The lightning striking distance—Revisited

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Abstract

First return stroke current waveforms measured by Berger [Methods and results of lightning records at Monte San Salvatore from 1963–1971 (in German), Bull. Schweiz. Elektrotech. ver. 63 (1972) 21403—21422] and Berger and Vogelsanger [Measurement and results of lightning records at Monte San Salvatore from 1955–1963 (in German), Bull. Schweiz. Elektrotech. ver. 56 (1965) 2–22] are used to estimate the charge stored in the lightning stepped leader channel. As opposed to previous charge estimates based on the entire current waveform, only the initial portion of measured current waveforms (100 ms in duration) was used in order to avoid the inclusion of any charges not involved in the effective neutralization of charges originally stored on the leader channel. The charge brought to ground by the return stroke within the first 100 ms, $Q_{f,100}$ (in C) is related to the first return stroke peak current, $I_{pf}$ (in kA), as $Q_{f,100} = 0.61 I_{pf}$. From this equation the charge distribution of the stepped leader as a function of the corresponding peak return stroke current is estimated. This distribution (along with the assumed average electric field of 500 kV/m in the final gap) is used to estimate the lightning striking distance $S$ (in meters) to a flat ground as a function of the prospective return stroke peak current $I$ (in kA): $S = 1.9 I_{pf}^{0.90}$. For the median first stroke peak current of 30 kA one obtains $S = 41$ m, while the traditional equation, $S = 10 I_{pf}^{0.65}$, gives $S = 91$ m. In our view, the new equation for striking distance provides a more physically realistic basis for the electro-geometric approach widely used in estimating lightning incidence to power lines and other structures.

Keywords: Lightning; Return strokes; Striking Distance; Stepped Leader

1. Introduction and literature review

As the stepped leader approaches the ground, the electric field at ground, particularly at the upper extremities of grounded objects, increases. When this field reaches the critical breakdown value, a connecting leader is launched toward the descending leader. The distance to the leader tip from a grounded structure when a connecting leader is initiated from this structure is called the striking distance. This distance depends on the electric field generated by the stepped leader, which in turn is determined by the distribution of charge on the stepped leader channel. After its initiation, the return stroke travels along the leader channel neutralizing this charge. It is customary in the practice of lightning protection to formulate the criterion for the onset of the upward connecting leader in terms of the return stroke peak current measured at the base of the lightning channel. This requires a relationship between the leader charge distribution and the return stroke peak current. We examine such relationships found in the literature and suggest a new one that better reflects the physics involved.

1.1. Golde [1,2]

Golde [1,2] was the first to suggest a relationship between the return-stroke peak current and the leader charge. In his derivation Golde assumed that the line charge density, $\rho_s$, on the vertical stepped-leader channel decreases exponentially with increasing height above ground,

$$\rho_s = \rho_{sz} e^{-z/\lambda},$$

where $\rho_{sz}$ is $\rho_s$ at $z = 0$ and $\lambda$ is the decay height constant ($\lambda = 1000$ m [1,2]). The total charge on the leader channel is
where $I_{pf}$ is the return stroke peak current in kA, $Q$ is in C and $k = 25$ kA/C. (Golde [26] suggested $k = 20$ kA/C.) Combining Eq. (3) with Eq. (2) one obtains:

$$\rho_{so} = 4.36 \times 10^{-5} I_{pf}.$$  

(4)

Golde did not give any justification for the assumed linear relationship (3) between the return stroke peak current and stepped-leader charge.

1.2. Eriksson [3]

Using current waveforms of first return strokes measured on the towers on Monte San Salvatore, Berger [4] found a relatively strong correlation between the first return stroke current peak, $I_{pf}$, and the charge brought to ground within 2 ms from the beginning of the return stroke (called impulse charge), $Q_{im}$. The relation can be represented by the following equation [3]:

$$I_{pf} = 10.6 Q_{im}^{0.7}.$$  

(5)

