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On the transmission line model for lightning return stroke representation

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[1] The widely used transmission-line (TL) model of lightning return stroke in a vertical channel is most rigorously represented by a vertical phased array of current sources that produce a spherical transverse electromagnetic (TEM) wave in the case of return-stroke speed v equal to the speed of light c. If the radius of the lightning channel were equal to zero, the equivalent representation could be obtained by applying a hypothetical infinitesimal source of pure spherical TEM wave at the bottom of the channel. A non-zero-radius vertical wire above ground excited by a practical source at its bottom end cannot support unattenuated current waves, and the associated electromagnetic field structure is non-TEM. *INDEX TERMS:* 0619 Electromagnetics: Electromagnetic theory; 0684 Electromagnetics: Transient and time domain; 3304 Meteorology and Atmospheric Dynamics: Atmospheric electricity; 3324 Meteorology and Atmospheric Dynamics: Lightning. **Citation:** Baba, Y., and V. A. Rakov, On the transmission line model for lightning return stroke representation, *Geophys. Res. Lett.*, *30*(24), 2294, doi:10.1029/2003GL018407, 2003.

1. Introduction

[2] The transmission-line (TL) model of lightning return stroke [*Uman and McLain*, 1969] has been widely used for lightning electromagnetic pulse (LEMP) calculations in various studies including lightning electromagnetic coupling with power and communication lines [e.g., *Zeddam and Degauque*, 1990; *Rachidi et al.*, 1996], estimation of lightning properties from measured electric and magnetic fields (and channel-base currents in the case of triggered lightning) [e.g., *Krider et al.*, 1996; *Uman et al.*, 2002], and the production of transient optical emissions (elves) in the lower ionosphere [e.g., *Krider*, 1994; *Rakov and Tuni*, 2003]. This model is often viewed [e.g., *Rakov and Uman*, 1998; *Gomes and Cooray*, 2000; *Thottappillil and Uman*, 2002] as incorporating a lumped current source at the channel base, which injects a specified current into the channel. The injected current wave is assumed to propagate upward along the channel without attenuation or distortion. *Thottappillil et al.* [2001] has recently derived simple analytical expressions for electric and magnetic fields predicted by the TL model for the case of a vertical perfectly conducting channel above flat perfectly conducting ground and a return-stroke speed v equal to the speed of light c. The derived expressions show that the electric and magnetic field waveshapes at all points in space are identical to the waveshape of current at the channel base. Further, the electric field has only a theta-directed component, magnetic field has only an azimuthal component, and the field magnitudes vary as the inverse distance from the channel base. Thus, in this special case ($v = c$), the electromagnetic field structure is a spherical transverse electromagnetic (TEM) wave. *Thottappillil et al.*'s [2001] results apply to a zero-radius wire excited at its bottom end by an infinitesimal source of pure spherical TEM wave. In this paper, we consider practical, non-zero-radius conductors and practical current sources.

[3] The structure of the paper is as follows. We first use the finite- difference time-domain (FDTD) method [*Yee*, 1966] for solving Maxwell's equations to demonstrate that a vertical phased array of current sources above perfectly conducting ground, activated as prescribed by the TL model with $v = c$, produces a spherical TEM wave, identical to that analytically derived for the TL model by *Thottappillil et al.* [2001]. Then, we apply the same approach to the case of a lumped current source at the bottom of a vertical perfectly conducting wire above perfectly conducting ground and show that the resultant field structure is non-TEM, as predicted by various other electromagnetic models [e.g., *Kordi et al.*, 2002, 2003; *Baba and Ishii*, 2003; *Grcev et al.*, 2003]. Finally, we apply the same FDTD method to the case of a vertical phased array of current sources (a TEM-field excitation) at the bottom of a vertical perfectly conducting wire above perfectly conducting ground and show that the electromagnetic field structure above the excitation region is non-TEM. We conclude that the most rigorous representation of the TL model, which requires an imposed current distribution along the entire lightning channel, is a phased array of current sources (implicitly used by *Thottappillil et al.* [2001] in their analytical derivations) and that a practical vertical conductor above ground excited at its bottom end by a practical lumped source cannot support unattenuated current waves. These conclusions also apply to the more realistic case of the TL model with $v < c$ (achieved by using a higher-permittivity medium).

