Applications of Electromagnetic Models of the Lightning Return Stroke

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Abstract—**Lightning return-stroke models are needed to study lightning effects on various objects and systems, as well as in characterizing the lightning electromagnetic environment. Reviewed here are models based on Maxwell's equations and referred to as electromagnetic models. In contrast to distributed-circuit and so-called engineering models, electromagnetic models of the lightning return stroke allow a self-consistent full-wave solution for both current distribution along the lightning channel and associated electromagnetic fields. In this paper, we review electromagnetic models with an emphasis on their applications.**

Index Terms—**Electromagnetic field, finite-difference time-domain method, lightning, lightning return-stroke model, method of moments.**

I. INTRODUCTION

LIGHTNING return-stroke models are needed to study lightning effects on various objects and systems and in characterizing the lightning electromagnetic environment. Clearly, conclusions drawn from these studies are influenced by the choice and validity of lightning return-stroke model employed. Rakov and Uman [1], based on governing equations, have categorized return-stroke models into four classes: 1) gas dynamic models, 2) electromagnetic models, 3) distributed-circuit models, and 4) "engineering" models. The latter can be viewed as equations relating the longitudinal channel current at any height and any time to the current at the channel origin (or corresponding line charge density equations). Additionally, lightning is represented in some studies by a lumped current source (see, for example, Baba and Rakov [2]). One can use electromagnetic, distributed circuit, and engineering models in studying lightning induced effects and in characterizing the lightning electromagnetic environment.

Engineering return-stroke models prescribe the longitudinal current along the lightning channel, based on the existing knowledge on evolution of return-stroke current waveform as it propagates from ground toward the cloud. The return-stroke wavefront speed in these models can be set arbitrarily since it is one of the input parameters. Engineering return-stroke models have been reviewed by Nucci *et al.* [3], Rakov and Dulzon [4], Thottappillil and Uman [5], Thottappillil *et al.* [6], Rakov and Uman [1], and Gomes and Cooray [7].

Distributed-circuit models of the lightning return stroke usually consider the lightning channel as an R-L-C transmission line (e.g., Gorin [8]; Baum and Baker [9]; Mattos and Christopoulos [10], [11]), where R, L, and C are series resistance, series inductance, and shunt capacitance, all per unit length, respectively. In an R-L-C transmission-line model, voltage and current are the solutions of the telegrapher's equations. Note that the telegrapher's equations can be derived from Maxwell's equations assuming that the electromagnetic waves guided by the transmission line have a transverse electromagnetic (TEM) field structure. Strictly speaking, the latter assumption is not valid for a vertical conductor above ground. Distributed-circuit models have been reviewed by Rakov and Uman [1]. There has lately been a renewed interest in developing distributed-circuit models (e.g., Theethayi and Cooray [12] and Visacro and De Conti [13]).

Electromagnetic return-stroke models are based on Maxwell's equations. These are relatively new and most rigorous (no TEM assumption) models suitable for specifying the source in studying lightning electromagnetic interaction with various systems and with the environment. In this class of models, Maxwell's equations are solved to yield the distribution of current along the lightning channel using numerical techniques, such as the method of moments (MoM) (Harrington [14]; Van Baricum and Miller [15]; Miller *et al.* [16]) and the finite-difference method (Yee [17]). In contrast to distributed-circuit and engineering models, electromagnetic return-stroke models allow a self-consistent full-wave solution for both current distribution along the lightning channel and associated electromagnetic fields. One of the advantages of the use of electromagnetic models, although it may be computationally expensive, is that one needs to employ neither an approximate equation such as the Cooray-Rubinstein formula (Cooray [18], [19]; Rubinstein [20]) to take into account field propagation effects, nor a model of field-to-conductor electromagnetic coupling such as Agrawal *et al.*'s model [21] in analyzing lightning-induced effects on electrical circuits. Baba and Rakov [22] have reviewed electromagnetic models of the lightning return stroke, discussing in particular lightning channel representations, excitation methods, and numerical procedures for solving Maxwell's equations.

Note that the so-called hybrid electromagnetic (HEM) model (e.g., Visacro *et al.* [23]), has recently been applied to representing lightning return strokes. It employs electric scalar and magnetic vector potentials for taking account of electromagnetic coupling but is formulated in terms of circuit quantities, voltages and currents. Since the HEM model, on the one hand, yields a non-TEM close electromagnetic field structure (as do electromagnetic models) and, on the other hand, apparently considers electric and magnetic fields as decoupled

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(as in distributed-circuit models), it occupies an intermediate place between electromagnetic and distributed-circuit models. Applications of HEM model to lightning return-stroke studies are described by Visacro and Silveira [24] and to analyze the interaction of lightning with grounded objects by Visacro and Silveira [25] and Silveira *et al.* [26]. Baba and Rakov [22] have shown that the current distribution along a vertical resistive wire, representing a lightning channel, predicted using the HEM model, is consistent with that obtained using electromagnetic models. Also, Silveira *et al.* [27] have shown that lightning-induced voltages on an overhead horizontal wire above perfectly conducting ground calculated using the HEM model agree reasonably well with those calculated using an electromagnetic model based on the Numerical Electromagnetic Code (NEC-2; Burke and Poggio [28]).

In this paper, we review applications of electromagnetic return-stroke models, with the HEM model being outside the scope of our review. A review on numerical procedures used in electromagnetic return-stroke models is found in the Appendix .

II. GENERAL CHARACTERIZATION OF ELECTROMAGNETIC MODELS

In this section, we briefly describe the classification of electromagnetic return-stroke models proposed or used as of today in terms of the channel representation and the excitation method.

A. *Representation of the Lightning Return Stroke Channel*

Electromagnetic models of the lightning return stroke could be classified into five types depending on channel representation:

1) a perfectly conducting/resistive wire in air above ground (e.g., Podgorski and Landt [29] and Kordi *et al.* [30]);
2) a wire surrounded by a dielectric medium (other than air) that occupies the entire half space above ground (the artificial dielectric medium is used only for finding current distribution along the lightning channel, which is then removed for calculating electromagnetic fields in air) (e.g., Moini *et al.* [31]);
3) a wire coated by a dielectric in air above ground (Kato *et al.* [32]);
4) a wire loaded by additional distributed series inductance in air above ground (e.g., Kato *et al.* [33] and Baba and Ishii [34]);
5) two parallel wires, which could be also viewed as a vertical coaxial structure, having additional distributed shunt capacitance in air (this fictitious configuration is used only for finding current distribution, which is then applied to a vertical wire in air above ground for calculating electromagnetic fields) (Bonyadi-ram *et al.* [35]).

In the following, we will review the return-stroke speed and channel characteristic impedance resulting from each of the five types of channel representation. The return-stroke speed largely determines the radiation field initial peak (e.g., Rakov and Dulzon [36]; Nucci *et al.* [3]), while the characteristic impedance of lightning channel influences the magnitude of lightning current and/or the current reflection coefficient at the top of strike object when a delta-gap electric-field or lumped voltage source (see Section II-B) is employed.