According to (5), a 25 kA peak current corresponds to a stepped-leader charge of 3.3 C. Based on Golde’s [1] suggestion that only the charge located on the lower portions of the leader channel is related to the peak current (a 25 kA peak current corresponds to a stepped leader charge of 1 C) and after comparing some of the measured striking distances with analytical results, Eriksson [3] modified the above relationship to:

$$I_{pf} = 29.4 Q_{im}^{0.7},$$  

(6)

where $I_{pf}$ is in kA and $Q$ in C. Eriksson [3] assumed that the charge is distributed linearly along a vertical leader channel of 5 km length. When this assumption is combined with (6) one obtains:

$$\rho_{so} = 3.2 \times 10^{-6} I_{pf}^{1.43}.$$  

(7)

The reference point for (6) is based on Golde’s assumption that a 25 kA peak current is associated with a stepped-leader charge of about 1 C.

1.3. Dellera and Garbagnati [5]

In some of the first return stroke currents measured by Berger [4] and Berger and Vogelsanger [6] one can observe a secondary peak (or a change in slope) appearing in the waveform after a few tens of microseconds from the beginning of the waveform. The time of occurrence of this secondary peak may change from one stroke to another. Dellera and Garbagnati [5] assumed that this subsidiary peak is associated with a return stroke current reflection from the upper end of the leader channel. They integrated the current waveforms that exhibited the secondary peak from different studies up to this subsidiary peak (or the change in slope) and assumed that the resultant charge was originally distributed uniformly along the leader channel. The length of the leader channel was calculated from the time to the subsidiary peak by assuming that the return stroke speed is a function of peak current and is given by the equation derived by Wagner [7]. From their analysis, Dellera and Garbagnati [5] obtained the following relationship between $\rho_{so}$ and $I_{pf}$:

$$\rho_{so} = 3.8 \times 10^{-5} I_{pf}^{0.68},$$  

(8)

where $\rho_{so}$ is in C/m and $I_{pf}$ is in kA.

1.4. Cooray [8]

Cooray [8] utilized a return-stroke model introduced by him for first return strokes to extract the relationship between the return stroke peak current and the charge per unit length at the bottom end of the leader channel. The result is given by the following equation:

$$\rho_{so} = 2.28 \times 10^{-6} + 1.46 \times 10^{-5} I_{pf} + 1.1 \times 10^{-7} I_{pf}^2,$$  

(9)

where $\rho_{so}$ is in C/m and $I_{pf}$ in kA. Since the above equation is based on a number of assumptions used in developing the return-stroke model, it is in need of independent confirmation.

2. The total stepped-leader charge as a function of peak current

We will use Berger’s return-stroke current waveforms to estimate the charge (e.g. Fig. 1) brought to ground by the return stroke within the first 100 $\mu$s of the discharge. The information we gather from this exercise will be used in turn to estimate the charge distribution of the stepped leader channel as follows.
The total positive charge that enters into the leader channel at the strike point (or the negative charge that goes into ground) during the return stroke can be divided into three components. The first part is the positive charge, \( Q_h \), that is necessary to neutralize the negative charge originally stored in the leader channel (see Fig. 2c). The second part, \( Q_s \), is the positive charge induced in the return stroke channel, which is essentially at ground potential, due to the background electric field produced by remaining cloud charges (see Fig. 2d). The third part is the additional positive charge spent to neutralize negative cloud charge that was not involved in the leader process (continuing current charge). The latter can be disregarded if the measured current is integrated up to the time of arrival of the return stroke at the point of initiation of the leader. This requires a reasonable assumption on the time taken by the return stroke to reach the point of initiation of the leader. Since, we utilize the data obtained by Berger [4] and Berger and Vogelsanger [6] in this analysis, it is important that this time is pertinent to their study in Switzerland.