2. Case 1: Vertical Phased Current Source Array

[4] In the TL model, the longitudinal current $I(z', t)$ in a straight and vertical lightning channel at an arbitrary height z' and time t is expressed as follows:

$$I(z',\ t) = I(0,\ t - z'/v), \tag{1}$$

where v is the return-stroke speed and $I(0, t)$ is the channel-base current. Equation (1) describes an imposed current distribution along the channel, and hence it is rigorously

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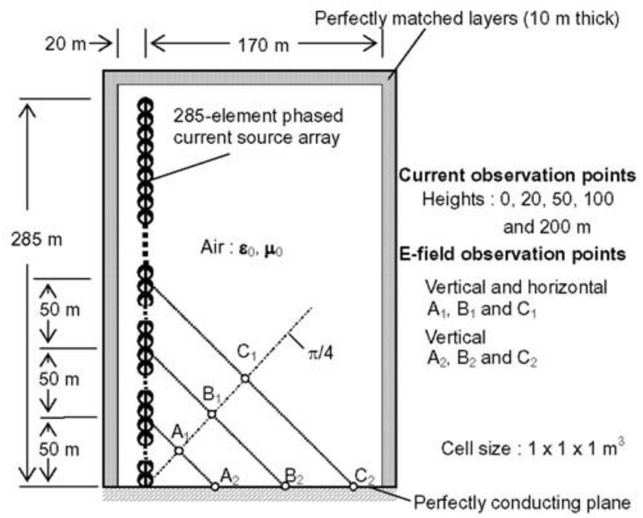

Figure 1. A vertical phased current-source array in air on a perfectly conducting plane. The working volume of $190 \times 40 \times 300$ m^3 is surrounded by perfectly matched layers (PML) of thickness 10 m, except for the bottom, perfectly conducting plane.

satisfied only by a vertical phased array of current sources that are progressively activated by an upward-moving return-stroke front. Note that such channel representation is valid for any v and does not require any unrealistic assumptions on the channel radius or on the source size. Typical values of v are one-third to one-half of c [e.g., *Rakov et al.*, 1992]. In this paper, we will assume that $v = c$, as done by *Thottappillil et al.* [2001] and *Kordi et al.* [2002, 2003], in order to simplify the analysis.

[5] Figure 1 shows a vertical phased array of 285 current sources on a perfectly conducting plane, whose electromagnetic field structure is analyzed using the FDTD method. The overall rectangular volume shown in Figure 1 is divided into $1 \times 1 \times 1$ m^3 cubic cells. Perfectly matched layers (PML) [*Berenger*, 1994] (absorbing boundaries) are set at the top and sides of the volume in order to avoid reflections there. At the bottom, the volume is limited by a perfectly conducting plane that is simulated by forcing the tangential components of electric field at the boundary to be zero. This latter method is also used to simulate perfectly conducting wires in Cases 2 and 3.

[6] Each current source produces a Gaussian pulse having a magnitude of 1 kA, which is turned on when an upward-moving return-stroke front reaches its altitude. In the FDTD analysis, the current source, for example, at $z' = 0$, is represented by specifying the four magnetic-field elements, $H_{x1}(t)$, $H_{x2}(t)$, $H_{y1}(t)$, and $H_{y2}(t)$, surrounding the element in which the current source is placed, as illustrated in Figure 2. The Gaussian current waveform at $z' = 0$ is expressed as

$$I(0,t) = \exp\left[-\left(\frac{\beta}{\tau_0}\right)^2 (t-\tau_0)^2\right] \tag{2}$$

where constants β and τ_0 control the width of the pulse. In the present analysis, β is set to 2.5 and τ_0 to 100 ns. The resultant half-peak width of the pulse is 67 ns. This current waveform is more convenient than a realistic lightning current waveform for demonstrating the effects studied here, primarily because of its well defined and easy-to-trace peak.

