994] for the TL model,

$$
\begin{equation*}
E_{\theta}(t)=\frac{Z_{0} I_{0}}{2 \pi R}\left[\frac{\beta \sin \theta}{1-\beta^{2} \cos ^{2} \theta}\right] u(t-R / c) \tag{3}
\end{equation*}
$$

Since $h_{u} \geq h_{d}$, only one condition, $\lambda \gg h_{u}$ or $(t-R / c) \ll \lambda / v_{u}$, needs to be satisfied for reducing the MTLE model to the TL model. For given $\lambda$ and $\beta$, the validity of approximation (3) depends on $\theta$, particularly when $\beta$ is close to unity. For example, if $\lambda=2000 \mathrm{~m}$ [Nucci et al., 1988] and $\beta=0.99$, approximation (3) is valid when $(t-R / c) \ll 132 \mathrm{~ns}$ for $\theta=8^{\circ}$ and when $(t-R / c) \ll 644 \mathrm{~ns}$ for $\theta=24^{\circ}$. If $\theta=90^{\circ}$, approximation (3) is valid when $(t-R / c) \ll \lambda / v$; that is, when $(t-R / c) \ll 6.7 \mu \mathrm{~s}$ if $\lambda=2000 \mathrm{~m}$ and $\beta=0.99$.
[7] As follows from equation (3), $E_{\theta}$ for the TL model is independent of the apparent channel length, although equation (3) is valid only in the far zone when $h_{u} \ll R$, with $h_{u}$ being a function of $\theta, \beta$, and $(t-R / c)$. (For example, $h_{u}=$ 15 km for $\theta=8^{\circ}, \beta=0.99$, and $(t-R / c)=1 \mu \mathrm{~s}$, and $h_{u}$ decreases to 3 or 0.3 km if $\theta$ increases to $24^{\circ}$ or $90^{\circ}$, respectively). On the other hand, $E_{\theta}$ in the MTLE model (see equation (2)) decreases with increasing $h_{u}$ and $h_{d}$ because the apparent currents, $I_{u}$ and $I_{d}$, at the actual and image fronts decrease exponentially as these fronts propagate in the upward and downward direction, respectively. According to Wait [1998], in order for the far zone approximation to be valid, distance $R$ must be much greater than attenuation distance $\lambda$, which he referred to as the "effective linear length of the radiating structure". This latter condition is satisfied in our calculations using the MTLE model. While the apparent channel length $h_{u}$ given by equation ( $2^{\prime \prime}$ ) may attain unrealistically large values for smaller $\theta$, larger $\beta$, and larger $(t-R / c)$, radiation from unrealistically large altitudes is negligible due to exponential current decay with height.

## 3. Analysis and Discussion

### 3.1. Waveforms and Angular Distributions of $E_{\theta}$ for a Typical First-Stroke Current

[8] As follows form equation 3, for a step-function current, the resultant electric field waveform for the TL model is also a step function. On the other hand, for the MTLE model the field instantaneously rises at $(t-R / c)=0$ to the same value as for the TL model and then decays exponentially, as shown in Figures 1a to 1d for different values of $\beta$ and $\theta$. In order to further characterize this decay, three values of retarded time $(t-R / c), 0,0.1$, and $1 \mu \mathrm{~s}$, were used in plotting the angular distributions of $E_{\theta}$ for the MTLE model. For the TL model, $E_{\theta}$ from equation (3) is the same for any value of $(t-R / c) \geq 0$, equal to $E_{\theta}$ predicted by the MTLE model for $(t-R / c)=0$. For both models, we assumed that $I_{0}=30 \mathrm{kA}$ (typical for negative first strokes in natural lightning) and $R=100 \mathrm{~km}$, so that


