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The lightning striking distance—Revisited

Vernon Cooray[a,*], Vladimir Rakov[b], Nelson Theethayi[a]

[a]Division for Electricity and Lightning Research, Uppsala University, Box-534, SE-75121 Uppsala, Sweden
[b]Department of Electrical and Computer Engineering, University of Florida, Box-16130, Gainesville, FL 32611-6130, USA

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Abstract

First return stroke current waveforms measured by Berger [Methods and results of lightning records at Monte San Salvatore from 1963–1971 (in German), Bull. Schweiz. Elektrotech. ver. 63 (1972) 21403—21422] and Berger and Vogelsanger [Measurement and results of lightning records at Monte San Salvatore from 1955–1963 (in German), Bull. Schweiz. Elektrotech. ver. 56 (1965) 2–22] are used to estimate the charge stored in the lightning stepped leader channel. As opposed to previous charge estimates based on the entire current waveform, only the initial portion of measured current waveforms (100 μs in duration) was used in order to avoid the inclusion of any charges not involved in the effective neutralization of charges originally stored on the leader channel. The charge brought to ground by the return stroke within the first 100 μs, $Q_{f,100\,\mu s}$ (in C) is related to the first return stroke peak current, I_{pf} (in kA), as $Q_{f,100\,\mu s} = 0.61\, I_{pf}$. From this equation the charge distribution of the stepped leader as a function of the corresponding peak return stroke current is estimated. This distribution (along with the assumed average electric field of 500 kV/m in the final gap) is used to estimate the lightning striking distance S (in meters) to a flat ground as a function of the prospective return stroke peak current I (in kA): $S = 1.9\, I_{pf}^{0.90}$. For the median first stroke peak current of 30 kA one obtains $S = 41$ m, while the traditional equation, $S = 10\, I_{pf}^{0.65}$, gives $S = 91$ m. In our view, the new equation for striking distance provides a more physically realistic basis for the electro-geometric approach widely used in estimating lightning incidence to power lines and other structures.

Keywords: Lightning; Return strokes; Striking Distance; Stepped Leader

1. Introduction and literature review

As the stepped leader approaches the ground, the electric field at ground, particularly at the upper extremities of grounded objects, increases. When this field reaches the critical breakdown value, a connecting leader is launched toward the descending leader. The distance to the leader tip from a grounded structure when a connecting leader is initiated from this structure is called the striking distance. This distance depends on the electric field generated by the stepped leader, which in turn is determined by the distribution of charge on the stepped leader channel. After its initiation, the return stroke travels along the leader channel neutralizing this charge. It is customary in the practice of lightning protection to formulate the criterion for the onset of the upward connecting leader in terms of the return stroke peak current measured at the base of the lightning channel. This requires a relationship between the leader charge distribution and the return stroke peak current. We examine such relationships found in the literature and suggest a new one that better reflects the physics involved.

1.1. Golde [1,2]

Golde [1,2] was the first to suggest a relationship between the return-stroke peak current and the leader charge. In his derivation Golde assumed that the line charge density, ρ_s, on the vertical stepped-leader channel decreases exponentially with increasing height above ground,

$$\rho_s = \rho_{so} e^{-z/\lambda}, \tag{1}$$

where ρ_{so} is ρ_s at $z = 0$ and λ is the decay height constant ($\lambda = 1000$ m [1,2]). The total charge on the leader channel is

*Corresponding author.
E-mail address: vernon.cooray@angstrom.uu.se (V. Cooray).

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given by

$$Q = \rho_{so}\lambda\left[1 - e^{-H/\lambda}\right], \tag{2}$$

where H is the total length of the channel. Golde [1] assumed that $H = 2.5 \times 10^3$ m. Using this charge distribution, Golde [1] calculated the electric field at ground as a function of height of the stepped leader tip above ground (directly above the field point). He assumed that a connecting leader is initiated from ground when the electric field there exceeds about 10^6 V/m. With this assumption he was able to estimate the maximum possible length of connecting leaders as a function of downward-moving leader charge. Golde also analysed the photographs of lightning strikes and concluded that the length of connecting leaders does not exceed a few tens of meters. Based on this conclusion he assumed that a typical peak current of 25 kA is associated with a stepped-leader charge of about 1 C. Such a leader produces a field of about 10^6 V/m at ground level when the leader tip is about 17 m above ground, thus limiting the length of connecting leaders of typical lightning first strokes to less than a few tens of meters. Further, he assumed that the return stroke peak current increases linearly with increasing leader charge,

$$I_{pf} = kQ, \tag{3}$$

where I_{pf} is the return stroke peak current in kA, Q is in C and $k = 25$ kA/C. (Golde [26] suggested $k = 20$ kA/C.) Combining Eq. (3) with Eq. (2) one obtains:

$$\rho_{so} = 4.36 \times 10^{-5} I_{pf}. \tag{4}$$

Golde did not give any justification for the assumed linear relationship (3) between the return stroke peak current and stepped-leader charge.

1.2. Eriksson [3]

Using current waveforms of first return strokes measured on the towers on Monte San Salvatore, Berger [4] found a relatively strong correlation between the first return stroke current peak, I_{pf}, and the charge brought to ground within 2 ms from the beginning of the return stroke (called impulse charge), Q_{im}. The relation can be represented by the following equation [3]:

$$I_{pf} = 10.6 Q_{im}^{0.7}. \tag{5}$$

According to (5), a 25 kA peak current corresponds to a stepped-leader charge of 3.3 C. Based on Golde's [1] suggestion that only the charge located on the lower portions of the leader channel is related to the peak current (a 25 kA peak current corresponds to a stepped leader charge of 1 C) and after comparing some of the measured striking distances with analytical results, Eriksson [3] modified the above relationship to:

$$I_{pf} = 29.4 Q^{0.7}, \tag{6}$$

where I_{pf} is in kA and Q in C. Eriksson [3] assumed that the charge is distributed linearly along a vertical leader channel of 5 km length. When this assumption is combined with (6) one obtains:

$$\rho_{so} = 3.2 \times 10^{-6} I_{pf}^{1.43}. \tag{7}$$

The reference point for (6) is based on Golde's assumption that a 25 kA peak current is associated with a stepped-leader charge of about 1 C.