[7] Currents and fields are calculated up to 1 μs with a time increment of 1.25 ns. This time increment is within the Courant stability limit [*Taflove and Hagness*, 2000] on time step, $\Delta t \leq \Delta s / c\sqrt{3}$, where $\Delta s = 1$ m is the size of cubic cell used.

[8] Figure 3a shows current waveforms at heights of 0, 20, 50, 100, and 200 m above the perfectly conducting plane. Figure 3b shows vertical and horizontal electric fields at points A_1, B_1, and C_1 and vertical electric fields at points A_2, B_2, and C_2 (horizontal electric fields at these points vanish), all calculated using the FDTD method. The straight line, on which points A_1, B_1, and C_1 and the base of the vertical phased current source array are located, crosses the bottom, perfectly conducting plane at an angle of 45 degrees (see Figure 1). The inclined distances between the channel base and points A_1, B_1, and C_1, are 35.3, 70.7, are 106 m,

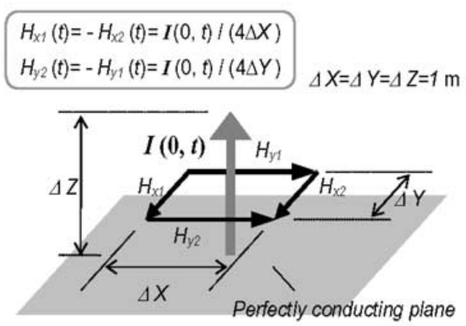

Figure 2. Specification of current source at $z' = 0$ in the FDTD procedure.

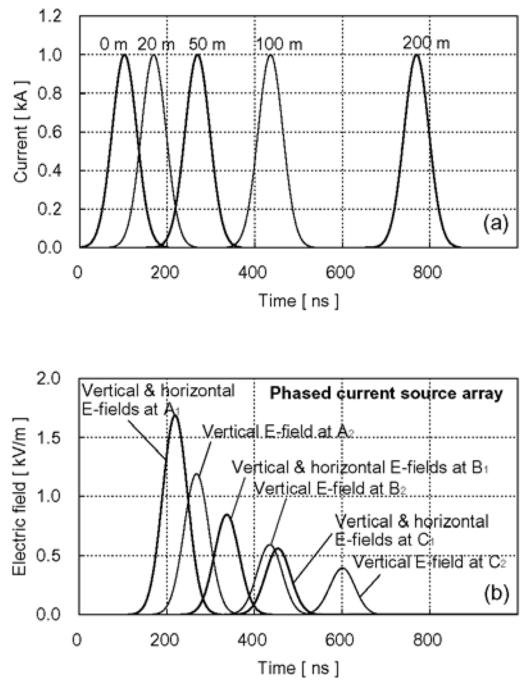

Figure 3. Waveforms of (a) current at different heights and (b) electric fields at points A_1, B_1, C_1, A_2, B_2, and C_2 calculated using the FDTD method for the vertical phased current source array shown in Figure 1.

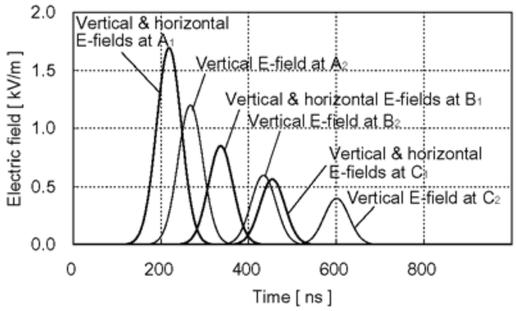

Figure 4. Waveforms of electric fields calculated using a simple expression analytically derived by *Thottappillil et al.* [2001] for the TL model with $v = c$.

respectively. The distances from the channel base to points A_2, B_2, and C_2 are 50, 100, and 150 m, respectively.

[9] Figure 4 shows electric fields at the same six observation points, but calculated using a simple expression analytically derived by *Thottappillil et al.* [2001, equation (19)] for the TL model. In Figures 3b and 4, an electric field pointing away from the vertical array and a downward electric field are defined as positive horizontal field and positive vertical electric field, respectively.