The time needed for the return stroke front to reach the point of leader initiation can be obtained from the following consideration:

1. The negative charge region in thunderclouds is located in the vicinity of the \(-10^\circ\) isotherm [23]. In temperate localities this isotherm is typically observed at a height of about 5 km from ground level. The height of the negative charge region in thunderstorms in Switzerland is probably close to this value. The tower used by Berger in his studies was about 70 m high, and it was located at the top of Monte San Salvatore at an altitude of 640 m above sea level. Since ground lightning flashes are probably initiated at the lower boundary of the negative charge region in the cloud where the electric field is higher than in the interior of the charge region, a reasonable estimate of the height of lightning initiation point in the cloud is 4 km. According to the measurements of Idone and Orville [15], the first return stroke speeds averaged over the bottom kilometre or so of the lightning channel are typically about \(10^8\) m/s. The observations also showed that the return stroke speed decreases with increasing height. According to the measurements of Schonland [9], the first return stroke speed close to the cloud base which was located at a height of about 2 km from ground level in South Africa is about \(5 \times 10^7\) m/s. Further, the minimum first return stroke speed measured in both the above-mentioned studies is \(2 \times 10^7\) m/s. From these observations one can safely conclude that the average speed of first return strokes over a 4-km channel may be close to \(5 \times 10^7\) m/s. Interestingly, Shao's [10], from VHF imaging of lightning channels in Florida, found, though for a single first return stroke, the average speed from ground to the point of initiation of the stepped leader to be \(5 \times 10^7\) m/s. For an average return-stroke speed of \(5 \times 10^7\) m/s, the charge brought to ground by Berger’s first return strokes during their travel from ground to the point of initiation of the stepped leader at 4 km (see above) occurs within \(80 \mu s\) of the beginning of current waveforms (or less if a higher average return-stroke speed is assumed).

2. Measurements of 3-MHz radiation associated with first return strokes show that the emission starts almost simultaneously with the initiation of the return stroke and ends more or less abruptly in about \(130 \mu s\) in temperate Sweden and in about \(200 \mu s\) in tropical Sri Lanka [11,12]. This observation provides indirect evidence that the travel time of the return stroke to the leader initiation point (origin of the flash) is about \(130 \mu s\) in temperate regions and it may be about \(200 \mu s\) in the tropics.

Based on these considerations it is assumed that the charge transported to ground by the first return stroke within \(100 \mu s\) of its initiation is approximately equal to the sum of the positive charge that is necessary to neutralize the negative charge originally stored in the leader channel and the positive charge induced in the return stroke channel. The charge of the stepped leader is equal in magnitude (but of opposite sign) to the former. This assumption is further discussed in Section 6. The total stepped-leader charge is needed for finding the charge distribution along the leader channel and then the striking distance.

First return stroke currents of Berger [4] and Berger and Vogelsanger [6] were integrated over the first \(100 \mu s\), and the results are depicted as a function of peak current in Fig. 1. Note that there is a strong linear correlation
between the two parameters with the correlation coefficient of about 0.94. The corresponding linear regression equation is given by

$$Q_{f,100 \mu s} = 0.061 I_{pf}. \tag{10}$$

where $Q_{f,100 \mu s}$ is the charge (in C) neutralized by the first return stroke within the first 100 $\mu$s, and $I_{pf}$ is the first return stroke peak current in kA. The next task is to infer the distribution of charge found from (10) along the stepped-leader channel.

3. The distribution of charge per unit length along the stepped leader channel as a function of peak current

In order to obtain the distribution of charge along the stepped-leader channel from the information given in the previous section, it is necessary to simplify the real problem. The real problem and its idealisation that we used below in the numerical simulation are illustrated in Fig. 2. It was assumed that the horizontal extent of the negative charge region in the cloud is large in comparison to the vertical distance between the ground and the charge region. Based on this assumption the cloud charge region was replaced by a perfectly conducting plane maintained at a given potential. This configuration simulates a uniform electric field between the cloud and the ground. The leader channel is simulated by a vertical lossy conductor of cylindrical geometry with a hemispherical tip. The losses are represented by a constant potential gradient. The well-known charge simulation method is used to obtain the charge distribution on the leader channel in a given electric field. It is of interest to note that the charge distribution induced on the stepped leader channel as it propagates towards the ground is identical to the charge distribution that would be induced on the lower half (below the point of initiation) of a vertical bi-directional leader developing in a uniform electric field. As the stepped leader extends towards the ground its charge distribution is determined by the background electric field generated by the cloud charges and any field enhancement caused by the presence of the ground, i.e. the proximity effect. As mentioned earlier, once the contact is established between the

