JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 108, NO. D17, 4542, doi:10.1029/2003JD003398, 2003

Application of the antenna theory model to a tall tower struck by lightning

Behzad Kordi,[1,2] Rouzbeh Moini,[3] Wasyl Janischewskyj,[4] Ali M. Hussein,[5,6]
Volodymyr O. Shostak,[7] and Vladimir A. Rakov[8]

Received 10 January 2003; revised 25 March 2003; accepted 21 May 2003; published 9 September 2003.

[1] The interaction of lightning with the 553-m high CN Tower in Toronto is modeled using the antenna theory model. A simple lossless wire structure is used to represent the tower. The return-stroke channel is modeled as a lossy vertical antenna attached to the tower top. The lossy antenna and the wire structure representing the tower are assumed to be fed at their junction point by a voltage source. The voltage waveform of this source is selected so that the source current resembles a typical lightning current waveform not influenced by the presence of the tall strike object. An electric field integral equation in the time domain is employed to calculate the lightning return stroke current distribution along the CN Tower and along the lightning channel. The equation is solved numerically using the method of moments. The lightning current flowing in the tower at the 474-m level above ground, predicted by the antenna theory (AT) model, compares favorably with the measurements conducted at the CN Tower. Once the temporal and spatial distributions of the current along the tower and along the lightning channel are determined, the corresponding remote electromagnetic fields are computed. Waveshapes of model-predicted electric and magnetic fields at a distance of 2 km from the tower are in good agreement with measurements. The contribution of the tower to the electric and magnetic fields at 2 km is about four to five times the contribution of the lightning channel. *INDEX TERMS:* 3304 Meteorology and Atmospheric Dynamics: Atmospheric electricity; 3324 Meteorology and Atmospheric Dynamics: Lightning; 3367 Meteorology and Atmospheric Dynamics: Theoretical modeling; *KEYWORDS:* lightning, atmospheric electricity, theoretical modeling

Citation: Kordi, B., R. Moini, W. Janischewskyj, A. M. Hussein, V. O. Shostak, and V. A. Rakov, Application of the antenna theory model to a tall tower struck by lightning, *J. Geophys. Res.*, *108*(D17), 4542, doi:10.1029/2003JD003398, 2003.

1. Introduction

[2] Direct lightning strikes to instrumented towers have been an important source of information on the lightning flash parameters. However, the presence of a vertically extended and grounded strike object in the lightning path can appreciably influence the return-stroke current waveforms and consequently the radiated electromagnetic fields. Lightning discharges to tall structures have recently received considerable attention (see *Rakov* [2001] for review), and several models have been proposed in the literature for the determination of the current distribution along the lightning current path (the tall structure and the lightning channel) [*Janischewskyj et al.*, 1996, 1998, 1999; *Motoyama et al.*, 1996; *Guerrieri et al.* 1998; *Rachidi et al.*, 2001, 2002; *Baba and Ishii*, 2001]. These models usually assume a current source located at the tip of the strike object, and the propagation of the lightning current wave is traced within the object and along the lightning channel, both represented by transmission lines with proper reflection coefficients at the extremities of the object. However, *Rachidi et al.* [2002] used a distributed-source representation of the lightning channel, and *Baba and Ishii* [2001] applied an electromagnetic model.

[3] *Janischewskyj et al.* [1996] estimated the reflection coefficients from currents measured at the CN Tower and associated modeling. They suggested that reflection coefficient values were apparently influenced by the front risetime of the measured current, which implies that the reflection coefficients may be functions of frequency. On the other hand, *Fuchs* [1998, Figures 10 and 11] found that current reflection coefficients at the bottom and at the top of the 160-m Peissenberg tower in Germany were apparently

[1]Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg, Manitoba, Canada.
[2]Formerly at the Department of Electrical Engineering, Amirkabir University of Technology, Tehran, Iran.
[3]Department of Electrical Engineering, Amirkabir University of Technology, Tehran, Iran.
[4]Department of Electrical and Computer Engineering, University of Toronto, Toronto, Ontario, Canada.
[5]Department of Electrical and Computer Engineering, Ryerson University, Toronto, Ontario, Canada.
[6]Also with the Department of Electrical and Computer Engineering, University of Toronto, Toronto, Ontario, Canada.
[7]Department of Electrical Engineering, Kyiv Polytechnic Institute, Kyiv, Ukraine.
[8]Department of Electrical and Computer Engineering, University of Florida, Gainesville, Florida, USA.

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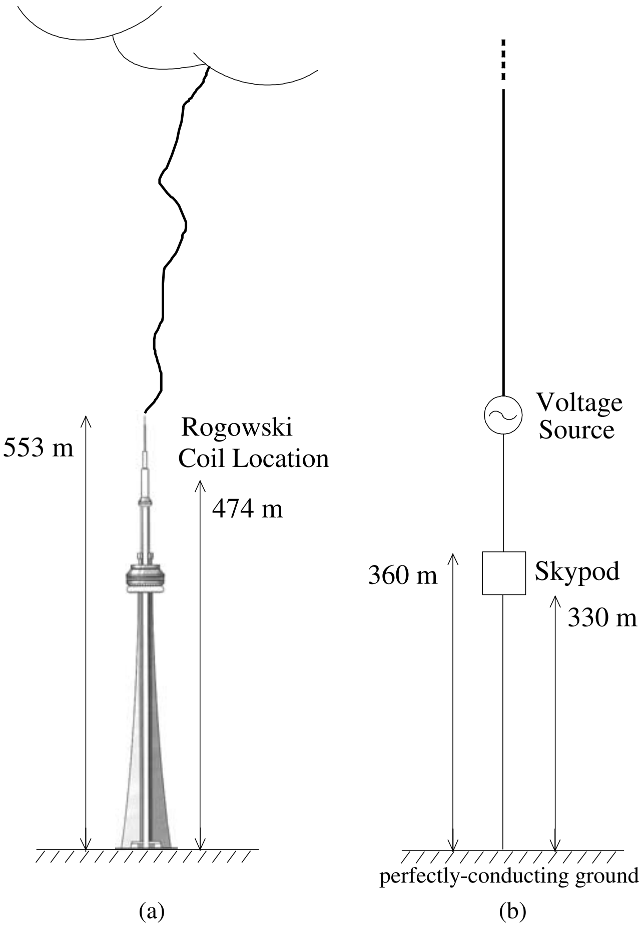