[10] The waveforms of Figure 3b (1 m × 1 m cross-section radiating structure, numerical results) are essentially identical to the corresponding waveforms in Figure 4 (zero-radius radiating structure, analytical results), which confirms the validity of the FDTD method used here. In both cases, the electromagnetic field structure is a spherical TEM wave.

3. Case 2: Vertical Conductor Above Ground Excited by a Lumped Current Source

[11] Figure 5 shows a perfectly conducting vertical wire excited by a lumped current source (1 m in height) at its base, whose electromagnetic field structure is analyzed using the FDTD method. The principal difference between this configuration (Figure 5) and that of Case 1 (Figure 1) is that in Case 1 the current is imposed along the entire channel length, while in Case 2 the current is imposed only at the bottom of the channel, and the current in other channel sections is determined by the solution of Maxwell's equations. The current waveform at the bottom of the wire is given by equation (2) with the same values of β and τ_0 as in Case 1.

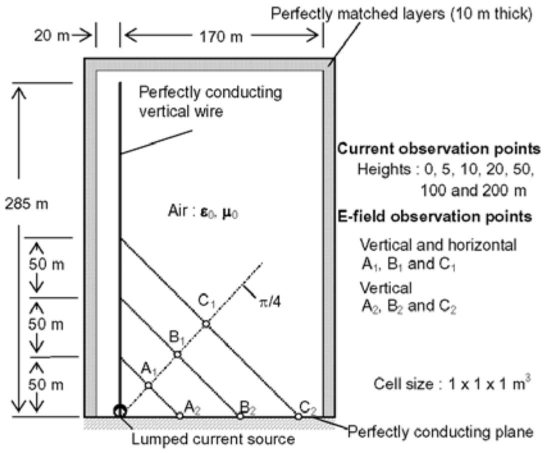

Figure 5. A perfectly conducting vertical wire in air on a perfectly conducting plane excited at the bottom by a lumped current source. Other conditions are the same as in Figure 1.

[12] Figure 6a, to be compared with Figure 3a, shows current waveforms on the vertical conductor observed at different heights. Figure 6b, to be compared with Figure 3b, shows waveforms of vertical and horizontal electric fields at points A_1, B_1, and C_1 and those of vertical electric fields at points A_2, B_2, and C_2.

[13] It is clear from Figure 6a that the current pulse amplitude attenuates significantly within the bottom 20 m or so and continues to decrease gradually above that height (the attenuation is stronger for shorter current pulses, larger cross-sections, and shorter current sources). This tendency is in contrast with Figure 3a (as expected) and in good agreement with the results obtained for a similar problem using thin cylindrical wires and the method of moments (MOM) [e.g., *Kordi et al.*, 2002, 2003; *Baba and Ishii*, 2003; *Grcev et al.*, 2003]. The attenuation of the current pulse amplitude is accompanied by lengthening its tail, so that the charge transfer is independent of height. The average propagation speed is 0.94*c* within the bottom 20 m and essentially c at larger heights.

[14] Further, in contrast with Figure 3b, the vertical electric field magnitudes at points A_1, B_1 or C_1 in Figure 6b are not equal to those of the horizontal electric field at the same point. This indicates the presence of a non-zero radial component of the electric field (and a non-radial component of Poynting vector) around the vertical conductor, in addition to the theta-directed component, the only electric field component observed in Case 1. Note that radial and theta components discussed here are for the spherical coordinate system cen-