Figure 1. Electric field versus time waveforms predicted by the MTLE model with $\lambda=2 \mathrm{~km}$ for $I=I_{0} u(t-$ $R / c), I_{0}=30 \mathrm{kA}, R=100 \mathrm{~km}$, and four values of $\beta=v / c, 0.3,0.5,0.9$, and 0.99 . Field values at $(t-R / c)=0$ for the MTLE model are equal to those at $(t-R / c) \geq 0$ for the TL model. (a) $\theta=10^{\circ}$, (b) $\theta=30^{\circ}$, (c) $\theta=60^{\circ}$, and (d) $\theta=90^{\circ}$.
$Z_{0} I_{0} /(2 \pi R)=18 \mathrm{~V} / \mathrm{m}$. This value of electric field would be observed, according to the TL model, at $R=100 \mathrm{~km}$ at ground surface $\left(\theta=90^{\circ}\right)$ if $v$ were equal to the speed of light $(\beta=1)$. For typical values of $v=c / 3$ to $v=c / 2$, the TL-model-predicted electric field would be 6 to $9 \mathrm{~V} / \mathrm{m}$, similar to experimentally observed values for first strokes [e.g., Rakov, 2001]. For the MTLE model, we set $\lambda=$ 2000 m [Nucci et al., 1988]. Polar plots of $\mathrm{E}_{\theta}$ as a function of $\theta$ are presented in Figure 2.
[9] As seen in Figure 2a [see also Krider, 1992, 1994], both the MTLE model at $(t-R / c)=0$ and the TL model predict that the maximum field is radiated along the ground surface $\left(\theta=90^{\circ}\right.$ ) for relatively low speeds ( $\beta \leq 0.707$ ), and, as the speed increases, the maximum field occurs at progressively smaller polar angles. Clearly, exponential current attenuation with height in the MTLE model results in a reduction in the electric field intensity after the initial peak. The field reduction appears to be more pronounced for larger $\beta$ and smaller polar angles (see Table 1), since both these factors lead to a greater radiating channel length and
hence smaller apparent current at the front, $I_{u}$ (see equations $\left(2^{\prime}\right),\left(2^{\prime \prime}\right)$, and ( $\left.2^{\prime \prime \prime}\right)$ ). Experimental evidence of current attenuation with height is presented in section 3.3.

### 3.2. Inferences for the Production of Elves

[10] Electromagnetic field pulses produced by lightning return strokes can cause breakdown of the neutral atmosphere at altitudes ranging from about 70 to 95 km where the gas density is considerably reduced (so that the electron mean free path is considerably increased) relative to that at ground level [e.g., Inan et al., 1991; Krider, 1994; Rowland et al., 1996]. This breakdown (heating of the electrons by the lightning electromagnetic pulse) is thought to be responsible for the transient optical emissions (enhanced airglow) observed at those altitudes and termed elves [Fuknunishi et al., 1996]. According to Fernsler and Rowland [1996], the breakdown first occurs at about 95 km , where the gas density is about $4 \times 10^{13} \mathrm{~cm}^{-3}$ (the electron mean free path is of the order of 1 m ) and the minimum field needed to increase the electron density by more than two e-folds (their definition of

breakdown) is about $15 \mathrm{~V} / \mathrm{m}$, with the field pulse duration being about $100 \mu \mathrm{~s}$. Using the critical value of field, $15 \mathrm{~V} / \mathrm{m}$, and integrating the corresponding Poynting vector for the TL model over the first $100 \mu \mathrm{~s}$, we obtained the value of energy density of $60 \mu \mathrm{~J} / \mathrm{m}^{2}$ needed for the production of elves.
[11] Elves have a duration of some hundreds of microseconds and a lateral extent of some hundreds of kilometers. They are usually associated with larger return strokes of either polarity. Fuknunishi et al. [1996] reported that the majority of their detected elves were associated with large positive lightning flashes. For their "typical example", the lightning peak current reported by the U.S. National Lightning Detection Network (NLDN) was +326 kA , and for their only event associated with a negative flash the peak current was -131 kA. Further, Inan et al. [1997] reported on two elves for which the corresponding lightning peak currents were +150 and +120 kA . Finally, Barrington-Leigh and Inan [1999] found that out of 39 events they identified as elves, $31 \%$ were associated with negative flashes. In their sample of 73 flashes with peak currents over $38 \mathrm{kA}, 38$ ( $52 \%$ ) had accompanying elves, and this fraction increased to $73 \%$ and $100 \%$ for peak currents exceeding 45 and 57 kA , respectively.
[12] In the following, we discuss combinations of return stroke current and speed needed for the production of elves according to the TL and MTLE models. It was assumed that elves are produced when the energy density (Poynting vector, $S=E_{\theta}^{2} / Z_{0}$, integrated over the first $100 \mu \mathrm{~s}$ ) at an altitude of 95 km exceeds a critical value of $60 \mu \mathrm{~J} / \mathrm{m}^{2}$ (see above). We expressed $R$ (radial distance from the origin of coordinates) as $H / \cos \theta$, where $H$ is constant altitude equal to 95 km , in equations (2) and (3) used to obtain the energy density expressions for the MTLE and TL models, respectively. We neglected any interaction of the lightning-generated electromagnetic pulse with the ionosphere, similar to the studies of Krider [1992, 1994] and Wait [1998]. Thus the current values given below should be viewed as lower limits. Results are shown in Figures 3a and 3b for four different values of return stroke speed, $0.3 c, 0.5 c, 0.9 c$, and $0.99 c$.
[13] It follows from Figure 3a that for the TL model and typical return stroke speeds of $0.3 c$ and $0.5 c$, the return stroke current as a function of polar angle exhibits minima, 151 and 82 kA , for polar angles of $44^{\circ}$ and $41^{\circ}$ (at horizontal distances of 91 and 83 km from the lightning channel), respectively. For higher return stroke speeds, the minimum current is smaller and occurs for smaller polar angles. For a relatively high speed of $0.9 c$, the minimum current is 23 kA for a polar angle of $24^{\circ}$ (a horizontal distance of 41 km ). If the speed were equal to $0.99 c$, the minimum current would be 7 kA , and the corresponding polar angle would be $8^{\circ}$ (a horizontal distance of 13 km ), implying that the majority of first and subsequent strokes would produce elves, which is apparently not the case. For a