1.3. Dellera and Garbagnati [5]

In some of the first return stroke currents measured by Berger [4] and Berger and Vogelsanger [6] one can observe a secondary peak (or a change in slope) appearing in the waveform after a few tens of microseconds from the beginning of the waveform. The time of occurrence of this secondary peak may change from one stroke to another. Dellera and Garbagnati [5] assumed that this subsidiary peak is associated with a return stroke current reflection from the upper end of the leader channel. They integrated the current waveforms that exhibited the secondary peak from different studies up to this subsidiary peak (or the change in slope) and assumed that the resultant charge was originally distributed uniformly along the leader channel. The length of the leader channel was calculated from the time to the subsidiary peak by assuming that the return stroke speed is a function of peak current and is given by the equation derived by Wagner [7]. From their analysis, Dellera and Garbagnati [5] obtained the following relationship between ρ_{so} and I_{pf}:

$$\rho_{so} = 3.8 \times 10^{-5} I_{pf}^{0.68}, \tag{8}$$

where ρ_{so} is in C/m and I_{pf} is in kA.

1.4. Cooray [8]

Cooray [8] utilized a return-stroke model introduced by him for first return strokes to extract the relationship between the return stroke peak current and the charge per unit length at the bottom end of the leader channel. The result is given by the following equation:

$$\rho_{so} = 2.28 \times 10^{-6} + 1.46 \times 10^{-5} I_{pf} + 1.1 \times 10^{-7} I_{pf}^2, \tag{9}$$

where ρ_{so} is in C/m and I_{pf} in kA. Since the above equation is based on a number of assumptions used in developing the return-stroke model, it is in need of independent confirmation.

2. The total stepped-leader charge as a function of peak current

We will use Berger's return-stroke current waveforms to estimate the charge (e.g. Fig. 1) brought to ground by the return stroke within the first 100 μs of the discharge. The information we gather from this exercise will be used in turn to estimate the charge distribution of the stepped leader channel as follows.

The total positive charge that enters into the leader channel at the strike point (or the negative charge that goes into ground) during the return stroke can be divided into three components. The first part is the positive charge, Q_l, that is necessary to neutralise the negative charge originally stored in the leader channel (see Fig. 2c). The second part, Q_i, is the positive charge induced in the return stroke channel, which is essentially at ground potential, due to the background electric field produced by remaining cloud charges (see Fig. 2d). The third part is the additional positive charge spent to neutralize negative cloud charge that was not involved in the leader process (continuing current charge). The latter can be disregarded if the measured current is integrated up to the time of arrival of the return stroke at the point of initiation of the leader. This requires a reasonable assumption on the time taken by the return stroke to reach the point of initiation of the leader. Since, we utilize the data obtained by Berger [4] and Berger and Vogelsanger [6] in this analysis, it is important that this time is pertinent to their study in Switzerland.

The time needed for the return stroke front to reach the point of leader initiation can be obtained from the following consideration:

(1) The negative charge region in thunderclouds is located in the vicinity of the -10° isotherm [23]. In temperate localities this isotherm is typically observed at a height of about 5 km from ground level. The height of the negative charge region in thunderstorms in Switzerland is probably close to this value. The tower used by Berger in his studies was about 70 m high, and it was located at the top of Monte San Salvatore at an altitude of 640 m above sea level. Since ground lightning flashes are probably initiated at the lower boundary of the negative charge region in the cloud where the electric field is higher than in the interior of the charge region, a reasonable estimate of the height of lightning initiation point in the cloud is 4 km.
According to the measurements of Idone and Orville [15], the first return stroke speeds averaged over the bottom kilometre or so of the lightning channel are typically about 10^8 m/s. The observations also showed that the return stroke speed decreases with increasing height. According to the measurements of Schonland [9], the first return stroke speed close to the cloud base which was located at a height of about 2 km from ground level in South Africa is about 5×10^7 m/s. Further, the minimum first return stroke speed measured in both the above-mentioned studies is 2×10^7 m/s. From these observations one can safely conclude that the average speed of first return strokes over a 4 km channel may be close to 5×10^7 m/s. Interestingly, Shao's [10], from VHF imaging of lightning channels in Florida, found, though for a single first return stroke, the average speed from ground to the point of initiation of the stepped leader to be 5×10^7 m/s. For an average return-stroke speed of 5×10^7 m/s, the charge brought to ground by Berger's first return strokes during their travel from ground to the point of initiation of the stepped leader at 4 km (see above) occurs within 80 μs of the beginning of current waveforms (or less if a higher average return-stroke speed is assumed).
(2) Measurements of 3-MHz radiation associated with first return strokes show that the emission starts almost simultaneously with the initiation of the return stroke and ends more or less abruptly in about 130 μs in temperate Sweden and in about 200 μs in tropical Sri Lanka [11,12]. This observation provides indirect evidence that the travel time of the return stroke to the leader initiation point (origin of the flash) is about 130 μs in temperate regions and it may be about 200 μs in the tropics.

Based on these considerations it is assumed that the charge transported to ground by the first return stroke within 100 μs of its initiation is approximately equal to the sum of the positive charge that is necessary to neutralise the negative charge originally stored in the leader channel and the positive charge induced in the return stroke channel. The charge of the stepped leader is equal in magnitude (but of opposite sign) to the former. This assumption is further discussed in Section 6. The total stepped-leader charge is needed for finding the charge distribution along the leader channel and then the striking distance.

First return stroke currents of Berger [4] and Berger and Vogelsanger [6] were integrated over the first 100 μs, and the results are depicted as a function of peak current in Fig. 1. Note that there is a strong linear correlation

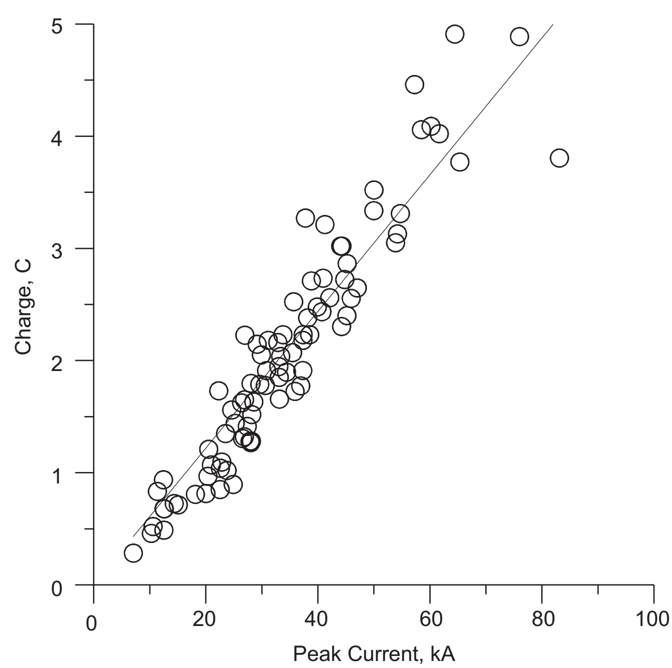

Fig. 1. The charge neutralized in the first 100 μs by the first return strokes studied by Berger [4] and Berger and Vogelsanger [6] as a function of return-stroke peak current.

between the two parameters with the correlation coefficient of about 0.94. The corresponding linear regression equation is given by

$$Q_{f,100\,\mu s} = .061\, I_{pf}, \tag{10}$$

where $Q_{f,100\,\mu s}$ is the charge (in C) neutralized by the first return stroke within the first 100 μs, and I_{pf} is the first return stroke peak current in kA. The next task is to infer the distribution of charge found from (10) along the stepped-leader channel.