Figure 1. (a) The CN Tower and (b) its wire model.

independent of either lightning current peak or maximum current rate of rise. Using CN Tower measurements at two different heights, *Bermudez et al.* [2001] have derived an expression for calculating the reflection coefficient as a function of frequency at the bottom of the tower. The effects of current reflections within the tower on the remote electromagnetic field have been studied by *Rachidi et al.* [2001]. They only considered the reflections of the current wave at the tower extremities, while there are more reflections in a complex tall object such as the CN Tower [*Janischewskyj et al.*, 2001]. Additionally, reflections can occur at the upward-propagating return-stroke front [*Shostak et al.*, 1999, 2000]. The use of the transmission line modeling approach requires the determination of reflection coefficients at the extremities of the strike object and at its major internal structural discontinuities, which is not always an easy task.

[4] In this paper, the Antenna Theory (AT) model, which has been used to simulate the lightning return stroke initiated at ground level [*Moini et al.*, 1997, 2000; *Kordi et al.*, 2002], is generalized to take into account the presence of a vertically extended strike object. This generalized AT model, similar to the original AT model, is based on solving the electric field integral equation (EFIE) in the time domain using the method of moments (MOM) [*Moini et al.*, 1998]. A sketch of the CN Tower struck by lightning and its representation in the AT model are shown in Figures 1a and 1b, respectively. The tower is represented by perfectly conducting vertical wires in which the wider part of the tower, the "Skypod", is simulated by a square wire loop (two-dimensional) inserted between two vertical wires. This wire loop is labeled "Skypod" in Figure 1b. Such representation of the tower is grossly simplified, but it allowed us to reproduce all the major reflections identified in the experimental data (see section 3). A perfectly conducting ground is assumed. The lightning return-stroke channel is modeled as a lossy vertical wire antenna, neglecting the effect of corona which was considered by *Moini et al.* [2000]. The resistance per unit channel length is taken as 0.07 Ω/m (the same value as that used by *Moini et al.* [2000] to reproduce very close electric field waveforms). The propagation speed of the current wave along the lightning channel is essentially equal to that along the tower which is equal to the speed of light. Although the return-stroke speed in the model used here is a factor of two to three higher than typically measured values, it has no effect on the distribution of current along the tower and will be shown to have relatively little effect on the fields 2 km away from the tower that are largely determined by currents in the tower. Further, our essentially identical wire representation of the lightning channel (resistive loading is rather small) and of the top portion of the tower does not facilitate reflections at the tower top. Such reflections are expected, but have not been identified in the experimental data (see Figure 2). The

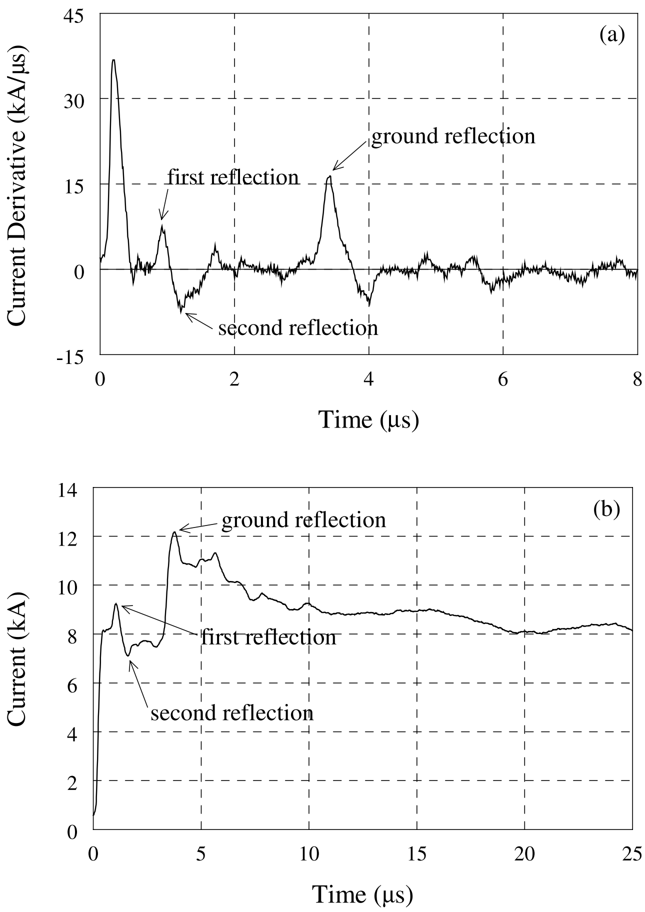

Figure 2. Typical lightning return-stroke (a) current derivative and (b) current measured on the CN Tower.

lightning return-stroke current is injected by a voltage source at the tip of the tower (see Figure 1b). The voltage source waveform is chosen so as to result in a typical CN Tower lightning current waveform not influenced by reflections within the tower. The current waveform at any specified segment along the current path (the tower and the return stroke channel) is found by solving an electric field integral equation (details of this solution are found in the work of *Moini et al.* [2000]). The resultant spatial and temporal distribution of the current is used to calculate the lightning-generated electromagnetic fields. Model-predicted current and field waveforms are compared with measurements. The current derivative waveforms used in the comparison were recorded by a Rogowski coil, placed at the 474-m level of the CN Tower, while the vertical component of the electric field and the azimuthal component of the magnetic field were measured 2 km north of the tower [*Hussein et al.*, 1995]. Further, the AT model is used to determine the contributions of the tower and the return-stroke channel to the total electromagnetic fields at 2 km.