Figure 2. (opposite) Polar plots of $E_{\theta}$ as a function of $\theta$ for $I=I_{0} u(t-R / c), I_{0}=30 \mathrm{kA}, R=100 \mathrm{~km}$, and five values of $\beta=v / c, 0.3,0.5,0.7,0.9$, and 0.99 . Vertical axis corresponds to $\theta=0^{\circ}$ and horizontal axis to $\theta=90^{\circ}$. (a) The MTLE model, $\lambda=2 \mathrm{~km},(t-R / c)=0$, and the TL model; (b) the MTLE model, $\lambda=2 \mathrm{~km},(t-R / c)=0.1 \mu \mathrm{~s}$; (c) the MTLE model, $\lambda=2 \mathrm{~km},(t-R / c)=1 \mu \mathrm{~s}$.

Table 1. $E_{\theta}$ as a Function of $\theta$ and $\beta=v / c$ as Predicted by the TL and MTLE $(\lambda=2 \mathrm{~km})$ Models for $I=I_{0} u(t-R / c), I_{0}=30 \mathrm{kA}$, $R=100 \mathrm{~km}$, and Three Values of $(t-R / c), 0,0.1$, and $1 \mu \mathrm{~s}$

| $\underline{\beta=v / c}$ |  | $E_{\theta}, \mathrm{V} / \mathrm{m}$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\theta^{\sigma}$ | $\begin{gathered} \text { MTLE Model, } \\ (t-R / c)=0, \text { and } \\ \text { TL Model } \end{gathered}$ | MTLE Model, $(t-R / c)=0.1 \mu \mathrm{~s}$ | MTLE Model, $(t-R / c)=1 \mu \mathrm{~s}$ |
| 0.3 | 10 | 1.0 | 1.0 | 0.97 |
| 0.3 | 30 | 2.9 | 2.9 | 2.8 |
| 0.3 | 60 | 4.8 | 4.8 | 4.6 |
| 0.3 | 90 | 5.4 | 5.4 | 5.2 |
| 0.5 | 10 | 2.1 | 2.0 | 1.8 |
| 0.5 | 30 | 5.5 | 5.5 | 5.0 |
| 0.5 | 60 | 8.3 | 8.2 | 7.6 |
| 0.5 | 90 | 9.0 | 8.9 | 8.3 |
| 0.7 | 10 | 4.2 | 4.0 | 3.1 |
| 0.7 | 30 | 10.0 | 9.7 | 8.0 |
| 0.7 | 60 | 12.4 | 12.3 | 10.9 |
| 0.7 | 90 | 12.6 | 12.5 | 11.3 |
| 0.9 | 10 | 13.1 | 11.7 | 4.5 |
| 0.9 | 30 | 20.6 | 19.5 | 12.1 |
| 0.9 | 60 | 17.6 | 17.2 | 14.4 |
| 0.9 | 90 | 16.2 | 16.0 | 14.2 |
| 0.99 | 10 | 62.6 | 34.9 | 0.89 |
| 0.99 | 30 | 33.6 | 30.5 | 13.2 |
| 0.99 | 60 | 20.4 | 19.9 | 16.1 |
| 0.99 | 90 | 17.8 | 17.6 | 15.4 |