3. The distribution of charge per unit length along the stepped leader channel as a function of peak current

In order to obtain the distribution of charge along the stepped-leader channel from the information given in the previous section, it is necessary to simplify the real problem. The real problem and its idealisation that we used below in the numerical simulation are illustrated in Fig. 2. It was assumed that the horizontal extent of the negative charge region in the cloud is large in comparison to the vertical distance between the ground and the charge region. Based on this assumption the cloud charge region was replaced by a perfectly conducting plane maintained at a given potential. This configuration simulates a uniform electric field between the cloud and the ground. The leader channel is simulated by a vertical lossy conductor of cylindrical geometry with a hemispherical tip. The losses are represented by a constant potential gradient. The well-known charge simulation method is used to obtain the charge distribution on the leader channel in a given electric field. It is of interest to note that the charge distribution induced on the stepped leader channel as it propagates towards the ground is identical to the charge distribution that would be induced on the lower half (below the point of initiation) of a vertical bi-directional leader developing in a uniform electric field. As the stepped leader extends towards the ground its charge distribution is determined by the background electric field generated by the cloud charges and any field enhancement caused by the presence of the ground, i.e. the proximity effect. As mentioned earlier, once the contact is established between the

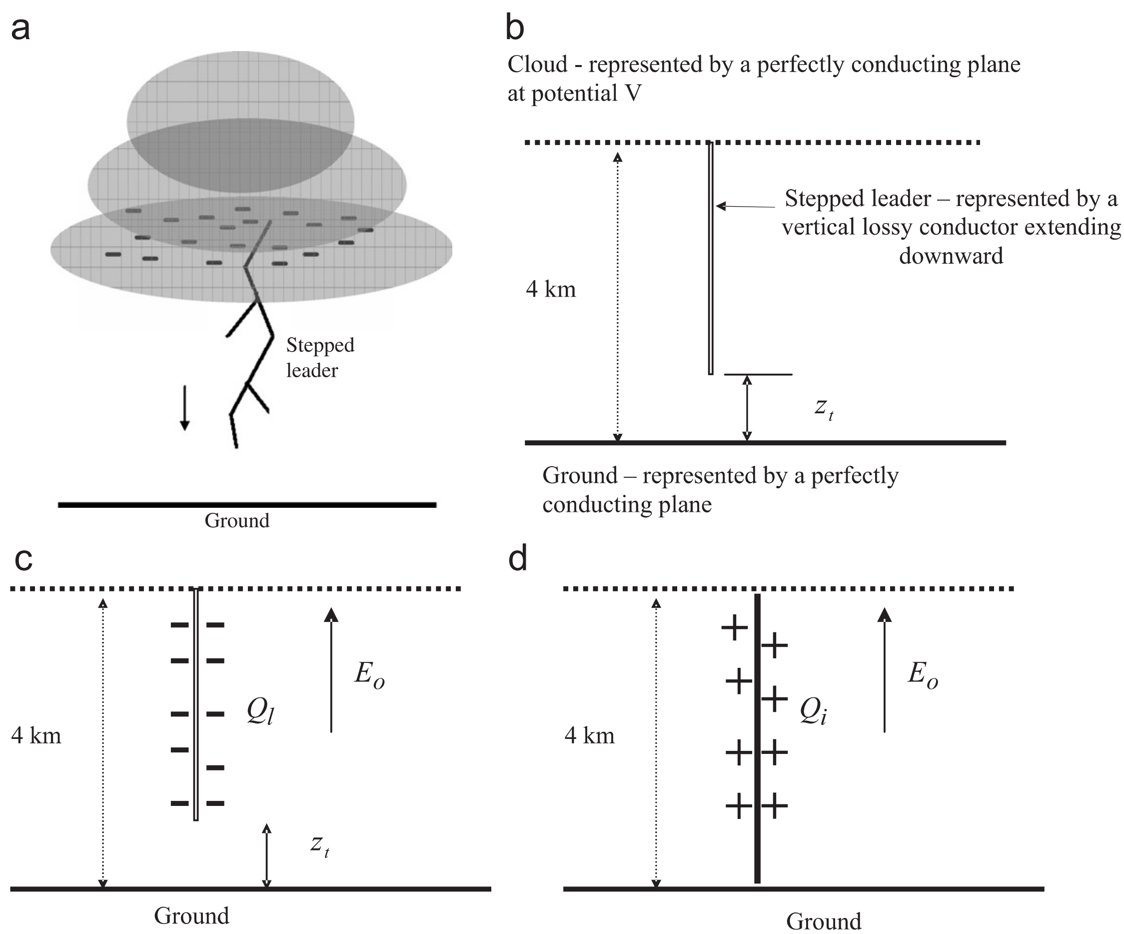

Fig. 2. (a) A sketch of the stepped leader approaching ground. (b) The idealisation used in the computation of charge distribution along the leader channel. (c) Situation just before the attachment process and return stroke (the negative charge density increases downwards). (d) Situation after the return stroke (the positive charge density increases upwards). In the figure z_t is the distance between the tip of the leader and the ground. In the case of lightning strike to a tower it represents the separation between the leader tip and the top of the tower. In general, z_t is greater than the striking distance, although in (c) it is implied to be equal to the striking distance. Q_l is the total charge deposited on the leader channel, and Q_i is the total charge induced on the fully-devloped return-stroke channel by the cloud electric field E_o.