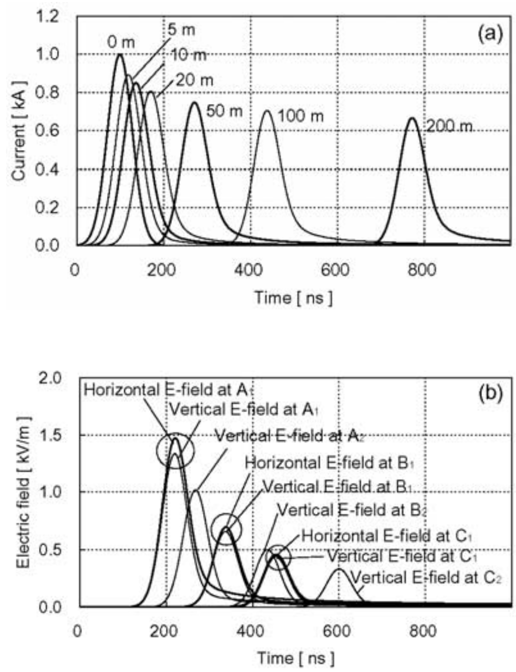

Figure 6. Waveforms of (a) current at different heights and (b) electric fields at points A_1, B_1, C_1, A_2, B_2, and C_2 calculated using the FDTD method for a perfectly conducting vertical wire in air on a perfectly conducting plane excited at its bottom by a lumped current source.

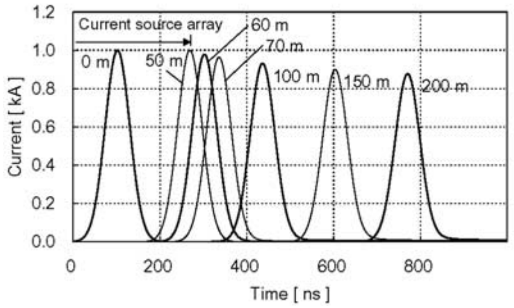

Figure 7. Current waveforms at different heights calculated using the FDTD method for a perfectly conducting vertical wire excited at its bottom by a vertical phased array of 50 current sources (Case 3).

tered at the channel base. Also, the vertical electric field on the perfectly conducting plane varies somewhat faster than the inverse distance, as opposed to Case 1 for which the inverse proportionality was observed. Thus, the electromagnetic field structure for Case 2 is non-TEM.

4. Case 3: Vertical Conductor Above Ground Excited by a Phased Current Source Array

[15] Figure 7 shows current waveforms at different heights calculated for a perfectly conducting vertical wire excited at its bottom by a vertical phased array of 50 current sources. The waveforms of the current sources are the same as those of the bottom 50 sources in Case 1 (Figure 1). Within a hemispherical shell 50 m in radius whose center is at the channel base, the electromagnetic field structure is a TEM wave during the first 300 ns or so, as follows from the analysis of Case 1. It is evident from Figure 7 that the current pulse does attenuate above the TEM-excitation region. Thus, an unattenuated current distribution generally requires that TEM wave excitation is maintained along the entire lightning channel length.

5. Concluding Remarks

[16] The transmission-line model of lightning return stroke in a vertical channel is most rigorously represented by a vertical phased array of current sources that produce a spherical TEM wave in the case of return-stroke speed v equal to the speed of light c. A perfectly conducting non-zero-radius vertical wire above ground excited by a practical source at its bottom end cannot support unattenuated current waves, and the associated electromagnetic field structure is non-TEM. The electromagnetic field structure of a vertical phased current source array differs from that of a vertical wire excited by a lumped current source also for the more practical case of $v < c$. A current pulse propagating along a vertical non-zero-radius wire above ground excited by a practical source cannot maintain its amplitude because such a wire generally cannot be viewed as a uniform transmission line. The characteristic impedance of a vertical cylinder above ground, as opposed to that of an inverted cone above ground, varies with height [e.g., *Menemenlis and Chun*, 1982], particularly near the ground, the region of primary interest when the generation of fields at early times is considered [e.g., *Thottappillil et al.*, 2001]. Lightning return-stroke models with an imposed current distribution along the entire channel represent a useful engineering tool, but should be used with caution in studying lightning physical processes.

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References

Baba, Y., and M. Ishii, Characteristics of electromagnetic return-stroke models, *IEEE Trans. Electromagn. Compat.*, *EMC-45*(1), 129–135, 2003.

Berenger, J. P., A perfectly matched layer for the absorption of electromagnetic waves, *J. Comput. Phys.*, *114*, 185–200, 1994.