2. Theory

[5] The response of conducting bodies, such as a wire structure, illuminated by an electromagnetic pulse is in fact equivalent to the diffraction of electromagnetic waves by metallic obstacles. The use of Maxwell's equations for this problem leads to time-space integral equations, which are to be satisfied by the currents induced in the structure.

[6] Let an incident electromagnetic wave illuminate a metallic object which is located above a perfectly conducting ground. This wave induces a current in the wire structure which will, in turn, produce a scattered field. The use of thin wire approximation results in an integro-differential equation by which the scattered field, $\mathbf{E^s}$, is linked to the current, I, induced in the wire structure. This relation can be expressed as [*Moini et al.*, 1998; *Kordi et al.*, 1998; *Miller et al.*, 1973]:

$$\mathbf{E^s}(\mathbf{r}, t) = \Gamma[I(r', t)]. \tag{1}$$

In this equation, $\mathbf{r}$ is the position vector that defines the location of the observation point, $\mathbf{r}'$ is the position vector that defines the location of the source, and $\mathbf{\Gamma}$ stands for the integro-differential operator derived from Maxwell's equations. The expression for $\mathbf{\Gamma}$ used in this study appears on the right-hand side of (15) of *Moini et al.* [2000]. The total electric field at any point in space is the sum of the applied and the scattered fields:

$$\mathbf{E^t} = \mathbf{E^a} + \mathbf{E^s}. \tag{2}$$

Applying the boundary condition on the tangential component of the electric field at the wire surface results in an equation relating the applied field to the induced current in the wire structure, as will be discussed next. The boundary condition for the case of a perfectly conducting wire can be expressed as:

$$\mathbf{s} \cdot (\mathbf{E^a} + \mathbf{E^s}) = 0, \tag{3}$$

where $\mathbf{s}$ is a unit vector tangential to the wire structure at any point on its surface and parallel to the wire axis [*Moini et al.*, 1998]. Substituting $\mathbf{E^s}$ ($\mathbf{r}$,t) given by (1) in (3) and rearranging terms, we obtain the following equation:

$$\mathbf{s} \cdot \mathbf{E^a} = -\mathbf{s} \cdot \Gamma[I(\mathbf{r}', t)]. \tag{4}$$

[7] Equation (4) is the electric field integral equation whose solution yields the induced current in the wire structure illuminated by an electromagnetic wave. The method of moments (MOM) has been applied to solve (4) [*Moini et al.*, 1998].

[8] For the transmitting antenna, which is the case being considered here, the applied field $\mathbf{E^a}$ is zero everywhere except for the segment at which the exciting voltage source is located. The following equation relates the applied field at the exciting segment and the corresponding source voltage:

$$\mathbf{E^a} = -\nabla V(\mathbf{r_o'}, t) = -(V/\Delta z)\hat{\mathbf{a}}_\mathbf{z}, \tag{5}$$

where $\mathbf{r_o'}$ is the position vector that defines the location of the exciting segment. It was assumed that $\mathbf{E^a}$ has only a vertical component, and its magnitude was determined as the ratio of the source voltage, V, and segment length, Δz, that is, the source was represented by the so-called delta-gap generator [e.g., *Balanis*, 1997]. Equation (4) can be easily modified in the MOM procedure to account for finite conductivity of some elements of the wire structure, as done for the lightning channel by *Moini et al.* [2000]. A more detailed description of the AT model is found in the work of *Moini et al.* [2000].

3. Measured Current Derivative, Current, and Field Waveforms

[9] The overwhelming majority of lightning discharges to the CN Tower are of the upward-initiated type, and therefore the presented experimental data are characteristic of subsequent (as opposed to first) strokes in downward-initiated lightning. All lightning events considered here transferred negative charge to ground through the tower. In order to discuss and interpret the overall structure of current and field waveforms typically observed at the CN Tower, we will consider relatively large lightning events for which the structure to be examined is most pronounced. Since we do not have correlated current and field records readily available for such events, we will use two different events for this purpose. The first one is represented by its current and current derivative in Figure 2, and the second one by its electric and magnetic fields in Figure 3. Then, for a comparison of model-predicted fields with measurements (see section 4) we will use a third, relatively small lightning event for which we do have correlated current and field measurements.

[10] A typical (in terms of waveshape) lightning return-stroke current derivative observed at the CN Tower and its corresponding current are presented in Figures 2a and 2b, respectively. The current derivative waveform was measured, while the current waveform was obtained by integrating the current derivative waveform numerically. Typical (in terms of waveshape) electric and magnetic field waveforms, measured for a different event, at a distance of 2 km north of the CN Tower are shown in Figures 3a and 3b, respectively. Current, current derivative, and field wave-