Type 1. The speed of current wave propagating along a vertical perfectly conducting/resistive wire is nearly equal to the speed of light, which is 1.5 to 2 times larger than typical measured values of return stroke wavefront speed: $c/3$ to $2c/3$ (e.g., Rakov [37]), a discrepancy that is the main deficiency of this channel representation. The characteristic impedance of the wire [0.4 to 0.7 kΩ for a 50-mm-radius vertical perfectly conducting wire according to Baba and Ishii [38]] is somewhat lower than the equivalent impedance of the natural lightning return-stroke channel [0.6 to 2.5 kΩ (Gorin and Shikilev [39])]. Note that a current wave suffers attenuation (dispersion) as it propagates along a vertical wire even if it has no ohmic losses (Baba and Rakov [40]). Further attenuation can be achieved by loading the wire by distributed series resistance.

Type 2. For a vertical wire surrounded by a dielectric medium of relative permittivity greater than 1, occupying the entire half space above flat ground, the speed of current wave is lower than c. When the relative permittivity is 9 or 2.25, the speed is $c/3$ or $2c/3$, respectively. The corresponding characteristic impedance ranges from 0.13 to $0.27\ \mathrm{k\Omega}$ $(0.4\ \mathrm{k\Omega}/\sqrt{9} = 0.13\ \mathrm{k\Omega}, 0.7\ \mathrm{k\Omega}/\sqrt{9} = 0.27\ \mathrm{k\Omega})$ for $c/3$, and 0.23 to 0.47 kΩ $(0.4\ \mathrm{k\Omega}/\sqrt{2.25} = 0.23\ \mathrm{k\Omega}, 0.7\ \mathrm{k\Omega}/\sqrt{2.25} = 0.47\ \mathrm{k\Omega})$ for $2c/3$. These characteristic impedance values (0.13 to 0.47 kΩ) are smaller than values of the expected equivalent impedance of the lightning return stroke channel (0.6 to 2.5 kΩ). Moini *et al.*. [31] and Shoory *et al.* [41] used a relative permittivity value of 5.3 to set the wave propagation speed at $0.43c$.

Type 3. *Kato et al.* [32] represented the lightning return-stroke channel by a vertical perfectly conducting wire, which was placed along the axis of a 4-m-radius dielectric cylinder of relative permittivity 200. This dielectric cylinder was surrounded by air. The speed of current wave propagating along the wire was about $0.7c$. Such a representation allows one to calculate both the distribution of current along the wire and the remote electromagnetic fields in a single, self-consistent procedure, while that of a vertical wire surrounded by an artificial dielectric medium occupying the entire half space (as in *Type 2* described above) requires two steps to achieve the same objective. Note that remote electromagnetic fields produced by a dielectric-coated wire in air (Type 3) can be influenced by the presence of coating. For the 4-m-radius dielectric cylinder used by Kato *et al.*, we estimate that the electric field is appreciably smaller than in the absence of the cylinder at 50 m or less and essentially the same at larger distances.

Type 4. The speed of current wave propagating along a vertical wire loaded by additional distributed series inductance of 17 and 2.6 $\mu\mathrm{H/m}$ in air is $c/3$ and $2c/3$, respectively, if the natural inductance of a vertical wire is assumed to be $L_0 = 2.1\ \mu\mathrm{H/m}$ (as evaluated by Rakov [42] for a 30-mm-radius wire at a height of 500 m above ground). The corresponding characteristic impedance ranges from 1.2 to 2.1 kΩ $(0.4\ \mathrm{k\Omega} \times \sqrt{(17+2.1)/2.1} = 1.2\ \mathrm{k\Omega}, 0.7\ \mathrm{k\Omega} \times \sqrt{(17+2.1)/2.1} = 2.1\ \mathrm{k\Omega})$ for

$c/3$, and 0.6 to 1.0 kΩ ($0.4\ \mathrm{k}\Omega \times \sqrt{(2.6+2.1)/2.1} = 0.6\ \mathrm{k}\Omega, 0.7\ \mathrm{k}\Omega \times \sqrt{(2.6+2.1)/2.1} = 1.0\ \mathrm{k}\Omega$) for $2c/3$. The characteristic impedance of the inductance-loaded wire (0.6 to 2.1 kΩ) is within the range of values of the expected equivalent impedance of the lightning return stroke channel. Note that additional inductance has no physical meaning and is invoked only to reduce the speed of current wave propagating along the wire to a value lower than the speed of light. The use of this representation also allows one to calculate both the distribution of current along the channel-representing wire and remote electromagnetic fields in a single, self-consistent procedure.

Type 5. The speed of current wave propagating along two parallel wires having additional distributed shunt capacitance in air is $0.43c$ when the additional capacitance is 50 pF/m, according to Bonyadi-ram *et al.* [35]. Each of these wires had a radius of 20 mm, and the separation between the wires was 30 m. This approach, similar to that in *Type 2* described above, uses a fictitious configuration for finding a reasonable distribution of current along the lightning channel, and then this current distribution is applied to the actual configuration (vertical wire in air above ground).

B. Excitations Methods

Methods of excitation used in electromagnetic models are listed below.

1) closing a charged vertical wire at its bottom end with a specified impedance (or circuit) (Podgorski and Landt [29]; Podgorski [43]);
2) a delta-gap electric-field source (e.g., Moini *et al.*, [44]; Chai *et al.* [45]) (same as a lumped voltage source); and
3) a lumped current source (e.g., Grcev *et al.* [46] and Noda *et al.* [47]).

Podgorski and Landt (1987 [29]), and *Podgorski* (1991 [43]) have represented a leader/return-stroke sequence by a pre-charged vertical resistive wire representing the lightning channel connected via a nonlinear resistor to the top of a vertical perfectly conducting wire representing the 553-m-high CN Tower, whose bottom was grounded. In their model, closing a charged vertical wire in a specified circuit constitutes excitation of the lightning return-stroke channel.

A delta-gap electric-field source can be placed at ground level (e.g., Moini *et al.* [44]) or at the top of a grounded strike object (e.g., Chai *et al.* [45]). This type of source generates a specified electric field, which is independent of current flowing through it. Since a delta-gap electric-field source has zero internal impedance, its presence in series with the lightning channel and strike object does not disturb any transient processes in them. If necessary, one could insert a lumped resistor in series with the delta-gap electric-field source to adjust the impedance seen by waves entering the channel from the strike object to a value consistent with the expected equivalent impedance of the lightning channel.