![Fig. 2.](image)

Fig. 2. (a) A sketch of the stepped leader approaching ground. (b) The idealisation used in the computation of charge distribution along the leader channel. (c) Situation just before the attachment process and return stroke (the negative charge density increases downwards). (d) Situation after the return stroke (the positive charge density increases upwards). In the figure $z_t$ is the distance between the tip of the leader and the ground. In the case of lightning strike to a tower it represents the separation between the leader tip and the top of the tower. In general, $z_t$ is greater than the striking distance, although in (c) it is implied to be equal to the striking distance. $Q_l$ is the total charge deposited on the leader channel, and $Q_i$ is the total charge induced on the fully-developed return-stroke channel by the cloud electric field $E_o$. 
negatively charged stepped leader and the ground the total
positive charge entering the channel from the ground
during the first 100 μs of the return stroke is equal to the
sum of the positive charge that is necessary to neutralize
the negative charge \( Q_l \) of the leader and the positive charge
\( Q_i \) induced on the channel due to the remaining negative
charges in the cloud. In the configuration shown in Fig.
2(b), the two components, labelled \( Q_i \) and \( Q_l \) in Figs. 2(c)
and (d), have approximately the same magnitudes for the
following reasons: (1) Lightning channels at the final stage
of both the leader and the return stroke are exposed to the
same background field \( E_0 \) (see Figs. 2c and d). (2) Lightning
channels at the final stages of both the leader
and the return stroke can be treated as steady-state arc
channels. (3) Since the potential gradient of an arc channel
is more or less independent of current [13], the final
potential gradient of the return-stroke channel is more or
less the same as that of the leader channel. (4) In the
absence of field enhancement caused by the ground, the
negative charge density (Fig. 2c) will increase linearly
towards the ground while the positive charge induced on
the return stroke channel (Fig. 2d) will increase linearly
towards the cloud. Thus, the two charge components will
have more or less the same magnitude but opposite signs.
The balance will be slightly disturbed by the field
enhancement caused by the ground. As a result, the
negative charge density near the leader tip will increase
almost exponentially when the tip is close to the ground
leading to a slight increase in the negative charge
component. Our calculations show, however, the difference
between \( Q_l \) and \( Q_i \) is less than about 10%. 

In order to obtain the leader charge distribution as a
function of return stroke peak current the following procedure was used. Different values of peak current
correspond to different values of cloud potential (Fig. 2b)
and, hence, to different values of \( E_0 \) (Fig. 2c). Further, the
leader charge distribution depends on the assumed value of
\( z_t \). Consider a return stroke peak current \( I_{pf} \). Since the total
charge injected into the channel from the ground during a
return stroke characterized by this peak current is about
0.061 \( I_{pf} \) (see (10)), the potential of the cloud in the
configuration shown in Fig. 2b is adjusted until the sum
\( Q_l + Q_i \) is equal to 0.061 \( I_{pf} \). The resultant leader charge
distribution is computed for different values of \( z_t \).

One of the input parameters required in the simulation is
the radius of the leader channel. The radius of the leader
channel is adjusted so that the average charge per unit
length the stepped leader channel, \( \rho_{av} \), and the leader
channel radius, \( R_l \), satisfy the equation:

\[
R_l = \frac{\rho_{av}}{2\pi \varepsilon_0 E_c}, \tag{11}
\]

\[
\rho_{av} = \frac{Q_l}{H}, \tag{12}
\]

where \( H \) is the length of the leader channel. This equation
is based on the Gauss’ law and on the assumption that the
air breakdown at the lateral surface of the leader channel (radial expansion of the corona sheath) con-
tinues until the electric field at the outer channel boundary, is equal to \( E_c = 3.0 \times 10^6 \) V/m, the electric
breakdown field at normal atmospheric conditions. It
should be pointed out that in reality the radius of the
leader channel varies as a function of height be-
cause the charge density along the leader channel decreases with height. In the calculations presented
in this paper the average radius of the leader channel is
obtained from 11 and 12 and the whole leader channel
is assumed to have this average radius along the whole
length.