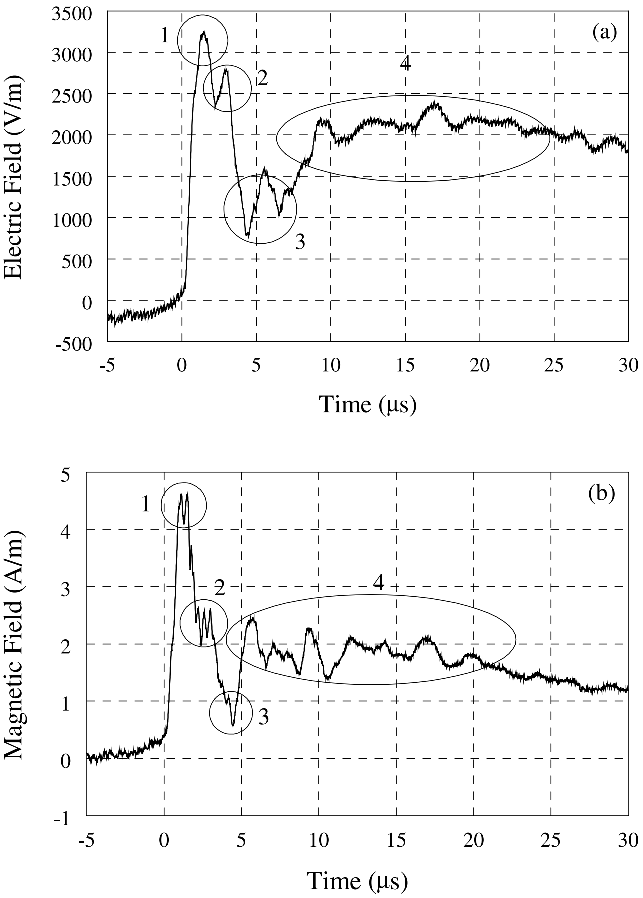

Figure 3. Typical lightning return-stroke (a) electric and (b) magnetic fields measured 2 km from the CN Tower. These fields do not correspond to the current shown in Figure 2b. 1, main peak; 2, second peak; 3, lowest point after the main peak; 4, plateau with oscillations.

forms have rather complex structure, as will be discussed next. In records of current and its derivative (see Figure 2), after the initial current rise and corresponding peak in current derivative, three pronounced features, related to major current reflections within the tower, are typically observed. The time analysis and interpretation of reflections labeled in Figure 2 is given in the work of *Janischewskyj et al.* [1997]. The first reflection is the consequence of the sudden increase in the diameter of the tower at the 360-m level (top of the Skypod). The second reflection (observed as a drop in the current and as a current derivative peak of opposite polarity relative to the polarity of the previous reflection) is due to the decrease of the tower's diameter at the 330 m level (bottom of the Skypod). Finally, the third one indicates the "ground reflection", occurring when the downward propagating current wave is reflected at the bottom of the tower and the upward-moving wave arrives at the current measurement point.

[11] Electric and magnetic field waveforms (see Figure 3) have even more complicated structure than that of the current, probably due to the influence of channel geometry. While the fields were measured at 2 km, their signatures look somewhat similar to those typically observed at larger distances (some tens of kilometers) [*Lin et al.*, 1979]. There are also some specific features in both the electric and magnetic field records (see Figure 3). The main peak (labeled "1") dominates, and sometimes it may have some structure (a few sub-peaks). On the falling part following the main peak, one can often observe the second peak (labeled "2") which in many records also has multiple sub-peaks. Furthermore, the lowest point after the main peak (labeled "3") of each of the field waveforms is observed within approximately 5 μs after the beginning of the waveform; sometimes it is associated with a zero-crossing and appears as a peak of opposite polarity. Finally, a "plateau" part with oscillations (labeled "4") is usually observed. For the electric field waveform this plateau usually follows a relatively slow-rise ramp, while for the magnetic field it follows a faster rise and decays faster than the electric field plateau. The described features of observed current, current derivative, and electric and magnetic filed waveforms will be compared, using a different lightning event, in section 4 with predictions of the AT model used to simulate lightning strikes to the CN Tower.

4. Application of the AT Model to Lightning Strikes to the CN Tower

[12] We first present computed current derivative and current waveforms at the 474-m level of the CN Tower

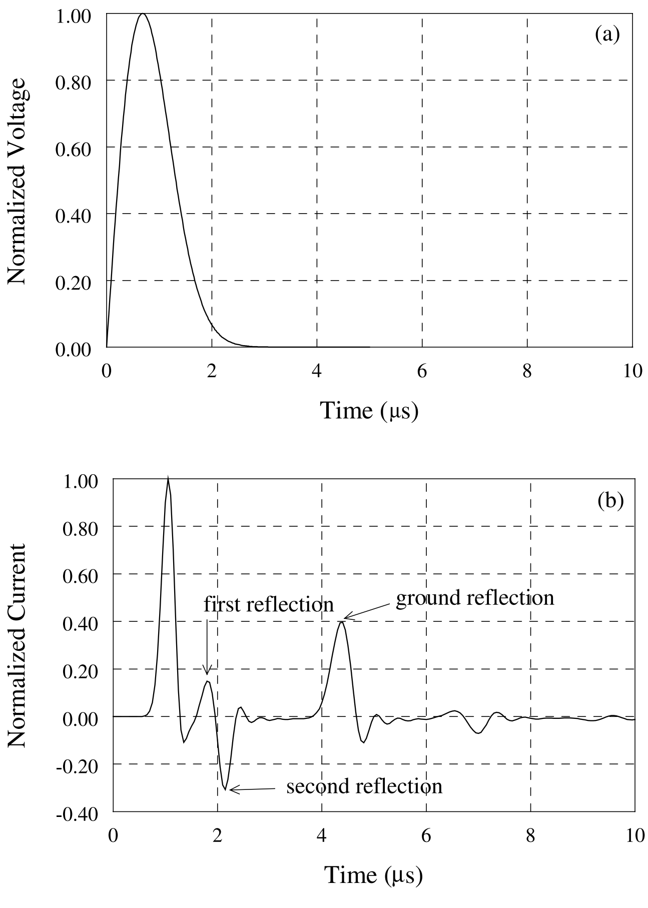

Figure 4. (a) The Gaussian pulse used as the voltage waveform and (b) the resultant normalized current waveform (predicted by the AT model) in the segment located at the 474-m level of the CN Tower.

and then computed electric and magnetic field waveforms at a distance of 2 km from the tower.