Similar to the delta-gap electric field source, a lumped current source can be placed at ground level (e.g., Grcev *et al.* [46]) or at the top of a grounded strike object (e.g., Noda *et al.* [47]). However, there is an important difference relative to the electric field (voltage) source. The use of a lumped current source inserted at the attachment point is justified only when reflected waves returning to the current source are negligible. This is the case for a branchless subsequent lightning stroke terminating on flat ground, in which an upward connecting leader is usually neglected and the return-stroke current wave propagates upward from the ground surface. The primary reason for the use of a lumped current source at the channel base is a desire to use directly the channel-base current, known from measurements for both natural and triggered lightning, as an input parameter of the model. When one employs a lumped ideal current source at the attachment point in analyzing lightning strikes to a tall grounded object, the lightning channel, owing to the infinitely large impedance of the ideal current source, is electrically isolated from the strike object, so that current waves reflected from ground cannot be directly transmitted to the lightning channel (only electromagnetic coupling is possible). Since this is physically unreasonable, a series ideal current source is not suitable for modeling of lightning strikes to tall grounded objects (Baba and Rakov [2]).

III. APPLICATIONS OF ELECTROMAGNETIC RETURN STROKE MODELS

In this section, we review applications of electromagnetic return-stroke models in studying lightning effects that result from

1) strikes to flat ground;
2) strikes to free-standing tall objects;
3) strikes to overhead power transmission lines; and
4) strikes to wire-mesh-like structures.

Table I gives a list of papers for each of these four configurations.

A. Strikes to Flat Ground

Moini *et al.* [31], [48]; Kordi *et al.* [30], [49]; Baba and Ishii [38]; Grcev *et al.* [46]; Aniserowicz [50]; Bonyadi-ram *et al.* [35], [51]; Maslowski [52], [53]; and Shoory *et al.* [41] have calculated waveforms of vertical electric and azimuthal magnetic fields due to lightning return strokes at different distances from the lightning channel attached to flat ground, and compared them with typical measured waveforms of electric and magnetic fields (Lin *et al.* [54] and Crawford *et al.* [55]). In these calculations, a typical subsequent-stroke waveform of channel-base current (Nucci *et al.* [3]) and a typical propagation speed of return-stroke wavefront ($0.43c$ in most cases) were used.

Features of measured field waveforms due to lightning return strokes (Nucci *et al.* [3] and Rakov and Uman [1]) include

1) a characteristic flattening in about 15 μs of vertical electric field tens to hundreds of meters from the strike point;
2) a sharp initial peak in both electric and magnetic fields measured beyond a kilometer or so;
3) a slow ramp following the initial peak for electric fields measured within a few tens of kilometers;
4) a hump following the initial peak in magnetic fields measured within several tens of kilometers;
5) zero crossing within tens of microseconds of the initial peak in both electric and magnetic fields beyond about 50 km.

TABLE I
LIST OF PAPERS ON APPLICATIONS OF ELECTROMAGNETIC MODELS OF THE LIGHTNING RETURN STROKE

Configuration	Reviewed journal papers	Other publications
Strike to flat ground	Moini *et al.* (1998 [44], 2000 [31]) Kordi *et al.* (2002 [30]) Baba and Ishii (2003 [38]) Pokharel *et al.* (2003 [56])* Baba and Rakov (2003 [73], 2005 [40]) Shoory *et al.* (2005 [41])*	Moini *et al.* (1997 [48]) Kato *et al.* (2001 [32]) Kordi *et al.* (2003 [49]) Grcev *et al.* (2003 [46]) Aniserowicz (2004 [50]) Bonyadi-ram *et al.* (2004 [51], 2005 [35]) Maslowski (2002 [52]; 2004 [53])* Tatematsu *et al.* (2004 [57])* Pokharel and Ishii (2006 [58])* Silveira *et al.* (2006 [27])
Strike to a free-standing tall object	Podgorski and Landt (1987 [29]) Baba and Ishii (2001 [34]) Kordi *et al.* (2003 [62]) Pokharel *et al.* (2004 [66])*	Podgorski (1991 [43]) Kato *et al.* (1999 [33]) Miyazaki and Ishii (2004 [63])* Petrache *et al.* (2005 [64])*
Strike to a power transmission line	Mozumi *et al.* (2003 [68])	Noda *et al.* (2005 [47])*
Strike to a wire-mesh-like structure		Chai *et al.* (1994 [45])* Miyazaki and Ishii (2005 [71], 2006 [72])

Papers with '*' consider lossy ground in finding current distribution along the lightning channel.

As an example, we present in Figs. 1 and 2 current profiles and corresponding fields computed using two different electromagnetic models. Fig. 1 shows current waveforms at different heights calculated by Moini *et al.* [31] using the MoM in the time domain for a vertical $0.07\text{-}\Omega/\mathrm{m}$ resistive wire that is excited at its bottom by a delta-gap electric-field source and surrounded by a dielectric medium of relative permittivity of 5.3, and those calculated by Shoory *et al.* [41] using the MoM in the frequency domain for a vertical $0.1\text{-}\Omega/\mathrm{m}$ resistive wire that is excited by a lumped current source. In both cases, for finding current distribution along the wire, the wire was surrounded by a dielectric medium of relative permittivity of 5.3 above flat perfectly conducting ground. The propagation speed of current wave is $0.43c$, where c is the speed of light. As seen in Fig. 1, current wave suffers both attenuation and dispersion as it propagates along the wire. Once the distribution of current along the wire is determined, the artificial dielectric medium is replaced with air for computing remote fields. Fig. 2(a)–(c) shows waveforms of vertical electric field on the surface of flat perfectly conducting ground at distances 0.5, 5, and 100 km, respectively, from the vertical lightning channel. Fig. 2(d) shows those of azimuthal magnetic field at distance 5 km from the channel. Both of the models are capable of reproducing features 1, 2, and 3 listed before. Note that Shoory *et al.* also used the MoM in the frequency domain for computing both currents and fields taking into account lossy ground.

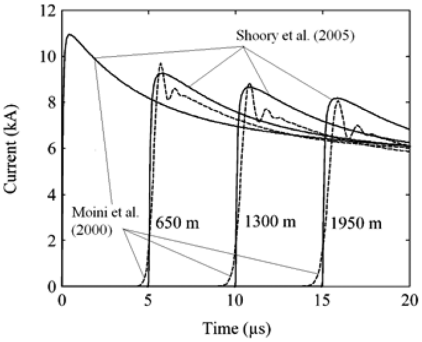
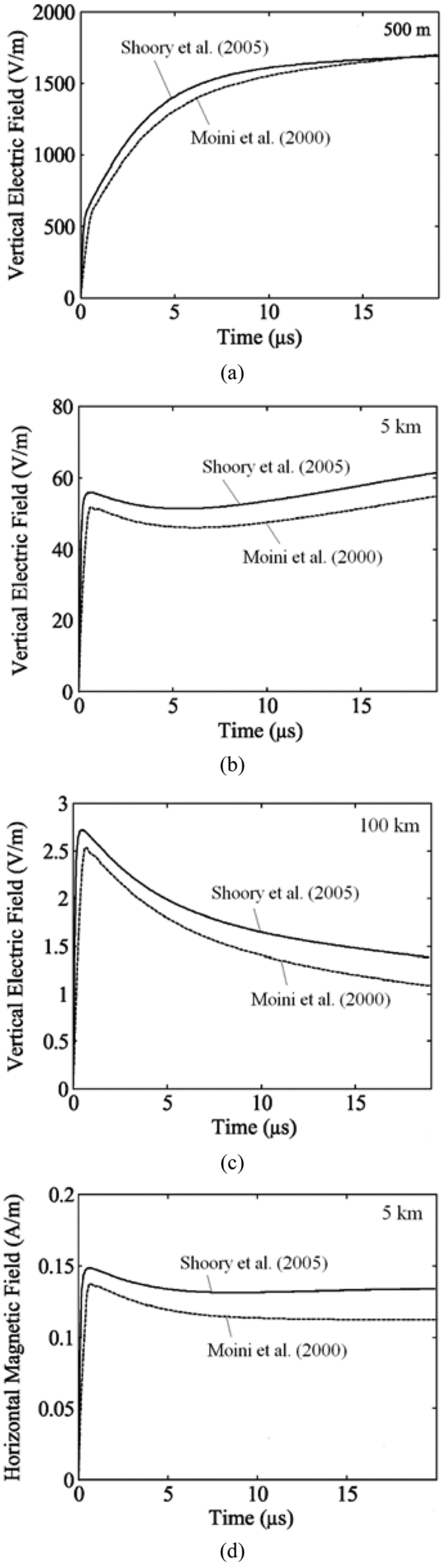