It is important to note that, in the configuration shown
in Fig. 2 where the background electric field, \( E_o \) generated
by the cloud is uniform, for a given \( |Q_l + Q_i| \), the estimated
leader charge distribution does not depend on potential
gradient, \( E_{z_l} \), of the leader channel. The reason for this is
that the charge distribution is determined by the difference
\( E_{z_l} - E_l \) and not by individual values of \( E_{z_l} \) and \( E_l \). However,
the cloud potential, and hence the value of \( E_{z_l} \) required to
dissipate a given amount of charge in the return stroke
(corresponding to a given value of peak current) are
influenced by the potential gradient of the leader. If the
leader channel were assumed to be perfectly conducting
\( (E_l = 0) \), the leader charge distribution would be a function of \( E_{z_l} \).

The leader charge distributions corresponding to a 30 kA
peak current for three values of the leader tip height above
ground \( z_t \) (see Fig. 2c) are shown in Fig. 3. The range of
variation of \( z_t \), from 10 to 100 m, in Fig. 3 was selected so
that to include the expected values of striking distance.
Note that the charge distribution is approximately linear
except near the tip of the leader. The abrupt increase of
charge density at the tip is caused partly by the presence of
the ground. (Note how the charge at the tip decreases with
increasing \( z_t \)). The charge distribution corresponding to
\( z_t = 50 \) m will be used for estimating the striking distance,
although the result is not sensitive to which of the three
leader charge distributions shown in Fig. 3 is used (see
Section 5).

Note that the charge distributions given in Fig. 3 are
valid for a fully developed stepped leader channel
with its tip near the ground. The charge distribution
along the leader channel when the leader tip is far
away from the ground is different from those given in
this figure. The charge distribution along the leader
channel when its tip is located at different heights from
the ground is depicted in Fig. 4. Note that as the leader
propagates downwards the highest charge density is
encountered at the channel element in which the leader
tip is located. As the leader tip moves downward the charge
density in that channel element decreases and finally
approaches the value corresponding to a fully extended
stepped leader.

The data shown in Fig. 4 can be summarized approxi-
mately by a single equation that describes how the charge
on the stepped leader channel varies as it propagates towards the ground. That equation is the following.

\[
\rho(\xi) = a_0 \left(1 - \frac{\xi}{H - z_0}\right) G(z_0) I_p + \frac{I_p(a + b\xi)}{1 + c\xi + d\xi^2} H(z_0)
\]
\[
z_0 \geq 10,
\]
\[
G(z_0) = 1 - (z_0/H),
\]
\[
H(z_0) = 0.3\alpha + 0.7\beta,
\]
\[
\alpha = e^{-(z_0/100)/75},
\]
\[
\beta = \left(1 - \frac{z_0}{H}\right).
\]

where \(z_0\) is the height of the leader tip above ground in meters (note that the above equation is valid for \(z_0 > 10\) m), \(H\) is the height of the cloud in meters, \(\rho(\xi)\) is the charge per unit length (in C/m), \(\xi\) is the length along the stepped leader channel with \(\xi = 0\) at the tip of the leader, \(I_p\) is the return stroke peak current in kA, \(a_0 = 1.476 \times 10^{-5}\), \(a = 4.857 \times 10^{-5}\), \(b = 3.9097 \times 10^{-6}\), \(c = 0.522\) and \(d = 3.73 \times 10^{-3}\).
Our calculations show that for a given charge per unit length at the bottom end of the fully developed stepped leader the charge distribution of the lower kilometre or so of the channel does not depend on \( H \). From there onwards the charge per unit length decreases linearly to zero at the top of the channel. As a result, Eq. (13), obtained for \( H = 4 \text{ km} \) can be used with any value of \( H \) provided that it is larger than about 3 km.

### 4. Testing the validity of the procedure to estimate the stepped-leader charge distribution

In order to test the validity of the procedure outlined in the previous section, we will apply the same procedure to triggered lightning strokes, use the resultant charge distribution for computing close electric fields and compare them with the measured ones (e.g., Crawford et al. [16]).