negatively charged stepped leader and the ground the total positive charge entering the channel from the ground during the first 100 μs of the return stroke is equal to the sum of the positive charge that is necessary to neutralize the negative charge Q_l of the leader and the positive charge Q_i induced on the channel due to the remaining negative charges in the cloud. In the configuration shown in Fig. 2(b), the two components, labelled Q_l and Q_i in Figs. 2(c) and (d), have approximately the same magnitudes for the following reasons: (1) Lightning channels at the final stage of both the leader and the return stroke are exposed to the same background field E_0 (see Figs. 2c and d). (2) Lightning channels at the final stages of both the leader and the return stroke can be treated as steady-state arc channels. (3) Since the potential gradient of an arc channel is more or less independent of current [13], the final potential gradient of the return-stroke channel is more or less the same as that of the leader channel. (4) In the absence of field enhancement caused by the ground, the negative charge density (Fig. 2c) will increase linearly towards the ground while the positive charge induced on the return stroke channel (Fig. 2d) will increase linearly towards the cloud. Thus, the two charge components will have more or less the same magnitude but opposite signs. The balance will be slightly disturbed by the field enhancement caused by the ground. As a result, the negative charge density near the leader tip will increase almost exponentially when the tip is close to the ground leading to a slight increase in the negative charge component. Our calculations show, however, the difference between $|Q_l|$ and $|Q_i|$ is less than about 10%.

In order to obtain the leader charge distribution as a function of return stroke peak current the following procedure was used. Different values of peak current correspond to different values of cloud potential (Fig. 2b) and, hence, to different values of E_0 (Fig. 2c). Further, the leader charge distribution depends on the assumed value of z_t. Consider a return stroke peak current I_{pf}. Since the total charge injected into the channel from the ground during a return stroke characterized by this peak current is about $0.061\,I_{pf}$ (see (10)), the potential of the cloud in the configuration shown in Fig. 2b is adjusted until the sum $|Q_l| + Q_i$ is equal to $0.061\,I_{pf}$. The resultant leader charge distribution is computed for different values of z_t.

One of the input parameters required in the simulation is the radius of the leader channel. The radius of the leader channel is adjusted so that the average charge per unit length the stepped leader channel, ρ_{av}, and the leader channel radius, R_l, satisfy the equation:

$$R_l = \frac{\rho_{av}}{2\pi\varepsilon_0 E_c}, \tag{11}$$

$$\rho_{av} = \frac{Q_l}{H}, \tag{12}$$

where H is the length of the leader channel. This equation is based on the Gauss' law and on the assumption that the air breakdown at the lateral surface of the leader channel (radial expansion of the corona sheath) continues until the electric field at the outer channel boundary, is equal to $E_c = 3.0 \times 10^6\,\mathrm{V/m}$, the electric breakdown field at normal atmospheric conditions. It should be pointed out that in reality the radius of the leader channel varies as a function of height because the charge density along the leader channel decreases with height. In the calculations presented in this paper the average radius of the leader channel is obtained from 11 and 12 and the whole leader channel is assumed to have this average radius along the whole length.

It is important to note that, in the configuration shown in Fig. 2 where the background electric field, E_o generated by the cloud is uniform, for a given $|Q_l| + Q_i$, the estimated leader charge distribution does not depend on potential gradient, E_l, of the leader channel. The reason for this is that the charge distribution is determined by the difference $E_o - E_l$ and not by individual values of E_o and E_l. However, the cloud potential, and hence the value of E_o, required to dissipate a given amount of charge in the return stroke (corresponding to a given value of peak current) are influenced by the potential gradient of the leader. If the leader channel were assumed to be perfectly conducting ($E_l = 0$), the leader charge distribution would be a function of E_o.

The leader charge distributions corresponding to a 30 kA peak current for three values of the leader tip height above ground z_t (see Fig. 2c) are shown in Fig. 3. The range of variation of z_t, from 10 to 100 m, in Fig. 3 was selected so that to include the expected values of striking distance. Note that the charge distribution is approximately linear except near the tip of the leader. The abrupt increase of charge density at the tip is caused partly by the presence of the ground. (Note how the charge at the tip decreases with increasing z_t). The charge distribution corresponding to $z_t = 50\,\mathrm{m}$ will be used for estimating the striking distance, although the result is not sensitive to which of the three leader charge distributions shown in Fig. 3 is used (see Section 5).

Note that the charge distributions given in Fig. 3 are valid for a fully developed stepped leader channel with its tip near the ground. The charge distribution along the leader channel when the leader tip is far away from the ground is different from those given in this figure. The charge distribution along the leader channel when its tip is located at different heights from the ground is depicted in Fig. 4. Note that as the leader propagates downwards the highest charge density is encountered at the channel element in which the leader tip is located. As the leader tip moves downward the charge density in that channel element decreases and finally approaches the value corresponding to a fully extended stepped leader.

The data shown in Fig. 4 can be summarized approximately by a single equation that describes how the charge