Fig. 1. Current waveforms at different heights calculated by Moini *et al.*[31] using the MoM in the time domain for a vertical $0.07\text{-}\Omega/\mathrm{m}$ resistive wire above flat perfectly conducting ground that is excited at its bottom by a lumped voltage source and surrounded by a dielectric medium of relative permittivity of 5.3, and those calculated by Shoory *et al.* [41] using the MoM in the frequency domain for a vertical $0.1\text{-}\Omega/\mathrm{m}$ resistive wire above flat perfectly conducting ground that is excited by a lumped current source and surrounded by the same dielectric medium. The propagation speed of current wave is $0.43c$, where c is the speed of light. Adapted from Shoory *et al.*[41].

Similar calculations of electric and magnetic fields were carried out by *Kordi et al.* [30], [49]; Baba and Ishii [38]; Grcev *et al.* [46]; Aniserowicz [50]; Bonyadi-ram *et al.* [35], [51]; Maslowski [53]. Kordi *et al.* [30], [49] employed the MoM in the time domain, while others used the MoM in the frequency domain. Lightning return-stroke channels of Grcev *et al.* [46]; Maslowski [53], and Bonyadi-ram *et al.* [51] were excited at their bottom by a lumped current source, while others were excited by a delta-gap electric-field source. Lightning return-stroke channels of Baba and Ishii [38], Aniserowicz [50], and Bonyadi-ram *et al.* [51] were represented by a vertical wire loaded by additional distributed series inductance in air, that of Grcev *et al.* [46] was represented by a vertical wire in an artificial dielectric medium (only for finding the distribution of current along the channel), and those of the others were represented by a vertical resistive or perfectly conducting wire in air.

Note that Maslowski [52] investigated the influence of branched and inclined lightning channel on lightning electromagnetic fields, and Baba and Ishii [38] studied the influence of a horizontal lightning channel in the thundercloud, connected to



(a)

(b)

(c)

(d)

Fig. 2. Waveforms of vertical electric field on the surface of flat perfectly conducting ground at distances (a) 0.5, (b) 5, and (c) 100 km from the vertical lightning channel, calculated by Moini *et al.* [31] using the MoM in the time domain and by Shoory *et al.* [41] using the MoM in the frequency domain, and (d) those of azimuthal magnetic field at distance 5 km from the channel. Adapted from Shoory *et al.* [41].

a vertical lightning channel attached to flat ground, on lightning electromagnetic fields.

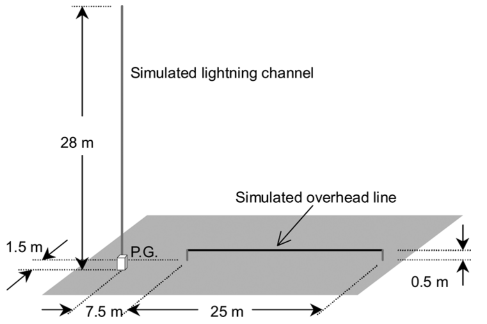


Fig. 3. Lightning interaction with a 25-m-long horizontal perfectly conducting wire above flat ground having conductivity 0.06 S/m simulated using the NEC-2 code. One end of the wire is located at distances $x = 7.5\ \mathrm{m}$ and $y = 0.75\ \mathrm{m}$ from a vertical lightning channel, as in Ishii *et al.* 's (1999 [59]) small-scale experiment. Both ends of the horizontal wire are terminated in 430-Ω resistance in parallel with 20-pF capacitance. The lightning channel is represented by a vertical wire loaded by distributed series resistance of 0.5 Ω/m and additional distributed series inductance of 6 $\mu\mathrm{H/m}$, with the current-wave propagation speed being about $0.43c$. Adapted from Pokharel *et al.* [56].

Moini *et al.* [44], Pokharel *et al.* [56], Tatematsu *et al.* [57], and Pokharel and Ishii [58] have calculated transient induced voltages on nearby overhead wires due to lightning strikes to flat ground. These studies will be reviewed.

Moini *et al.* [44] have calculated transient voltages on overhead perfectly-conducting wires of different geometries such as parallel and nonparallel wires above flat perfectly conducting ground using the MoM in the time domain. In order to find the distribution of current along the lightning channel, they represented it by a vertical perfectly-conducting wire, which was excited at its bottom by a delta-gap electric-field source and surrounded by a dielectric medium having a relative permittivity of 4, occupying the entire half space above ground. The propagation speed of current wave along the wire was about $0.5c$. Using the resultant distribution of current along this channel-representing vertical wire and replacing the artificial dielectric medium with air, they calculated transient voltages induced on the overhead wires. The authors conclude that, in calculating induced effects on nonuniform wires or complex-shape wires, scattering-theory approach is more appropriate than that based on field-to-conductor electromagnetic coupling models (e.g., Agrawal *et al.* [21]) that are based on transmission line theory (telegrapher's equations with source terms).