4.1. Computation of Current in the Tower

[13] As stated in the Introduction, the CN Tower is represented by a thin-wire structure and the return stroke channel by a lossy wire antenna, with the exciting voltage source being located at the tower tip. Once the waveform of this voltage source is specified, the current distribution along the lightning channel and along the tower is determined by solving the electric field integral equation using the method of moments. The wire structure representing the tower and the simulated lightning channel were divided into segments of 5 m in length. All segments had a radius of 10 cm.

[14] As a first step, in order to find the response of the wire structure shown in Figure 1b, a narrow Gaussian pulse shown in Figure 4a was employed as the voltage waveform of the source. Figure 4b shows the resultant normalized current waveform at the segment located at the 474-m level of the CN Tower at which the waveform shown in Figure 2a was measured. Similar to Figure 2a, three major reflections are seen in Figure 4b. The first is due to the structural discontinuity at the top of the Skypod, and the cause for the second one is the structural discontinuity at the bottom of the Skypod. The most pronounced reflection is from the ground. The occurrence times (at 474 m) of the initial peak and of the three reflections are listed in Table 1. The computed round-trip times were found by measuring the time interval between each reflection and the initial peak in Figure 4b, while the expected round-trip times were found using known distances between the 474-m level and the structural discontinuities and the speed of light. Although the 5-m segment length, corresponding to the propagation time of 0.017 μs, used in the AT model limits the accuracy of the calculated times, they are very close to expected times.

[15] Now, we consider the measured current derivative and current waveforms (as previously noted, current was obtained by integrating the current derivative numerically), shown in Figures 5a and 5b, respectively, of an event that occurred in July 1999. The corresponding measured electric and magnetic fields are shown in Figures 6a and 6b, respectively. In order to apply the AT model, we assume a current waveform at the lightning channel base that is not influenced by the strike object. The source current waveform was specified to match the initial rising portion of the measured current waveform, with the tail portion being set

Table 1. Propagation Times Within the CN Tower

Features of Current Derivative Waveform (Figure 4b)	Occurrence Time (μs) at 474 m (From Figure 4b)	Round-Trip Time 2τ (μs): Antenna Theory Model (From Figure 4b)	Round-Trip Time 2τ (μs): Transmission Line Model ($2\tau = 2\,\ell/c$)[a]
First peak	1.05	-	-
First reflection	1.80	0.75	0.76
Second reflection	2.15	1.10	1.13
Ground reflection	4.40	3.35	3.36

[a] ℓ is the distance from the 474-m level to a structural discontinuity, ℓ = 114 m for the top of the Skypod, ℓ = 174 m for the bottom of the Skypod, and ℓ = 504 m for ground (current paths along the roof and bottom of the Skypod are included).

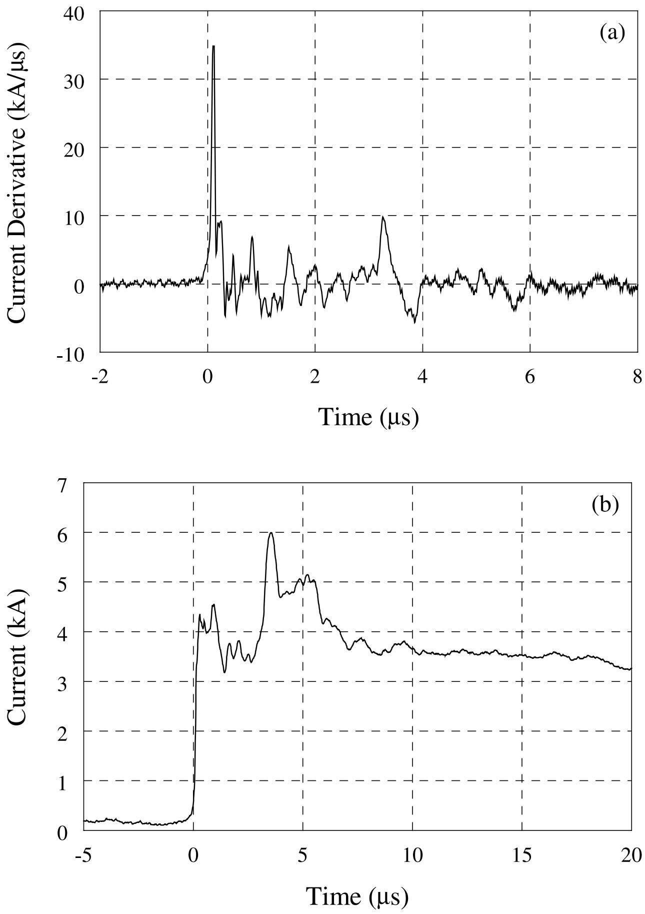

Figure 5. Lightning return-stroke (a) current derivative and (b) current of an event observed at the CN Tower in July 1999.

to match that of the measured current waveform disregarding the multiple reflections. The current and current derivative peaks are 4.7 kA and 25 kA/μs, respectively. This waveform, specified using well-known Heidler's formula [*Heidler et al.*, 1999] and depicted in Figure 7, was used as the source current. The source voltage was determined by the current waveform shown in Figure 7 and by the input impedances of the lightning channel and the tower (for more details see *Moini et al.* [2000]). The solution of the electric field integral equation gives current as a function of time and height above ground for both the tower and the lightning channel, this current being then used as the input to the field calculation procedure. Figure 8 depicts the calculated current derivative and current waveforms 474 m above ground, to be compared with measurements shown in Figure 5.