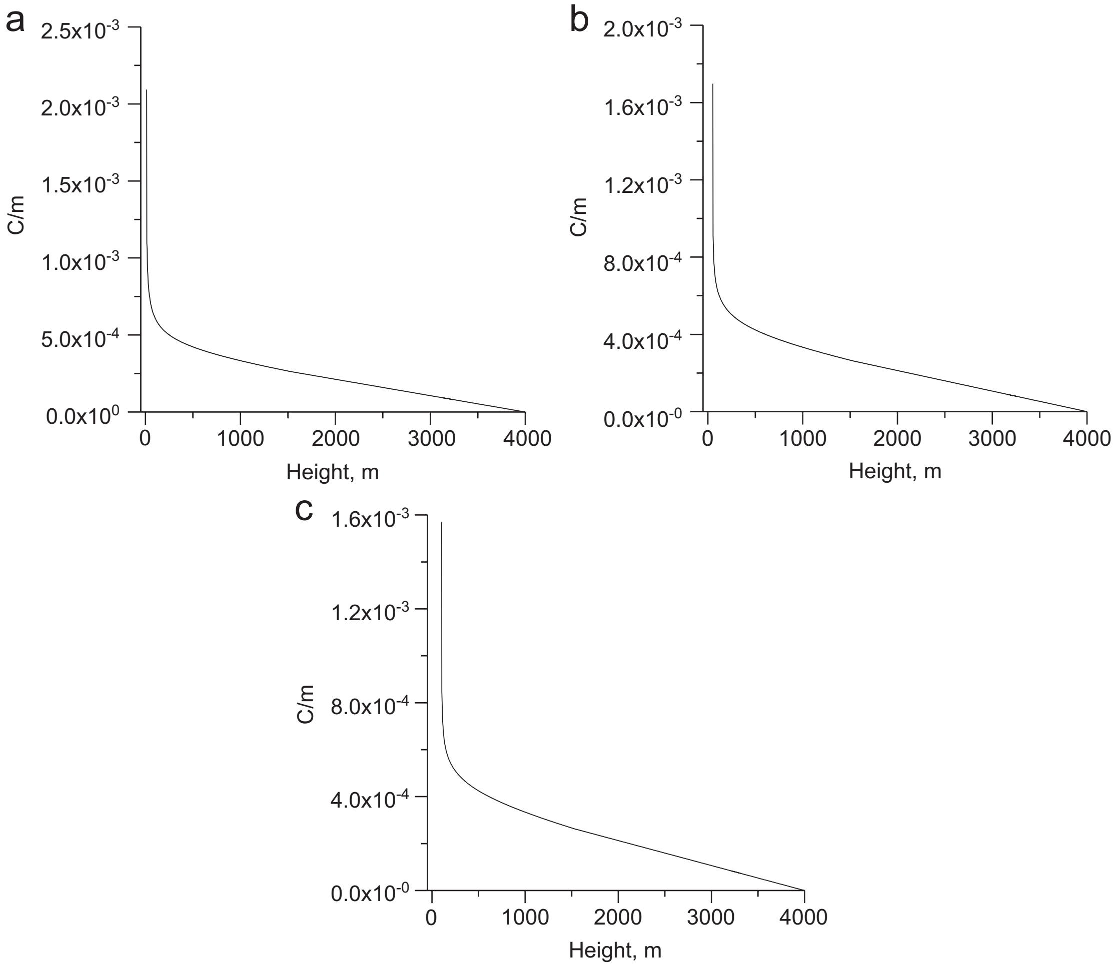

Fig. 3. The leader charge distribution obtained for 30 kA peak current for different heights, z_t, of the leader tip above ground: (a) $z_t = 10$ m, (b) $z_t = 50$ m and (c) $z_t = 100$ m.

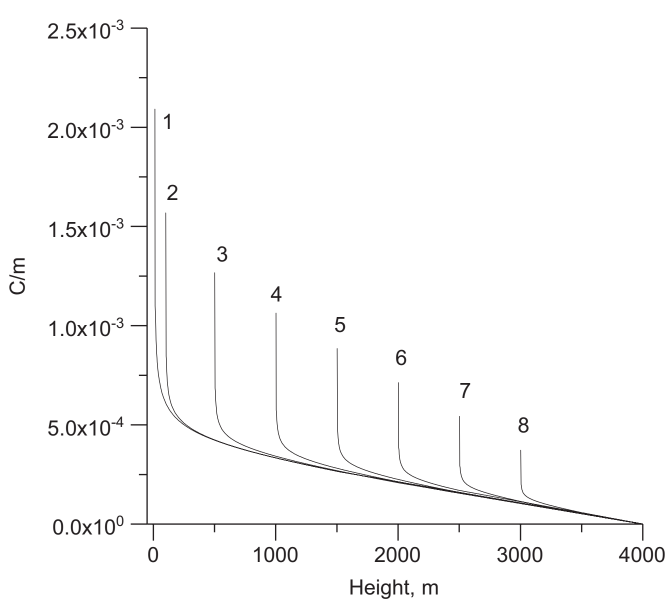

Fig. 4. Charge distribution along the leader channel when the leader tip is at a height of: (1) 10 m, (2) 100 m, (3) 500 m, (4) 1000 m, (5) 1500 m, (6) 2000 m, (7) 2500 m and (8) 3000 m from ground level.

on the stepped leader channel varies as it propagates towards the ground. That equation is the following.

$$\rho(\xi) = a_o\left(1 - \frac{\xi}{H - z_o}\right) \quad G(z_o)I_p + \frac{I_p(a + b\xi)}{1 + c\xi + d\xi^2}H(z_o)$$
$$z_o \geqslant 10, \tag{13}$$

$$G(z_o) = 1 - (z_o/H), \tag{14}$$

$$H(z_o) = 0.3\alpha + 0.7\beta, \tag{15}$$

$$\alpha = e^{-(z_o - 10)/75}, \tag{16}$$

$$\beta = \left(1 - \frac{z_o}{H}\right), \tag{17}$$

where z_o is the height of the leader tip above ground in meters (note that the above equation is valid for $z_o > 10$ m), H is the height of the cloud in meters, $\rho(\xi)$ is the charge per unit length (in C/m), ξ is the length along the stepped leader channel with $\xi = 0$ at the tip of the leader, I_p is the return stroke peak current in kA, $a_o = 1.476 \times 10^{-5}$, $a = 4.857 \times 10^{-5}$, $b = 3.9097 \times 10^{-6}$, $c = 0.522$ and $d = 3.73 \times 10^{-3}$.

Our calculations show that for a given charge per unit length at the bottom end of the fully developed stepped leader the charge distribution of the lower kilometre or so of the channel does not depend on H. From there onwards the charge per unit length decreases linearly to zero at the top of the channel. As a result, Eq. (13), obtained for $H = 4$ km can be used with any value of H provided that it is larger than about 3 km.

4. Testing the validity of the procedure to estimate the stepped-leader charge distribution

In order to test the validity of the procedure outlined in the previous section, we will apply the same procedure to triggered lighting strokes, use the resultant charge distribution for computing close electric fields and compare them with the measured ones (e.g., Crawford et al. [16]).

It is known [18,24,25] that individual strokes in a multiple-stroke ground flash tap negatively charged regions that are displaced primarily horizontally from each other. Thus the vertical length of the dart leader channels involved in subsequent strokes in Berger's study may only be slightly larger than the 4 km length assumed for first strokes. Let us assume 5 km as a representative value of the dart-leader length. The optically observed average return-stroke speed over the bottom 2 km or so of the channel is about 10^8 m/s [14,15], and it does not change much along the lightning channel. If we assume that this speed is maintained along the entire channel, then the return-stroke front will reach the height of 5 km in 50 μs. An average return stroke speed of about 10^8 m/s along the entire length of the dart leader channel is also supported by the observations of Shao [10] who found that the average subsequent return stroke speeds over channel lengths of 10–15 km range from 0.5×10^8 to 1.5×10^8 m/s. Based on these observations, we integrated Berger's subsequent return stroke currents up to 50 μs, and plotted the results as a function of peak current in Fig. 5. Similar to Fig. 1, one can observe a strong linear relationship between the subsequent stroke peak current, I_{ps}, and the charge dissipated within the first 50 μs, $Q_{s,50\,\mu s}$. The results can be represented by the following equation:

$$Q_{s,50\,\mu s} = 0.028\, I_{ps}, \tag{18}$$

where the charge is in C and the peak current in kA.