Pokharel *et al.* [56] have calculated transient voltages on a 25-m-long horizontal overhead perfectly-conducting wire above flat ground having conductivity 0.06 S/m shown in Fig. 3, using the Numerical Electromagnetic Code (NEC-2) (Burke and Poggio [28]) that is based on the MoM in the frequency domain. They represented the lightning return-stroke channel by a 28-m-long vertical 0.5-Ω/m resistive wire having additional distributed series inductance of 6 $\mu\mathrm{H/m}$. The wire was excited at its bottom by a delta-gap electric-field source in series with 750-Ω lumped resistance. The propagation speed of current wave along the wire was about $0.43c$. Induced voltages were computed within first 300 ns, so that they are not influenced by reflections from the open end of 28-m-long vertical wire. Fig. 4(a) and (b) shows calculated induced-voltage waveforms

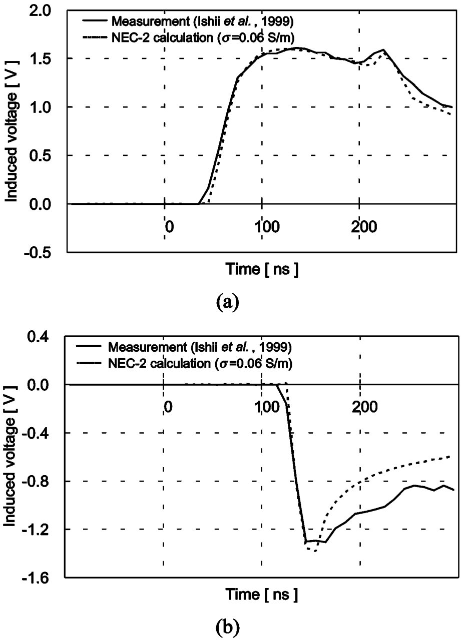


Fig. 4. Waveforms of voltage induced at the (a) close and (b) remote ends of the 25-m-long horizontal wire above flat ground measured by Ishii *et al.*(1999 [59]) and those calculated, using the NEC-2 code, by Pokharel *et al.* for ground conductivity 0.06 S/m [56]. Adapted from Pokharel *et al.* [56].

at the close and remote ends of the horizontal wire, respectively, and those measured by Ishii *et al.* [59]. Calculated waveforms agree well with corresponding measured waveforms. This work showed for the first time that voltages induced on an overhead wire above lossy ground could be calculated reasonably accurately using the NEC-2 code.

Similar to Pokharel *et al.* [56], Tatematsu *et al.* [57] have calculated transient voltages on an overhead perfectly-conducting wire above flat perfectly-conducting and lossy ground using the FDTD method and shown that FDTD-calculated voltages agree well with the voltages calculated using field-to-conductor electromagnetic coupling models of Rusck [60] and Agrawal *et al.* [21]. They represented the lightning return-stroke channel by a vertical perfectly conducting wire having additional distributed series inductance of 10 $\mu\mathrm{H/m}$ above flat ground having conductivity of 1 mS/m. The wire was excited at its bottom by a lumped current source. The propagation speed of current wave along the wire was about $0.31c$. This work showed for the first time that voltages induced on an overhead wire above lossy ground could be calculated reasonably accurately using the FDTD method.

Pokharel and Ishii [58] have calculated transient voltages on a 500-m-long horizontal overhead perfectly-conducting wire above flat perfectly-conducting ground, using the thin-wire time-domain (TWTD) code (see Van Baricum and Miller [15]), based on the MoM in the time domain. A nonlinear element simulating a surge arrester was connected between the wire at its center point and ground. The lightning return-stroke channel was represented by a vertical 0.6-Ω/m resistive wire having additional distributed series inductance of 6 $\mu\mathrm{H/m}$ that was excited at its bottom by a delta-gap electric-field source. The propagation speed of current wave along the wire was about $0.48c$. The use of TWTD code allows one to incorporate nonlinear elements, but makes it impossible to consider the frequency-dependent effects of lossy ground.

B. Strikes to Free-Standing Tall Objects

Podgorski and Landt [29] and Podgorski [43], using the modified TWTD code (Van Baricum and Miller [15]) that is based on the MoM in the time domain, have represented a lightning strike to the 553-m-high CN Tower by a precharged resistive (0.7 Ω/m) vertical wire connected via a nonlinear resistance (10 kΩ prior to the attachment and 3 Ω after the attachment) to the top of the CN Tower. The latter was represented by a perfectly conducting wire. The calculated waveform of current near the top of the tower was found to be similar to the corresponding measured waveform.

Kato et al. [33] have calculated waveforms of lightning current and associated electric and magnetic fields 200 m from the strike point, assuming perfectly conducting ground, due to a lightning strike to the 553-m-high CN Tower and to the 168-m-high Peissenberg Tower using the MoM in the time domain, and compared them with corresponding measured waveforms. Baba and Ishii [34] have calculated electric and magnetic fields 2 km and 630 m from the strike point, assuming perfectly conducting ground, due to lightning strikes to the CN Tower and the 200-m-high Fukui chimney using the NEC-2 code, and compared those with corresponding measured waveforms. Fig. 5 shows NEC-2-calculated waveforms for the Fukui-chimney case with corresponding waveforms measured by Goshima *et al.* [61]. NEC-2-calculated waveforms agree well with corresponding measured waveforms. Kordi *et al.* [62] have calculated waveforms of lightning current and associated electric and magnetic fields at 2 km due to a lightning strike to the CN Tower, assuming perfectly conducting ground and using the MoM in the time domain, and compared them with corresponding measured waveforms. Miyazaki and Ishii [63] have calculated, using the NEC-2 code, lightning current and associated electric and magnetic fields on the surface of ground having conductivity of 1 mS/m at distances ranging from 100 m to 500 km from the lightning channel due to lightning strikes to tall towers whose heights ranged from 60 to 240 m. Petrache *et al.* [64] have studied, using the NEC-4 (Burke [65]), influence of ground conductivity (1, 10 mS/m, and ∞) on the lightning current in the CN Tower and associated electric and magnetic fields 2 km away from the tower. In these five works, except for Kordi *et al.*'s one, the lightning return-stroke channel was represented by a vertical wire having additional distributed series inductance, and the lightning channel and the tall strike object were excited by a delta-gap electric-field source inserted between them. In Kordi *et al.*'s work [62], the lightning channel was represented by a resistive wire in air and excited by a delta-gap electric-field source.

Pokharel *et al.* [66] have calculated, using the NEC-2 code, induced voltages on an overhead wire due to a lightning strike to the 200-m-high Fukui chimney and compared those with corresponding measured voltage waveforms (Michishita *et al.* [67]). They represented the lightning channel by a vertical wire having distributed series resistance of 1 Ω/m and additional distributed series inductance 9 $\mu\mathrm{H/m}$. The lightning channel and the chimney were excited by a delta-gap electric-field