[16] A comparison of the calculated and measured currents reveals that, despite the simplicity of the model, there exists a reasonable agreement between the measured and calculated current waveforms. In particular, the model is capable of reproducing the major reflections which are seen in the measurements.

4.2. Computation of Electric and Magnetic Fields

[17] The current distributions in the tower and in the lightning channel are used to compute the remote electro-

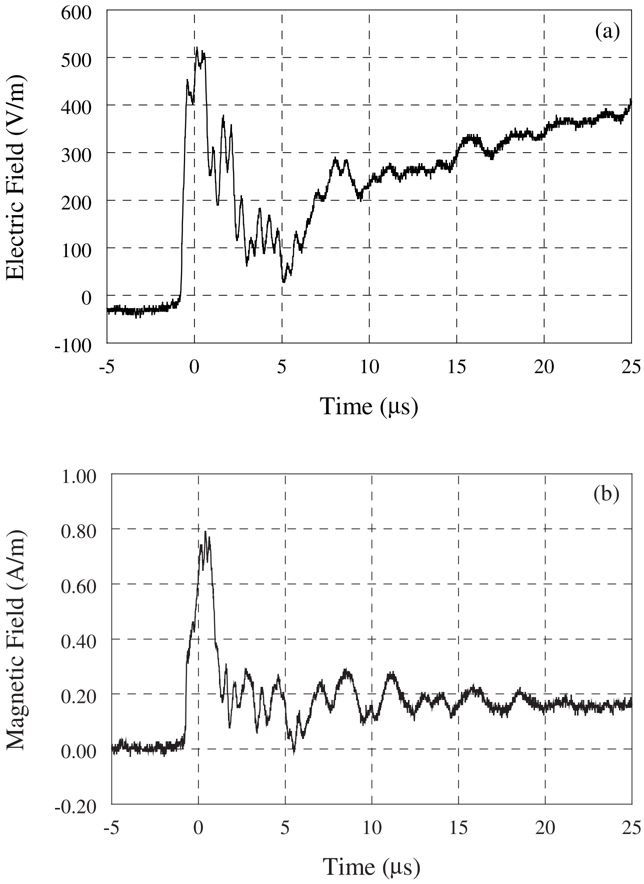

Figure 6. Lightning return-stroke (a) electric and (b) magnetic fields measured 2 km north of the CN Tower corresponding to measured current derivative and current shown in Figure 5.

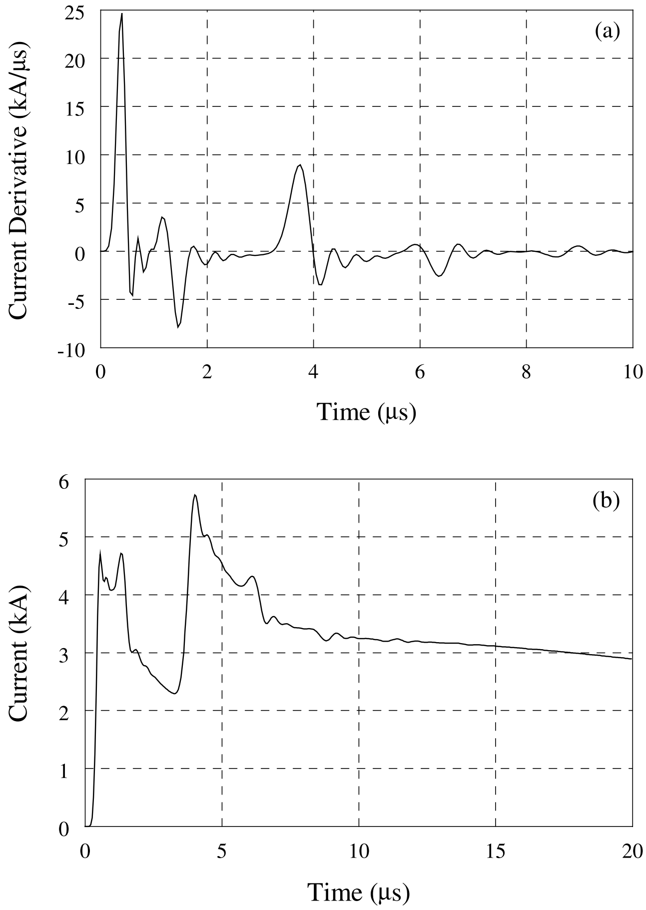

Figure 8. The calculated (a) current derivative and (b) current waveforms at the 474-m level of the CN Tower. The corresponding fields at 2 km are shown in Figures 9 through 11.

magnetic fields. Consider a segment of the wire of length dz' located at height z' from the ground and oriented along the z axis, whose current is $I(z', t)$. The vertical component of the electric field and the azimuthal component of the magnetic field at a point (r, φ, z) are related to the current of that segment by (6) and (7) [*Master and Uman*, 1983]:

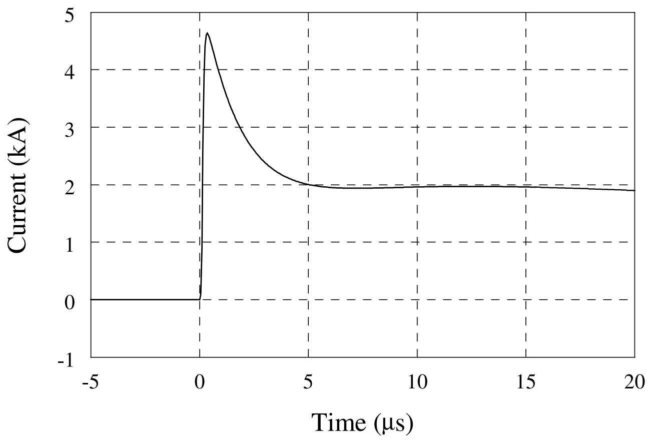

Figure 7. The source current waveform specified using Heidler's function [*Heidler et al.*, 1999], that was used for calculating the current and fields presented in Figures 8 through 11.