The same procedure as before (see Section 3) is used to obtain the charge distribution along the leader channel corresponding to different peak currents. The charge distribution on the dart leader channel when the tip of the leader is at different heights from ground level can be obtained from Eqs. (13)–(17) using $a_o = 5.09 \times 10^{-6}$, $a = 1.325 \times 10^{-5}$, $b = 7.06 \times 10^{-6}$, $c = 2.089$, and $d = 1.492 \times 10^{-2}$.

Once the charge distribution along the leader channel is known, the close electric field at a given point at ground level can be calculated and compared with measurements. For a vertical dart leader channel of length H the electric field, E_z, at distance D from the ground strike point is given by

$$E_z = \int_o^H \rho(z) \frac{z\,\mathrm{d}z}{2\pi\varepsilon_o \left(D^2 + z^2\right)^{3/2}}, \tag{19}$$

where $\rho(z)$ is given by Eqs. (13)–(17) using the values of coefficients given above and ε_o is the permittivity of free space. The electric fields of dart leaders at 50 and 110 m as a function of ensuing return stroke peak current, calculated using the above equation, are represented by solid lines in Fig. 6. Also shown in Fig. 6 are corresponding experimental data (crosses and broken lines) for triggered lightning in Florida (1997–1999) as reported by Crawford et al. [16]. Note that the calculated fields agree with the measurements within about 20%. This is in support of the procedure used in this paper to obtain the charge distribution on the leader channel as a function of peak current, if the characteristics of the processes involved for Berger's subsequent strokes in Switzerland and for triggered lightning strokes in Florida are similar.

Interestingly, triggered-lightning data of Crawford et al. [16] indicate an inverse distance dependence of the close leader electric field change, which is consistent with a more or less uniform leader charge density distribution along the bottom kilometre or so of the channel. On the other hand, in our model the leader charge density decreases linearly with height with some additional charge at the leader tip (see Fig. 3). A fairly good agreement between the model predictions and measurements seen in Fig. 6 probably implies that close electric fields that are determined by charges on a relatively short segment of the channel (some

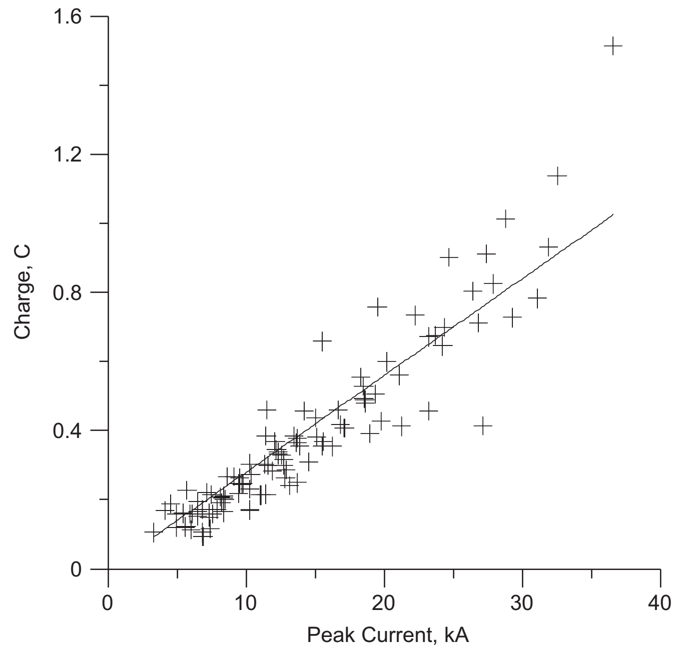

Fig. 5. The charge dissipated in the first 50 μs by the subsequent return strokes studied by Berger [4] and Berger and Vogelsanger [6] as a function of peak current.

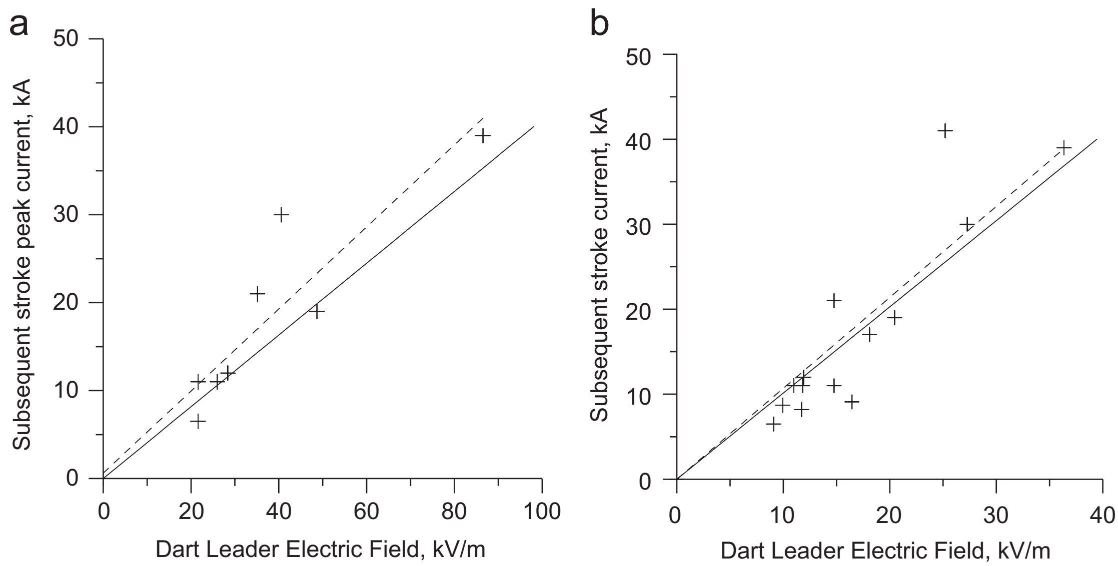

Fig. 6. Comparison of the dart leader electric fields calculated at: (a) 50 m, and (b) 110 m from the lightning channel base with the measurements reported by Crawford et al. [16]. The solid line represents the calculations and the crosses represent the measured fields. The dotted line gives the best fit to the experimental data.

hundreds of meters in length) are essentially independent of the type of charge density distribution. As a result, a non-uniform charge density distribution produces essentially the same close electric fields as the uniform one, as long as the average charge per unit length on the bottom portion of the channel is approximately same. The charge distribution derived here could also be used in various return stroke models that require the charge distribution along the leader channel as input [20,21].