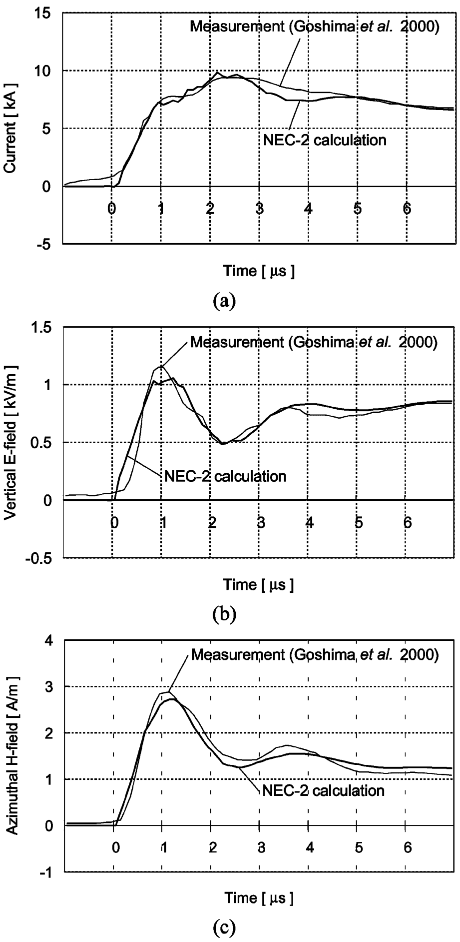


Fig. 5. Waveforms of (a) current at the top of the 200-m-high Fukui chimney, (b) vertical electric field, and (c) azimuthal magnetic field, 630 m away from the chimney, calculated by Baba and Ishii [34] using the NEC-2 code and assuming perfectly conducting ground, and those measured by Goshima *et al.* [61]. The lightning channel was represented by a vertical conductor having distributed series resistance of 1 Ω/m and additional distributed series inductance of 3 $\mu\mathrm{H}/\mathrm{m}$, with the current-wave propagation speed being about $0.5c$. The 200-m-high chimney is represented by a vertical perfectly conducting wire. The lightning channel and the chimney are excited by a delta-gap electric-field source in series with 400-Ω lumped resistance. Adapted from Baba and Ishii [34].

source in series with lumped resistance of 100 Ω inserted between them. The propagation speed of current wave along the channel was about $0.37c$. Fig. 6 shows the plan view of the overhead wire and the chimney. Fig. 7 shows waveforms of current at the top of the chimney and voltages induced on the overhead wire near the terminations, measured by Michishita *et al.* [67]. Fig. 8 shows those calculated by Pokharel *et al.* [66] assuming ground conductivity to be 0.02 S/m, for which best agreement with measured waveforms was found.

Podgorski [43] investigated the lightning current waveform in the CN Tower connected to a vertical lightning channel, which was represented by a 0.7-Ω/m resistive wire having many 0.7- or 7-Ω/m resistive twigs (representing radial corona discharge). Kato *et al.* [33]) studied the influence of inclined lightning channel attached to the Peissenberg Tower on lightning electromagnetic field. Pokharel *et al.* [66] investigated the influence of inclined lightning channel attached to the Fukui chimney on lightning-induced voltages.

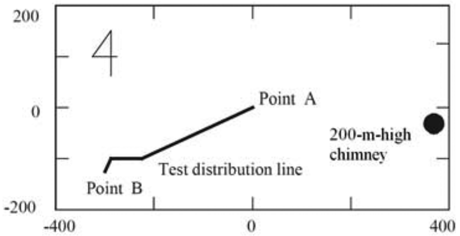
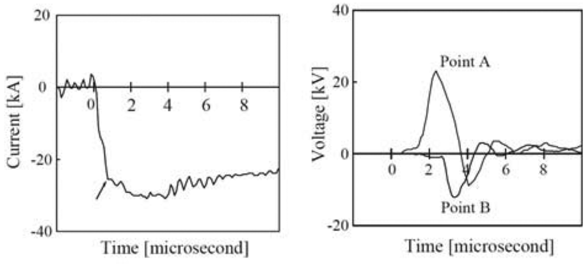


Fig. 6. Plan view of a single overhead wire and a nearby 200-m-high chimney. Voltages on the overhead wire induced by a lightning strike to the chimney were calculated using the NEC-2 code by Pokharel *et al.* [66]. The lightning channel was represented by a vertical conductor having distributed series resistance of 1 Ω/m and additional distributed series inductance of 9 $\mu\mathrm{H}/\mathrm{m}$, with the current-wave propagation speed being about $0.37c$. The 200-m-high chimney was represented by a vertical perfectly conducting wire. The lightning channel and the chimney were excited by a delta-gap electric-field source in series with lumped resistance of 100 Ω inserted between them. Adapted from Pokharel *et al.* [66]. Vertical and horizontal scales are in meters.



(a) (b)

Fig. 7. Waveforms of (a) current at the top of the 200-m-high chimney and (b) voltages induced on the overhead wire near its terminations, measured by Michishita *et al.* [67]. Adapted from Pokharel *et al.* [66].

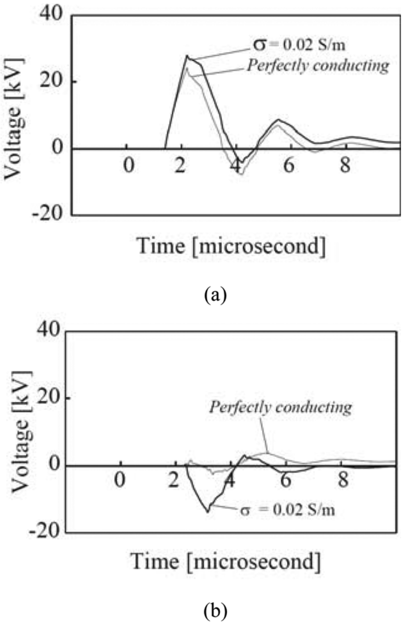


Fig. 8. Waveforms of voltage induced on the overhead wire near the terminations (a) closer to the chimney, and (b) farther from it, calculated by Pokharel *et al.* [66] assuming ground conductivity 0.02 S/m. Adapted from Pokharel *et al.* [66].

C. Strikes to Overhead Power Transmission Lines

Mozumi *et al.* [68] have calculated, using the TWTD code, voltages across insulators of a 500-kV double-circuit power transmission line with two overhead ground wires located above perfectly conducting ground, in the case that the line tower is struck by lightning and thereby back-flashover occurs across the insulator of one phase. In order to analyze back-flashover using the TWTD code, they modified it to incorporate a flashover model (Motoyama [69]. For the TWTD calculations, the lightning return-stroke channel was represented by a vertical perfectly conducting wire of radius 0.1 m in air. The lightning channel and the tower were excited by a delta-gap electric-field source in series with 5 kΩ lumped resistance inserted between them.

Noda et al. [47] have calculated, using the FDTD method, voltages across insulators of a 500-kV double-circuit power transmission line, located above ground having conductivity of 10 mS/m, in the case that the line tower is struck by lightning. In their calculations, a lightning return-stroke channel was represented by a 0.23-m-radius (Noda *et al.* [70]) vertical perfectly conducting wire having additional distributed series inductance of 10 $\mu\mathrm{H/m}$, and the speed of current wave propagating along the wire was $0.33c$. The lightning channel and the tower were excited by a lumped current source inserted between them.

In these two works, towers were represented by more realistic structures (including crossarms and slanted bars) than tall objects described in Section III-B.

D. Strikes to Wire-Mesh-Like Structures

Chai *et al.* [45] have studied, using the NEC-2 code, the electromagnetic environment inside of a wire-array lightning protection system for a launch vehicle, located above ground having conductivity of 10 mS/m, when the protection system was subjected to a direct lightning strike. In their analysis, lightning channel was represented by a resistive wire, and the excitation was accomplished using a delta-gap electric-field source.