Gomes, C., and V. Cooray, Concepts of lightning return stroke models, *IEEE Trans. Electromagn. Compat.*, *EMC-42*(1), 82–96, 2000.

Grcev, L., F. Rachidi, and V. A. Rakov, Comparison of electromagnetic models of lightning return strokes using current and voltage sources, in *Proc. of 12th Int. Conf. on Atmospheric Electricity*, 593–596, Versailles, France, 2003.

Kordi, B., R. Moini, and V. A. Rakov, Comment on 'Return stroke transmission line model for stroke speed near and equal that of light' by R. Thottappillil, J. Schoene, and M. A. Uman, *Geophys. Res. Lett.*, *29*(10), 1369, doi:10.1029/2001GL014602, 2002.

Kordi, B., R. Moini, and V. A. Rakov, Comparison of lightning return stroke electric fields predicted by the transmission line and antenna theory models, in *Proc. of 15*th *Int. Zurich Symp. on Electromagn. Compat.*, 102P2, 551–556, Zurich, 2003.

Krider, E. P., On the peak electromagnetic fields radiated by lightning return strokes toward the middle-atmosphere, *J. Atmos. Electr.*, *14*, 17–24, 1994.

Krider, E. P., C. Leteinturier, and J. C. Willett, Submicrosecond fields radiated during the onset of first return strokes in cloud-to-ground lightning, *J. Geophys. Res.*, *101*(D1), 1589–1597, 1996.

Menemenlis, C., and Z. T. Chun, Wave propagation on nonuniform line, *IEEE Trans. Power Apparatus and Systems*, *PAS-101*(4), 833–839, 1982.

Rachidi, F., C. A. Nucci, M. Ianoz, and C. Mazzetti, Influence of a lossy ground on lightning-induced voltages on overhead lines, *IEEE Trans. Electromagn. Compat.*, *EMC-38*(3), 250–264, 1996.

Rakov, V. A., R. Thottappillil, and M. A. Uman, On the empirical formula of Willett et al. relating lightning return stroke peak current and peak electric field, *J. Geophys. Res.*, *97*, 11,527–11,533, 1992.

Rakov, V. A., and M. A. Uman, Review and evaluation of lightning return stroke models including some aspects of their application, *IEEE Trans. Electromagn. Compat.*, *EMC-40*(4), 403–426, 1998.

Rakov, V. A., and W. Tuni, Lightning electric field intensity at high altitudes: Inferences for production of elves, *J. Geophys. Res.*, *108*(D20), doi:10.1029/2002JD003349, 2003.

Taflove, A., and S. C. Hagness, *Computational Electrodynamics: The Finite-Difference Time-Domain Method*, Artech House Pub., Boston, 2000.

Thottappillil, R., J. Schoene, and M. A. Uman, Return stroke transmission line model for stroke speed near and equal that of light, *Geophys. Res. Lett.*, *28*(18), 3593–3596, 2001.

Thottappillil, and Uman, M. A., Reply to the 'Comment on "Return stroke transmission line model for stroke speed near and equal that of light' by R. Thottappillil, J. Schoene, and M. A. Uman" by B. Kordi, R. Moini, and V. A. Rakov, *Geophys. Res. Lett.*, *29*, doi:10.1029/2002GL014758, 2002.

Uman, M. A., and D. K. McLain, The magnetic field of the lightning return stroke, *J. Geophys. Res.*, *74*, 6899–6910, 1969.

Uman, M. A., J. Schoene, V. A. Rakov, K. J. Rambo, and G. H. Schnetzer, Correlated time derivatives of current, electric field intensity, and magnetic flux density for triggered lightning at 15 m, *J. Geophys. Res.*, *107*(D13), doi:10.1029/2000JD000249, 2002.

Yee, K. S., Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media, *IEEE Trans. Antennas Propagat.*, *AP-14*(3), 302–307, 1966.

Zeddam, A., and P. Degauque, Current and voltage induced on telecommunication cables by a lightning stroke, in *Lightning Electromagnetics*, edited by R. L. Gardner, Hemisphere, New York, 377–400, 1990.

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