5. The striking distance of stepped leaders as a function of return stroke peak current

In order to obtain the striking distance we will evaluate the distance from ground to the tip of the stepped leader when the average potential gradient between the leader tip and the ground is equal to 500 kV/m. As shown in Section 3, the charge distribution on the leader channel depends on the distance between the tip of the leader and the ground. Thus, in general, one cannot use the charge distribution derived for a fixed value of z_t in evaluating the striking distance. However, it is possible to demonstrate that the charge distributions obtained with z_t equal to some tens of meters can be used in evaluating the striking distance without a significant error. For example, the striking distance obtained for two charge density distributions using $z_t = 10$ and 100 m are depicted in Fig. 7. Note that the striking distance is insensitive to the value of z_t. The reason for this is that the striking distance depends on the average electric field in the gap between the leader tip and the ground. The differences in the charge distributions for different values of z_t do not influence this average electric field significantly. Thus, in evaluating the striking distance one can use the charge distribution obtained for, say,

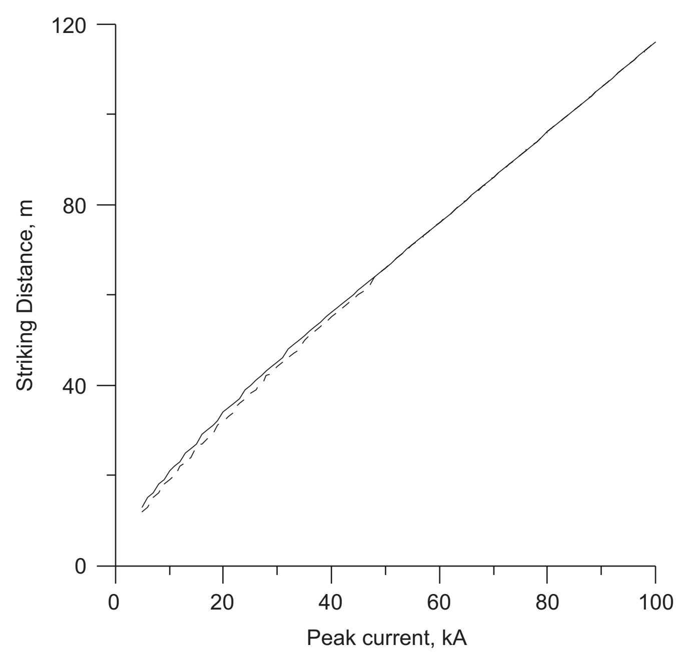

Fig. 7. Striking distance calculated using the charge density distributions corresponding to $z_t = 10$ m (solid line) and $z_t = 100$ m (dashed line).

$z_t = 50$ m, with the result being insensitive to the choice of z_t in the range (10–100 m) covering the range of expected values of striking distance. In a recent paper an attempt was made to relate the striking distance to the leader potential instead of the more conventional approach of relating it to the leader charge [22]. The conventional approach is adopted in the present paper.

From the results obtained in this paper it is difficult to derive an explicit relationship between the charge per unit length on the leader channel and the return stroke peak

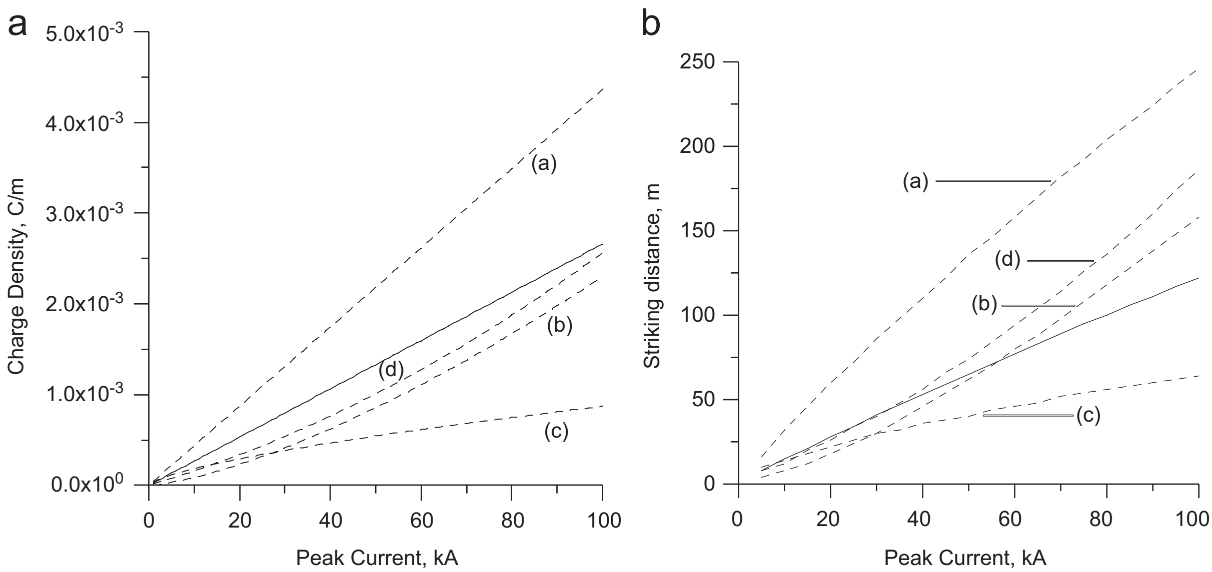

Fig. 8. (a) Charge density–current relationships obtained here (solid line) and proposed by different authors (dashed lines). (a) Golde [1], (b) Eriksson [3], (c) Dellera and Garbagnati [5], (d) Cooray [8]. The solid line corresponds to the average charge density over the lowest 100 m of the leader channel as predicted by Eq. (20) of the present study. (b) The striking distance based on the charge relationships obtained here (solid line) and proposed by different authors (dashed lines). (a) Golde [1], (b) Eriksson [3], (c) Dellera and Garbagnati [5], (d) Cooray [8].

current because the charge density in the vicinity of the leader tip vary very rapidly with height. However, if we take the average charge density over the bottom hundred meters of the channel, the charge that is most important in the formation of the return stroke peak current, its relationship to the return stroke peak current can be expressed as:

$$\rho_{av} = 2.7 \times 10^{-5}\, I_{\mathrm{p}}, \tag{20}$$

where ρ_{av} is the average (over the bottom 100 m) leader charge density in C/m, and I_{p} is the return stroke peak current in kA. The relationship given in (20) is depicted in Fig. 8(a). Previously suggested the charge density-current relationships given by Eqs. (4), (7), (8) and (9) are also depicted in Fig. 8(a).