$$dE_z(r,\varphi,z,t) = \frac{dz'}{4\pi\varepsilon_0}\left[\frac{2(z-z')^2-r^2}{R^5}\int_0^t i(z',\tau - R/c)d\tau + \frac{2(z-z')^2-r^2}{cR^4} i(z', t-R/c) - \frac{r^2}{c^2R^3}\frac{\partial i(z',t-R/c)}{\partial t}\right], \quad (6)$$

$$dH_\varphi(r,\varphi,z,t) = \frac{dz'}{4\pi}\left[\frac{r}{R^3} i(z', t-R/c) + \frac{r}{cR^2}\frac{\partial i(z',t-R/c)}{\partial t}\right]. \quad (7)$$

[18] In (6) and (7), c is the speed of light in free space. The electric field is composed of three terms. These terms are related to the integral of the current, the current itself, and the current derivative and are called electrostatic, induction, and radiation terms, respectively. The magnetic field contains only two terms: magnetostatic (or induction) and radiation. Total electric and magnetic fields are found by integrating (6) and (7), respectively, along both the tower and the lightning channel. The presence of the ground is taken into account by using the image theory.

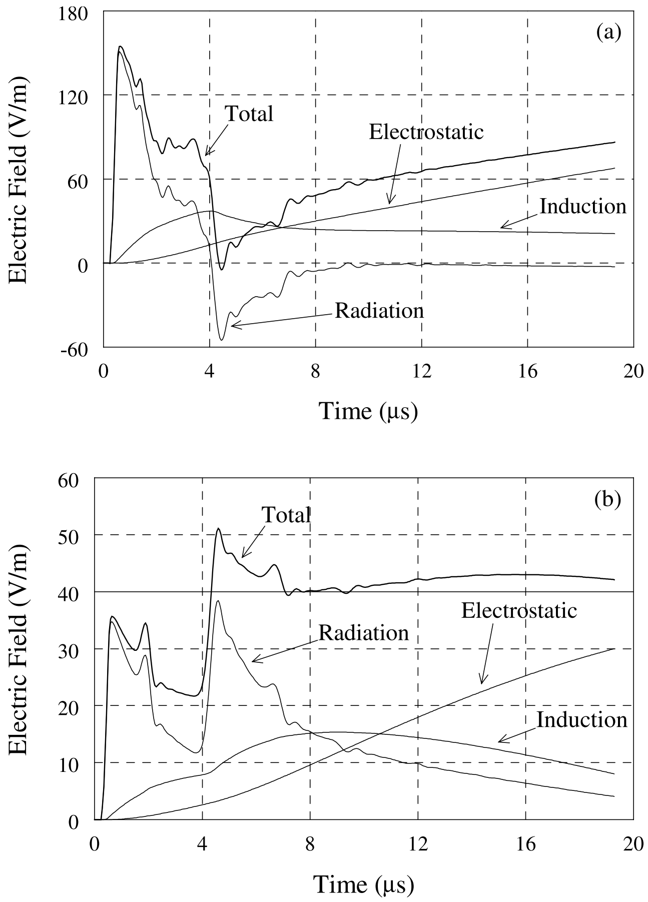

Figure 9. The calculated electric field at 2 km. (a) The tower contribution. (b) The channel contribution ($v = c$). The input current waveform is shown in Figure 7.

[19] Calculated electric and magnetic fields are shown in Figure 11. (The contribution of the horizontal wire segments of the tower model to the electric and magnetic fields has been ignored.) In Figures 9, the electric field has been computed at a distance of 2 km from the tower (the distance between the electric and magnetic field sensors and the CN Tower) at the ground level. In this figure, the electrostatic, induction, and radiation components of the contributions of the tower and of the channel are shown separately. is the same as Figure 9 but for the magnetic field. In Figure 11, the total electric and magnetic fields along with the tower and channel contributions are shown. Examining these figures, one can observe that there is a pronounced change in the fields at 4 to 5 μs. This field change is apparently associated with the first ground reflection and is subtractive for the tower contribution (Figures 9a and 10a) and additive for the channel contribution (Figures 9b and 10b).

5. Discussion

[20] There is a good agreement between the waveshapes of model-predicted and measured electric and magnetic fields. On the other hand, there are significant discrepancies in terms of field magnitude, the model-predicted fields being smaller than measured. Specifically, the mismatch is about 60% for the electric field peak and about 35% for the magnetic field peak. It is likely that these discrepancies are largely due to calibration errors (including the antenna enhancement factor for electric field; sensor was located on the roof of a building), which resulted in the measured fields being overestimated. Another possible reason is the fact that the computed current and current derivative peaks are somewhat smaller (by 10% and 25%, respectively) than measured ones.

[21] As seen in Figure 11, the contribution of the tower is about four (H) to five (E) times the contribution of the channel and accounts for about 80% of the total initial field peak. For a realistic return-stroke speed lower than the speed of light ($v < c$) the contribution from the lightning channel would be even smaller. According to *Janischewskyj et al.* [1999], who used an "engineering" model of the lightning return stroke, the contribution from the CN Tower to the total electric field at a distance of 2 km from the tower is about a factor of two greater than the contribution from the lightning channel. It is worth noting that for the "engineering" model relative contributions from the tower and from the channel depend on the assumed return-stroke speed.