Miyazaki and Ishii [71] have calculated, using the NEC-4 code, distributions of lightning current inside buildings of height ranging from 10 to 40 m located above perfectly conducting ground and directly struck by lightning. They represented the vertical lightning channel attached to the building top by a vertical wire having distributed series resistance and additional distributed series inductance, and the building (including internal electric power wires) by perfectly-conducting-wire grid. The internal power wires were directly connected to the building in their calculations in order to simulate the condition when surge protective devices inserted between the building and the power wires were operating. They inserted a delta-gap electric-field source between the lightning channel and building. They showed that magnitudes of current flowing in electric power wires on the top and bottom floors were largest regardless of the building height. Miyazaki and Ishii [72] have also calculated, using the NEC-4 code, time derivatives of magnetic field inside a 30-m-high building located above perfectly conducting ground and directly struck by lightning. They represented the vertical lightning channel attached to the building in the same manner as Miyazaki and Ishii [71] (see above). Miyazaki and Ishii [72] showed that time derivatives of magnetic field in upper parts of building were largest but could be reduced by installing a finer conducting mesh on the building roof.

IV. SUMMARY

Electromagnetic models have been used to calculate the distribution of current along a vertical lightning channel attached to flat ground and associated remote electromagnetic fields. In this application, lightning return-stroke channel was represented by a vertical resistive/perfectly conducting wire in air, a vertical wire surrounded by a dielectric medium with permittivity higher than that of air (only for finding the distribution of current along the channel), a vertical wire with dielectric coating in air, or a vertical wire having additional distributed series inductance. The use of additional distributed shunt capacitance was also considered. The channel was excited at its bottom by a lumped current source or by a delta-gap electric-field source. Voltages induced on a nearby overhead conductor due to a lightning strike to flat ground were analyzed using electromagnetic models. These models allow a self-consistent full-wave solution for current distribution along the lightning channel, associated electromagnetic fields, and lightning electromagnetic coupling effects on various systems.

In analyzing lightning strikes to free-standing tall objects, a vertical wire having additional distributed series inductance excited by a delta-gap electric-field source is typically used to represent a lightning return-stroke channel. Transient voltages on a nearby distribution line due to a lightning strike to a tall grounded object have been reasonably accurately reproduced using this class of models.

Voltages on an overhead power transmission line due to a direct lightning strike to its tower top have also been analyzed using electromagnetic return-stroke models. In contrast with the circuit-theory approach, electromagnetic coupling between the lightning channel and power transmission line was included in the analysis, although it is expected to make a relatively small, 10–15%, contribution to the line voltage.

Electromagnetic environments inside wire-like structures, including lightning protection system and tall building, have also been analyzed using this class of models.

It is well known that one needs to take account of lossy ground for calculating lightning-induced voltages. The MoM in the frequency domain and the FDTD method readily allow specification of lossy ground and therefore are particularly suitable in analyzing lightning-induced voltages. The use of these methods, although computationally expensive, requires neither an approximate equation such as the Cooray-Rubinstein formula to take into account field propagation effects nor a field-to-conductor electromagnetic coupling model (e.g., Agrawal *et al.* [21]). These methods can also handle configurations containing nonparallel wires above ground.

Remote vertical electric and azimuthal magnetic fields generated by lightning return strokes are calculated reasonably accurately using the MoM in the time domain, as well as the MoM in the frequency domain and the FDTD method.

APPENDIX
NUMERICAL PROCEDURES USED IN ELECTROMAGNETIC MODELS OF THE LIGHTNING RETURN STROKE

In this section, we briefly describe numerical procedures used in electromagnetic models of the lightning return stroke, which include (in chronological order of their usage in electromagnetic models):

1) the MoM in the time domain;
2) the MoM in the frequency domain;
3) the FDTD method.

1. Methods of Moments (MoMs) in the Time and Frequency Domains

a) MoM in the Time Domain: The MoM in the time domain (Van Baricum and Miller [15] and Miller *et al.* [16]) is widely used in analyzing responses of thin-wire metallic structures to external time-varying electromagnetic fields. The entire conducting structure representing the lightning channel is modeled by a combination of cylindrical wire segments whose radii are much smaller than the wavelengths of interest. The so-called electric-field integral equation for a perfectly conducting thin wire in air (see Fig. 9), assuming that current I and charge q are confined to the wire axis (thin-wire approximation) and that the boundary condition on the tangential electric field on the surface of the wire (this field must be equal to zero) is fulfilled, is given by

$$\hat{\boldsymbol{s}} \cdot \boldsymbol{E}_{inc}(\boldsymbol{r}, t) = \frac{\mu_0}{4\pi} \times \int_C \left[\begin{array}{l} \frac{\hat{\boldsymbol{s}} \cdot \hat{\boldsymbol{s}}'}{R} \frac{\partial I(s', t')}{\partial t'} \\ + c \frac{\hat{\boldsymbol{s}} \cdot \boldsymbol{R}}{R^2} \frac{\partial I(s', t')}{\partial s'} - c^2 \frac{\hat{\boldsymbol{s}} \cdot \boldsymbol{R}}{R^3} q(s', t') \end{array} \right] ds' \quad \text{(A.1)}$$

where

$$q(s', t') = -\int_{-\infty}^{t'} \frac{\partial I(s', \tau)}{\partial s'} d\tau$$

C is an integration path along the wire axis, $\boldsymbol{E}_{inc}$ denotes the incident electric field that induces current I, $\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}'$, $\boldsymbol{r}$ and t denote the observation location (a point on the wire surface) and time, respectively, $\boldsymbol{r}'$ and $\boldsymbol{t}'$ denote the source location (a point on the wire axis) and time, respectively, s and s' denote the distance along the wire surface at $\boldsymbol{r}$ and that along the wire axis at $\boldsymbol{r}'$, $\hat{\boldsymbol{s}}$ and $\hat{\boldsymbol{s}}'$ denote unit vectors tangent to path $\boldsymbol{C}$ in (A.1) at $\boldsymbol{r}$ and $\boldsymbol{r}'$, μ_0 is the permeability of vacuum, and c is the speed of light. Through numerically solving (A.1), which is based on Maxwell's equations, the time-dependent current distribution along the wire structure (lightning channel), excited by a lumped source, is obtained.

The TWTD code (Van Baricum and Miller [15]) (available from the Lawrence Livermore National Laboratory) is based on the MoM in the time domain. One of the advantages of the use of the time-domain MoM is that it can incorporate nonlinear effects such as the lightning attachment process (e.g., Podgorski and Landt [29]), although it does not allow lossy ground and wires buried in lossy ground to be incorporated.