The striking distance as a function of peak current obtained using the charge density distributions given by Eqs. (4), (7), (8) and (9) are depicted in Fig. 8(b) together with the results obtained in this paper. Note that the striking distances obtained here from Eqs. (4) and (7) differ from those calculated by Golde [1] and Eriksson [3], respectively, using the same equations. The reason for this discrepancy is the following. Even though Golde [2] defines the striking distance as the distance when the average potential gradient across the striking distance is 500 kV/m, in order to simplify the computation he assumed that the striking distance is reached when the electric field at ground level attains 500 kV/m. This simplification results in an average potential gradient larger than 500 kV/m across the striking distance. The striking distance published by Eriksson [3] is the striking distance to a 60 m tower and not to a flat ground and hence is not directly comparable to our results. The results obtained in the present study can be summarised by the following equation for the striking distance, S (in meters), as a function of peak current, I (in kA; all subscripts dropped):

$$S = 1.9\, I^{0.90}. \tag{21}$$

It follows from Eq. (21) that a 30 kA peak current is associated with a striking distance to flat ground of about 41 m, while the traditional equation, $S = 10\, I^{0.65}$ [26], gives $S = 91$ m. In our view, Eq. (21) yields a more physically realistic value of striking distance. A more practical case of striking distance to an object protruding above ground is a subject of future studies. The striking distance to flat ground considered here may be applicable to the case of a tower-like object, if it is redefined as the distance to the upper end of upward connecting leader that effectively increases the height of the object at the time when the common streamer zone of the descending and upward connecting leaders is formed [27].

6. Discussion of the assumptions made in this study

In deriving the relationship between the stepped leader charge and the return-stroke peak current several simplifying assumptions are made. We discuss these assumptions below. Some of the assumptions can be justified using available experimental data and the testing of the validity of others awaits additional experimental data.

It is important to note that the charge given by (10) (1.8 C for the median first-stroke peak current of 30 kA) is not the total charge that is being brought to ground during the return stroke. The median value of the total charge brought to ground during return strokes is in the range of 4–5 C [4,6,18,19]. On the other hand Eq. (10) gives only the charge transferred to the ground during the first 100 μs. By integrating the same current waveforms as those used in our study but over a time period of 2 ms Berger obtained 4.5 C as a typical value (the impulse charge). This means that about 2.7 C of charge has to be contributed by the charge neutralisation processes taking place in branches and on in-cloud horizontal channel sections, or by neutralisation processes taking place in the corona sheath after the first 100 μs.

Note that in order to obtain the striking distance what is really needed is the charge distribution on the first few hundred meters of the stepped leader channel. Contributions to the electric field in the final gap from charges stored on the higher channel sections are negligible. The entire channel was considered here in order to obtain a self-consistent formulation of the problem (see Fig. 2).

6.1. Uniform background electric field

We assumed that the electric field between the cloud charge region and the ground is uniform. This assumption is valid if the cloud charge region has a very large horizontal extent, compared to its height above ground. Some justification for this assumption is found in Willett et al. [17] who have analysed the vertical profile of the electric field below mature thunderclouds. Their data show that after an initial increase of the field within the first few tens of meters (caused by a corona charge layer), the electric field remains more or less constant with altitude over the first 1–2 km. Unfortunately, no measurements are presented for higher altitudes.

6.2. Electrostatic approach

In calculating the charge distribution on the leader channel it is assumed that the leader charge is always in equilibrium with the background electric field. That is, the calculated charge distribution corresponds to a steady state condition whereas in reality the leader process is dynamic. In fact, during each step of the leader the charge distribution along the whole channel has to be readjusted so that it conform to the background electric field and the channel potential gradient. On the other hand, the speed of the stepped leader is much lower than the speed of light, and it is reasonable to assume that at a given time the charge distribution along the channel is close to the steady state one. Without this assumption, the calculated charge per unit length would be shifted towards lower values.

6.3. Channel branches

In the calculations any channel branches were neglected. In reality the charge dissipated by the return stroke is originated partly from the branches and partly from the main channel. The contribution of the branches to the charge dissipated within the first 100 μs depends on how fast the return stroke front will travel along the branches and the amount of charge located on them. Neglecting the contribution of branches to the return stroke charge leads to an overestimation of the charge per unit length on the leader channel.

6.4. 100-μs integration time

The charge of the leader channel resides mainly on the corona sheath. The time that is necessary to remove charge from the corona sheath depends on the rate at which the central core of the leader channel is brought to ground potential during the return stroke, the age of the corona sheath, the radius of the corona sheath and the speed of propagation of the discharge channels that neutralize the corona sheath. At present not much information is available concerning any of these processes and, therefore, one has to regard the estimated leader charge density as a lower limit because it is based on the assumption that the leader charge is neutralised within a few tens of microseconds (actually this time varies from 100 μs at ground level to zero at the assumed leader origination point). A longer neutralisation time would require a longer current integration time than the 100 μs adopted here, and this will necessarily increase the estimated leader charge density. We cannot rule out the possibility that the neutralisation time of the corona sheath is longer than the 100 μs assumed in this paper. Thus, it would be appropriate to treat the striking distance estimated here as a lower limit. In lightning protection, it is the lower limit of the striking distance that is of primary interest, because it will set limits on the maximum spacing between the air terminals on a structure to be protected.

7. Conclusions

By evaluating the charge dissipated by the first return strokes studied by Berger within the first 100’s the charge stored on the stepped leader channel is estimated. This charge, Q_f,100 μs (in C), is related to the return stroke peak current, I_{pf} (in kA) by the equation Q_f,100 μs $= 0.61\, I_{pf}$. Based on electrostatic considerations, the distribution of the charge along the leader channel is found. This in turn is used (along with the assumed electric field of 500 kV/m in the final gap) to estimate the striking distance of the stepped leader to flat ground as a function of the prospective return stroke peak current: $S = 1.9\, I_{pf}^{0.90}$.

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