It is known [18,24,25] that individual strokes in a multiple-stroke ground flash tap negatively charged regions that are displaced primarily horizontally from each other. Thus the vertical length of the dart leader channels involved in subsequent strokes in Berger’s study may only be slightly larger than the 4 km length assumed for first strokes. Let us assume 5 km as a representative value of the dart-leader length. The optically observed average return-stroke speed over the bottom 2 km or so of the channel is about \( 10^8 \text{ m/s} \) [14,15], and it does not change much along the lightning channel. If we assume that this speed is maintained along the entire channel, then the return-stroke front will reach the height of 5 km in 50 \( \mu \text{s} \). An average return stroke speed of about \( 10^8 \text{ m/s} \) along the entire length of the dart leader channel is also supported by the observations of Shao [10] who found that the average subsequent return stroke speeds over channel lengths of 10–15 km range from \( 0.5 \times 10^8 \) to \( 1.5 \times 10^8 \text{ m/s} \). Based on these observations, we integrated Berger’s subsequent return stroke currents up to 50 \( \mu \text{s} \), and plotted the results as a function of peak current in Fig. 5. Similar to Fig. 1, one can observe a strong linear relationship between the subsequent stroke peak current, \( I_{ps} \), and the charge dissipated within the first 50 \( \mu \text{s} \), \( Q_{50,50\mu s} \). The results can be represented by the following equation:

\[
Q_{50,50\mu s} = 0.028 I_{ps},
\]

where the charge is in C and the peak current in kA.

The same procedure as before (see Section 3) is used to obtain the charge distribution along the leader channel corresponding to different peak currents. The charge distribution on the dart leader channel when the tip of the leader is at different heights from ground level can be obtained from Eqs. (13)–(17) using \( a_0 = 5.09 \times 10^{-6} \), \( a = 1.325 \times 10^{-5} \), \( b = 7.06 \times 10^{-6} \), \( c = 2.089 \), and \( d = 1.492 \times 10^{-2} \).

Once the charge distribution along the leader channel is known, the close electric field at a given point at ground level can be calculated and compared with measurements. For a vertical dart leader channel of length \( H \) the electric field, \( E_z \), at distance \( D \) from the ground strike point is given by

\[
E_z = \int_0^H \rho(z) \frac{dz}{2\pi\epsilon_0 (D^2 + z^2)^{3/2}},
\]

where \( \rho(z) \) is given by Eqs. (13)–(17) using the values of coefficients given above and \( \epsilon_0 \) is the permittivity of free space. The electric fields of dart leaders at 50 and 110 m as a function of ensuing return stroke peak current, calculated using the above equation, are represented by solid lines in Fig. 6. Also shown in Fig. 6 are corresponding experimental data (crosses and broken lines) for triggered lightning in Florida (1997–1999) as reported by Crawford et al. [16]. Note that the calculated fields agree with the measurements within about 20%. This is in support of the procedure used in this paper to obtain the charge distribution on the leader channel as a function of peak current, if the characteristics of the processes involved for Berger’s subsequent strokes in Switzerland and for triggered lightning strokes in Florida are similar.

Interestingly, triggered-lightning data of Crawford et al. [16] indicate an inverse distance dependence of the close leader electric field change, which is consistent with a more or less uniform leader charge density distribution along the bottom kilometre or so of the channel. On the other hand, in our model the leader charge density decreases linearly with height with some additional charge at the leader tip (see Fig. 3). A fairly good agreement between the model predictions and measurements seen in Fig. 6 probably implies that close electric fields that are determined by charges on a relatively short segment of the channel (some
hundreds of meters in length) are essentially independent of the type of charge density distribution. As a result, a non-uniform charge density distribution produces essentially the same close electric fields as the uniform one, as long as the average charge per unit length on the bottom portion of the channel is approximately same. The charge distribution derived here could also be used in various return stroke models that require the charge distribution along the leader channel as input [20,21].