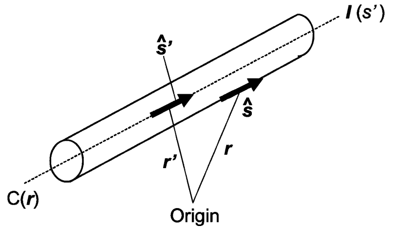


Fig. 9. Thin-wire segment for MoM-based calculations. Current is confined to the wire axis, and the tangential electric field on the surface of the wire is set to zero.

b) MoM in the Frequency Domain: The MoM in the frequency domain (Harrington [14]) is widely used in analyzing the electromagnetic scattering by antennas and other metallic structures. In order to obtain the time-varying responses, Fourier and inverse Fourier transforms are employed. The electric-field integral equation derived for a perfectly conducting thin wire in air (see Fig. 9) in the frequency domain is given by

$$-\hat{\boldsymbol{s}} \cdot \boldsymbol{E}_{inc}(\boldsymbol{r}) = \frac{j\eta}{4\pi k} \int_C I(s') \left(k^2 \hat{\boldsymbol{s}} \cdot \hat{\boldsymbol{s}}' - \frac{\partial^2}{\partial s \partial s'} \right) g(\boldsymbol{r}, \boldsymbol{r}')\, ds' \quad \text{(A.2)}$$

where

$$g(\boldsymbol{r}, \boldsymbol{r}') = \exp\left(\frac{-jk\,|\boldsymbol{r} - \boldsymbol{r}'|}{|\boldsymbol{r} - \boldsymbol{r}'|} \right), \quad k = \omega\sqrt{\mu_0 \varepsilon_0}, \quad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

ω is the angular frequency, μ_0 is the permeability of vacuum, and ε_0 is the permittivity of vacuum. Other quantities in (A.2) are the same as those in (A.1). Current distribution along the lightning channel can be obtained numerically solving (A.2).

This method allows lossy ground and wires in lossy ground (for example, grounding of a tall strike object) to be incorporated into the model. The commercially available numerical electromagnetic codes [e.g., NEC-2 (Burke and Poggio [28], and NEC-4 (Burke [65])] are based on the MoM in the frequency domain.

2. Finite-Difference Time-Domain (FDTD) Method

The FDTD method (Yee [17]) employs a simple way to discretize Maxwell's equations in differential form. In the Cartesian coordinate system, it requires discretization of the entire space of interest into small cubic or rectangular-parallelepiped cells. Cells for specifying or computing electric field (electric field cells) and magnetic field cells are placed relative to each other as shown in Fig. 10. Electric and magnetic fields of the cells are calculated using the discretized Maxwell's equations (A.3) and (A.4), shown at the top of the next page. Equation (A.3), which is based on Ampere's law, is an equation updating z component of electric field, $E_z(i, j, k + 1/2)$, at point $x = i\Delta x$, $y = j\Delta y$, and $z = (k + 1/2)\Delta z$, and at time $t = n\Delta t$. Equation (A.4), which is based on Faraday's law, is an equation updating x component of magnetic field, $H_x(i, j - 1/2, k + 1/2)$, at point $x = i\Delta x$, $y = (j - 1/2)\Delta y$, and $z = (k + 1/2)\Delta z$, and at time $t = (n + 1/2)\Delta t$. Equations updating x and y components of electric field, and y and z components of magnetic field can be written in a similar manner. Note that $\sigma(i, j, k + 1/2)$ and $\varepsilon(i, j, k + 1/2)$ are the conductivity and permittivity at point $x = i\Delta x, y = j\Delta y$, and $z = (k + 1/2)\Delta z$, respectively, $\mu(i, j - 1/2, k + 1/2)$ is the permeability at point $x = i\Delta x$, $y = (j - 1/2)\Delta y$, and $z = (k + 1/2)\Delta z$. By updating electric and magnetic fields at every point using (A.3) and (A.4), transient fields throughout the computational domain are

$$\begin{aligned} E_z^n\left(i,j,k+\frac{1}{2}\right) = {} & \frac{\frac{1-\sigma\left(i,j,k+\frac{1}{2}\right)\Delta t}{\left[2\varepsilon\left(i,j,k+\frac{1}{2}\right)\right]}}{\frac{1+\sigma\left(i,j,k+\frac{1}{2}\right)\Delta t}{\left[2\varepsilon\left(i,j,k+\frac{1}{2}\right)\right]}} \\ & \times E_z^{n-1}\left(i,j,k+\frac{1}{2}\right) \\ & + \frac{\frac{\Delta t}{\varepsilon\left(i,j,k+\frac{1}{2}\right)}}{\frac{1+\sigma\left(i,j,k+\frac{1}{2}\right)\Delta t}{\left[2\varepsilon\left(i,j,k+\frac{1}{2}\right)\right]}} \frac{1}{\Delta x \Delta y} \\ & \times \begin{bmatrix} H_y^{n-(1/2)}\left(i+\frac{1}{2},j,k+\frac{1}{2}\right)\Delta y - H_y^{n-(1/2)}\left(i-\frac{1}{2},j,k+\frac{1}{2}\right)\Delta y \\ -H_x^{n-(1/2)}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)\Delta x + H_x^{n-(1/2)}\left(i,j-\frac{1}{2},k+\frac{1}{2}\right)\Delta x \end{bmatrix} \end{aligned} \tag{A.3}$$

$$\begin{aligned} H_x^{n+(1/2)}\left(i,j-\frac{1}{2},k+\frac{1}{2}\right) = {} & H_x^{n-(1/2)}\left(i,j-\frac{1}{2},k+\frac{1}{2}\right) \\ & + \frac{\Delta t}{\mu\left(i,j-\frac{1}{2},k+\frac{1}{2}\right)} \frac{1}{\Delta y \Delta z} \\ & \times \begin{bmatrix} -E_z^n\left(i,j,k+\frac{1}{2}\right)\Delta z + E_z^n\left(i,j-1,k+\frac{1}{2}\right)\Delta z \\ +E_y^n\left(i,j-\frac{1}{2},k+1\right)\Delta y - E_y^n\left(i,j-\frac{1}{2},k\right)\Delta y \end{bmatrix}. \end{aligned} \tag{A.4}$$

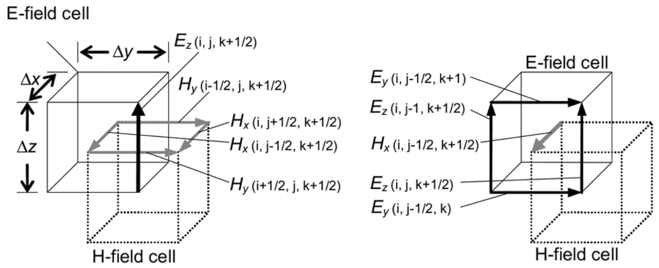


Fig. 10. Placement of electric-field and magnetic-field cells for solving discretized Maxwell's equations using the FDTD method.

obtained. Since the material constants of each cell can be specified individually, a complex inhomogeneous medium can be analyzed easily.

In order to analyze fields in unbounded space, an absorbing boundary condition has to be set on each plane which limits the space to be analyzed, so as to avoid reflections there. The FDTD method allows one to incorporate wires buried in lossy ground, such as strike-object grounding electrodes (Noda *et al.* [47]), and nonlinear effects.

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