typical negative first-stroke current of 30 kA , elves would be produced only if the return stroke speed were greater than about $2.5 \times 10^{8} \mathrm{~m} / \mathrm{s}$.
[14] Results for the MTLE model (see Figure 3b) suggest that, compared to the TL model, considerably larger currents are required for the production of elves. Even for the highest return stroke speeds (approaching the speed of light) considered here the minimum current is about 200 kA . This possibly indicates that the current attenuation distance, $\lambda=$ 2 km, suggested by Nucci et al. [1988] and adopted here for the MTLE model is too small. A larger value of $\lambda$ would yield a lower (more realistic) minimum current required for the production of elves. For example, for $\lambda=6 \mathrm{~km}$ the minimum current is about 110 kA .

### 3.3. Experimental Evidence of Current Attenuation With Height

[15] Return stroke peak current attenuation with height above ground for negative subsequent strokes in natural lightning has been inferred from return stroke luminosity profiles by Jordan and Uman [1983] and Jordan et al. [1992, 1995]. Because of the lack of branches, the light profile along a subsequent return stroke channel is relatively simple, usually showing gradual intensity decay with height. Jordan and Uman [1983] found for seven subsequent return strokes an exponential decrease of the luminosity peak with height with a decay constant of $0.6-0.8 \mathrm{~km}$, resulting in a luminosity peak decrease at a height of 480 m to $45-55 \%$ of its value at ground and to $9.7-17 \%$ at 1400 m . Further, Jordan et al. [1995] reported, from a different experiment, a decrease to $33 \%$ at 600 m and to $19 \%$ at 1100 m for one subsequent return stroke. Finally, from their analysis of the light profiles of three dart leader/return stroke sequences, Jordan et al.
[1997] found that in two out of three events the return stroke luminosity peak at 480 m and 1400 m decayed to, respectively, $70-75 \%$ and $25-30 \%$ of its value at the bottom of the channel and in the third event to, respectively, $90-95 \%$ and about $70 \%$ of the channel-bottom value. Wang et al. [1999], using the ALPS optical imaging system, observed a considerable decrease in the light pulse peak within the first tens of meters of propagation for return strokes in triggered lightning. Wang et al. [2000], also using ALPS, observed considerable attenuation and dispersion of the light pulses associated


Figure 3. Return stroke current as a function of polar angle needed to produce elves for four values of $\beta=v / c, 0.3$, $0.5,0.9$, and 0.99 . (a) The TL model; (b) the MTLE model, $\lambda=2 \mathrm{~km}$. Note that there is singularity at $\theta=0$ when $\beta=1$ since the current wave front arrives at the observation point at the same instant as the electromagnetic signal from this wave front (both travel at the speed of light).
with two negative first return strokes in natural lightning, as these pulses propagated along the branched channels.

## 4. Summary

[16] 1. At early times, when $(t-R / c) \ll \lambda / v_{u}$ where $v_{u}$ depends on $\beta$ and $\theta$, the MTLE model is indistinguishable from the TL model. At later times, the exponential current attenuation with height in the MTLE model results in a considerable reduction in the electric field intensity relative to that predicted by the TL model. The field reduction appears to be more pronounced for larger $\beta$, and smaller polar angles, both these factors leading to a smaller apparent current at the upward-propagating return stroke front.
[17] 2. For the TL model and typical return stroke speeds of $0.3 c$ and $0.5 c$, the return stroke current needed to produce elves as a function of polar angle exhibits minima, 151 and 82 kA , for polar angles of $44^{\circ}$ and $41^{\circ}$, respectively. These minimum currents appear to be consistent with observations of elves and their associated lightning return stroke currents found in the literature. For a typical negative first-stroke current of 30 kA , elves would be produced only if the return stroke speed were greater than about $2.5 \times 10^{8} \mathrm{~m} / \mathrm{s}$.
[18] 3. Besides the return stroke current and propagation speed, the likelihood of the production of elves can be also influenced by the current attenuation with height. Results for the MTLE model with $\lambda=2 \mathrm{~km}$ suggest that, even for the highest return stroke speeds, the minimum current required for the production of elves is about 200 kA .
[19] Acknowledgments. This research was supported in part by NSF grant ATM-0003994. The authors would like to thank B. A. DeCarlo for his help with calculations and E. P. Krider, M. A. Uman, and three anonymous reviewers for their comments and suggestions that helped us considerably improve the manuscript.

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