5. The striking distance of stepped leaders as a function of return stroke peak current

In order to obtain the striking distance we will evaluate the distance from ground to the tip of the stepped leader when the average potential gradient between the leader tip and the ground is equal to 500 kV/m. As shown in Section 3, the charge distribution on the leader channel depends on the distance between the tip of the leader and the ground. Thus, in general, one cannot use the charge distribution derived for a fixed value of \( z_t \) in evaluating the striking distance. However, it is possible to demonstrate that the charge distributions obtained with \( z_t \) equal to some tens of meters can be used in evaluating the striking distance without a significant error. For example, the striking distance obtained for two charge density distributions using \( z_t = 10 \) and 100 m are depicted in Fig. 7. Note that the striking distance is insensitive to the value of \( z_t \). The reason for this is that the striking distance depends on the average electric field in the gap between the leader tip and the ground. The differences in the charge distributions for different values of \( z_t \) do not influence this average electric field significantly. Thus, in evaluating the striking distance one can use the charge distribution obtained for, say, \( z_t = 50 \) m, with the result being insensitive to the choice of \( z_t \) in the range (10–100 m) covering the range of expected values of striking distance. In a recent paper an attempt was made to relate the striking distance to the leader potential instead of the more conventional approach of relating it to the leader charge [22]. The conventional approach is adopted in the present paper.

From the results obtained in this paper it is difficult to derive an explicit relationship between the charge per unit length on the leader channel and the return stroke peak current.

![Fig. 6. Comparison of the dart leader electric fields calculated at: (a) 50 m, and (b) 110 m from the lightning channel base with the measurements reported by Crawford et al. [16]. The solid line represents the calculations and the crosses represent the measured fields. The dotted line gives the best fit to the experimental data.](image)

![Fig. 7. Striking distance calculated using the charge density distributions corresponding to \( z_t = 10 \) m (solid line) and \( z_t = 100 \) m (dashed line).](image)
current because the charge density in the vicinity of the leader tip vary very rapidly with height. However, if we take the average charge density over the bottom hundred meters of the channel, the charge that is most important in the formation of the return stroke peak current, its relationship to the return stroke peak current can be expressed as:

\[ \rho_{av} = 2.7 \times 10^{-5} I_p, \]  

(20)

where \( \rho_{av} \) is the average (over the bottom 100 m) leader charge density in C/m, and \( I_p \) is the return stroke peak current in kA. The relationship given in (20) is depicted in Fig. 8(a). Previously suggested the charge density-current relationships given by Eqs. (4), (7), (8) and (9) are also depicted in Fig. 8(a).

The striking distance as a function of peak current obtained using the charge density distributions given by Eqs. (4), (7), (8) and (9) are also depicted in Fig. 8(b). Note that the striking distances obtained here from Eqs. (4) and (7) differ from those calculated by Golde [1] and Eriksson [3], respectively, using the same equations. The reason for this discrepancy is the following. Even though Golde [2] defines the striking distance as the distance when the average potential gradient across the striking distance is 500 kV/m, in order to simplify the computation he assumed that the striking distance is reached when the electric field at ground level attains 500 kV/m. This simplification results in an average potential gradient larger than 500 kV/m across the striking distance. The striking distance published by Eriksson [3] is the striking distance to a 60 m tower and not to a flat ground and hence is not directly comparable to our results. The results obtained in the present study can be summarised by the following equation for the striking distance, \( S \) (in meters), as a function of peak current, \( I \) (in kA; all subscripts dropped):

\[ S = 1.9 I^{0.90}. \]  

(21)

It follows from Eq. (21) that a 30 kA peak current is associated with a striking distance to flat ground of about 41 m, while the traditional equation, \( S = 10 I^{0.65} \) [26], gives \( S = 91 \) m. In our view, Eq. (21) yields a more physically realistic value of striking distance. A more practical case of striking distance to an object protruding above ground is a subject of future studies. The striking distance to flat ground considered here may be applicable to the case of a tower-like object, if it is redefined as the distance to the upper end of upward connecting leader that effectively increases the height of the object at the time when the common streamer zone of the descending and upward connecting leaders is formed [27].

6. Discussion of the assumptions made in this study

In deriving the relationship between the stepped leader charge and the return-stroke peak current several simplifying assumptions are made. We discuss these assumptions below. Some of the assumptions can be justified using available experimental data and the testing of the validity of others awaits additional experimental data.