6. Conclusions

[22] The antenna theory model for the lightning return stroke initiated at ground level is extended to analyze the case of lightning strikes to the CN Tower. In this approach,

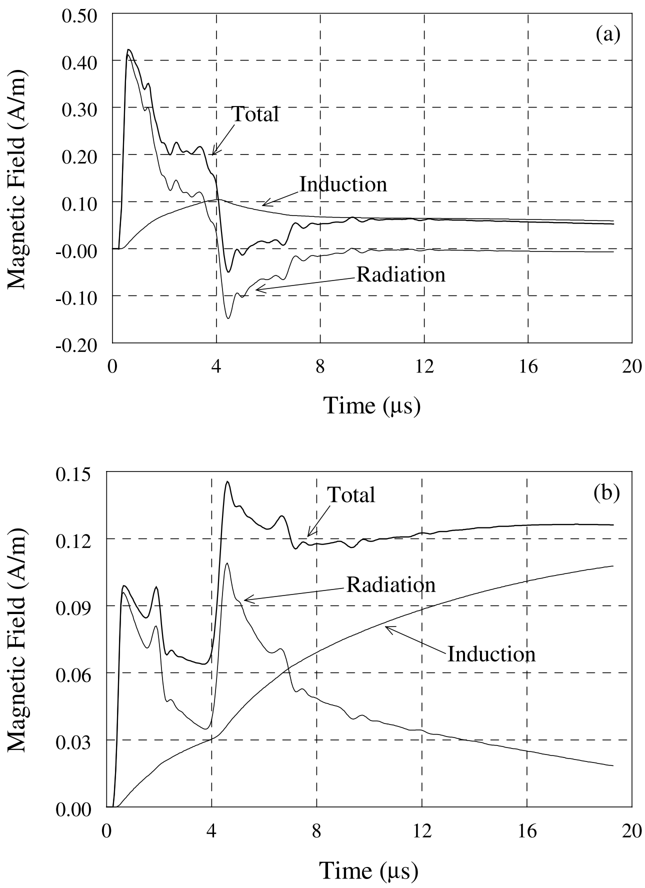

Figure 10. The calculated magnetic field at 2 km. (a) The tower contribution. (b) The channel contribution ($v = c$). The input current waveform is shown in Figure 7.

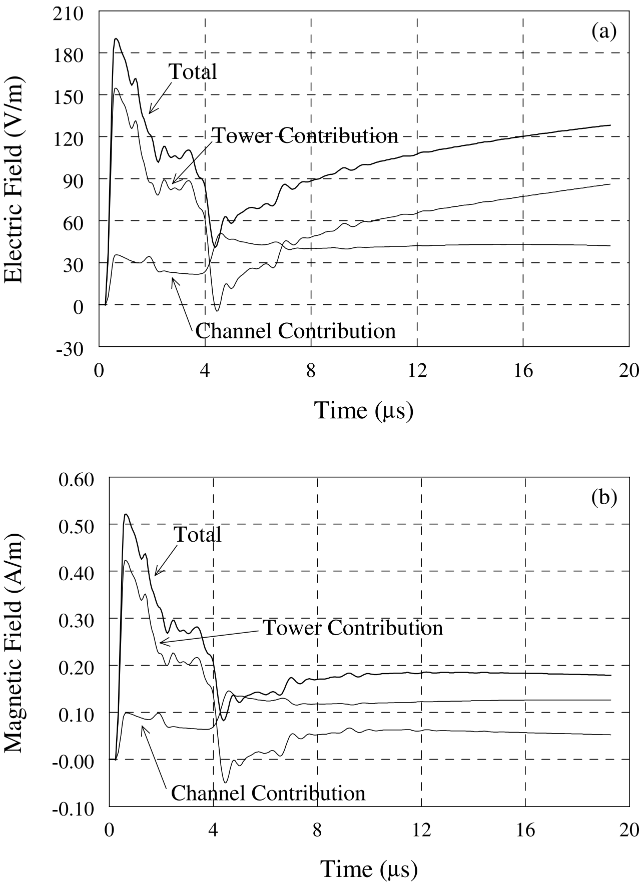

Figure 11. The total calculated (a) electric field and (b) magnetic field at 2 km and their components due to the tower and the lightning channel ($v = c$) contributions. The input current waveform is shown in Figure 7.

the tower is represented by a wire structure and the channel by a lossy vertical wire antenna fed at the tower tip by a voltage source. Solving the electric field integral equation yields the current as a function of time at each segment of the tower and the channel. Despite the simplicity of the model, the waveshapes of calculated current, current derivative, and electromagnetic fields compare well with those measured at the 474-m level of the CN Tower (for current) and at a distance of 2 km from the tower (for fields), although the field magnitudes are not well matched. The contribution from the tower to the electric and magnetic fields at 2 km is a factor of four to five (or more for a realistic return-stroke speed lower than the speed of light) greater than the contribution from the lightning channel.

[23] **Acknowledgments.** This research was supported in part by the Natural Sciences and Engineering Research Council of Canada, Amirkabir University of Technology, and the National Science Foundation (Grant ATM-0003994). The authors would like to thank Y. Baba, M.A. Uman, and three anonymous reviewers for a number of useful comments.

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A. M. Hussein, Department of Electrical and Computer Engineering, Ryerson University, 350 Victoria St., Toronto, ON M5B 2K3, Canada.

W. Janischewskyj, Department of Electrical and Computer Engineering, University of Toronto, 10 King's College Road, Toronto, ON M5S 3G4, Canada.

B. Kordi, Department of Electrical and Computer Engineering, University of Manitoba, 15 Gillson St., Winnipeg, MB R3T 5V6, Canada.

R. Moini, Department of Electrical Engineering, Amirkabir University of Technology, 424 Hafez Ave., Tehran 15914, Iran.

V. A. Rakov, Department of Electrical and Computer Engineering, University of Florida, Gainesville, Fl., USA.

V. O. Shostak, Department of Electrical Engineering, Kyiv Polytechnic Institute, KPI-1670, 37 Prospect Peremohy, Kyiv, 03056, Ukraine.