It is important to note that the charge given by (10) (1.8 C for the median first-stroke peak current of 30 kA) is not the total charge that is being brought to ground during the return stroke. The median value of the total charge brought to ground during return strokes is in the range of 4–5 C [4,6,18,19]. On the other hand Eq. (10) gives only the
charge transferred to the ground during the first 100 µs. By integrating the same current waveforms as those used in our study but over a time period of 2 ms Berger obtained 4.5 C as a typical value (the impulse charge). This means that about 2.7 C of charge has to be contributed by the charge neutralisation processes taking place in branches and on in-cloud horizontal channel sections, or by neutralisation processes taking place in the corona sheath after the first 100 µs.

Note that in order to obtain the striking distance what is really needed is the charge distribution on the first few hundred meters of the stepped leader channel. Contributions to the electric field in the final gap from charges stored on the higher channel sections are negligible. The entire channel was considered here in order to obtain a self-consistent formulation of the problem (see Fig. 2).

6.1. Uniform background electric field

We assumed that the electric field between the cloud charge region and the ground is uniform. This assumption is valid if the cloud charge region has a very large horizontal extent, compared to its height above ground. Some justification for this assumption is found in Willett et al. [17] who have analysed the vertical profile of the electric field below mature thunderclouds. Their data show that after an initial increase of the field within the first few tens of meters (caused by a corona charge layer), the electric field remains more or less constant with altitude over the first 1–2 km. Unfortunately, no measurements are presented for higher altitudes.

6.2. Electrostatic approach

In calculating the charge distribution on the leader channel it is assumed that the leader charge is always in equilibrium with the background electric field. That is, the calculated charge distribution corresponds to a steady state condition whereas in reality the leader process is dynamic. In fact, during each step of the leader the charge distribution along the whole channel has to be readjusted so that it conform to the background electric field and the channel potential gradient. On the other hand, the speed of the stepped leader is much lower than the speed of light, and it is reasonable to assume that at a given time the charge distribution along the channel is close to the steady state one. Without this assumption, the calculated charge per unit length would be shifted towards lower values.

6.3. Channel branches

In the calculations any channel branches were neglected. In reality the charge dissipated by the return stroke is originated partly from the branches and partly from the main channel. The contribution of the branches to the charge dissipated within the first 100 µs depends on how fast the return stroke front will travel along the branches and the amount of charge located on them. Neglecting the contribution of branches to the return stroke charge leads to an overestimation of the charge per unit length on the leader channel.

6.4. 100-µs integration time

The charge of the leader channel resides mainly on the corona sheath. The time that is necessary to remove charge from the corona sheath depends on the rate at which the central core of the leader channel is brought to ground potential during the return stroke, the age of the corona sheath, the radius of the corona sheath and the speed of propagation of the discharge channels that neutralize the corona sheath. At present no much information is available concerning any of these processes and, therefore, one has to regard the estimated leader charge density as a lower limit because it is based on the assumption that the leader charge is neutralised within a few tens of microseconds (actually this time varies from 100 µs at ground level to zero at the assumed leader origination point). A longer neutralisation time would require a longer current integration time than the 100 µs adopted here, and this will necessarily increase the estimated leader charge density. We cannot rule out the possibility that the neutralisation time of the corona sheath is longer than the 100 µs assumed in this paper. Thus, it would be appropriate to treat the striking distance estimated here as a lower limit. In lightning protection, it is the lower limit of the striking distance that is of primary interest, because it will set limits on the maximum spacing between the air terminals on a structure to be protected.

7. Conclusions

By evaluating the charge dissipated by the first return strokes studied by Berger within the first 100’s the charge stored on the stepped leader channel is estimated. This charge, $Q_f$, 100 µs (in C), is related to the return stroke peak current, $I_{pf}$ (in kA) by the equation $Q_f, 100 \mu s = 0.61 I_{pf}$. Based on electrostatic considerations, the distribution of the charge along the leader channel is found. This in turn is used (along with the assumed electric field of 500 kV/m in the final gap) to estimate the striking distance of the stepped leader to flat ground as a function of the prospective return stroke peak current: $S = 1.9 I_{pf}^{0.90}$.

References


