Lightning strikes to tall objects: Currents inferred from far electromagnetic fields versus directly measured currents

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[1] We have derived far-field-to-current conversion factors for lightning strikes to tall objects for (1) the initial peak current at the object top, (2) the largest peak current at the object top, and (3) the peak current at the object bottom. These far-field-to-current conversion factors are needed for proper interpretation of peak currents reported by lightning detection networks and are each expressed here as the product of (a) the far-field-to-current conversion factor for lightning strikes to flat ground based on the transmission line model and (b) an appropriate correction factor to account for the transient process in the strike object. The correction factors for the three considered cases are: (1) $f_{tall\_ini.top} = v/(v + c)$, (2) $f_{tall\_top} = [1 + \rho_{bot}(1 + \rho_{top})]\, v/(v + c)$, and (3) $f_{tall\_bot} = (1 + \rho_{bot})\, v/(v + c)$, where ρ_{bot} and ρ_{top} are current reflection coefficients at the object bottom and at the object top for upward-propagating waves, respectively. **Citation:** Baba, Y., and V. A. Rakov (2007), Lightning strikes to tall objects: Currents inferred from far electromagnetic fields versus directly measured currents, *Geophys. Res. Lett.*, *34*, L19810, doi:10.1029/2007GL030870.

1. Introduction

[2] Modern lightning detection (or locating) systems provide lightning return stroke peak currents estimated from measured magnetic field peaks. Direct measurements of lightning currents on tall towers are used for testing the validity of field-to-current conversion equations [e.g., *Diendorfer et al.*, 1998; *Diendorfer and Pichler*, 2004; *Lafkovici et al.*, 2006]. The field and current peaks are usually assumed to be proportional to each other, with the proportionality coefficient (field-to-current conversion factor) being determined for strikes to flat ground. Therefore, these conversion factors, in general, are not applicable to strikes to tall (electrically long) objects. Further, in the case of strikes to tall objects, as a result of transient process in the object, current waveforms can differ significantly at different heights along the object and can exhibit more than one peak (typically, secondary peak is larger than the initial one). In such cases, the field-to-current conversion factor depends on current measurement location (e.g., near the top or bottom of the tower) and on whether initial or largest current peak is used. Thus, for calibrating lightning locating systems (LLSs) using current measurements on towers, it is necessary to know appropriate far-field-to-current conversion factors.

[3] *Lafkovici et al.* [2006] have shown that current peaks measured directly near the top (at a height of about 500 m) of the 553-m CN Tower in Toronto for 21 strokes in 7 presumably upward flashes initiated from the tower in 2005 are considerably (by a factor of two to three) smaller than the corresponding current peaks estimated by the North American Lightning Detection Network (NALDN). Thus, the NALDN, which is calibrated using triggered-lightning strokes terminating on small-height grounded objects, tended to significantly overestimate currents in strokes terminating on the CN Tower.

[4] In contrast, *Diendorfer et al.* [1998] reported that current peaks measured directly near the top of the 160-m Peissenberg tower in Germany for 44 return strokes and initial-stage pulses, all greater than about 3 kA, in 8 flashes terminating on the tower in 1997–1998 were on average 0.88 of the corresponding current peaks estimated by the Austrian lightning detection system (ALDIS). Also, *Diendorfer and Pichler* [2004] reported that current peaks measured directly near the top of the 100-m Gaisberg tower in Austria for 334 return strokes terminating on the tower in 2000–2003 were on average 0.98 of the corresponding current peaks estimated by the European lightning detection network (EUCLID). Further, they found a good agreement between directly measured and EUCLID-reported peak currents for 173 initial-stage pulses (larger than 2 kA). Essentially all Peissenberg and Gaisberg tower flashes were of upward type; that is, they were typically composed of the initial stage and one or more leader/return stroke sequences.

[5] In this paper, we will derive far-field-to-current conversion factors for lightning strikes to tall objects for (1) the initial peak current at the object top, (2) the largest peak current at the object top, and (3) the peak current at the object bottom. We will use these conversion factors in examining the reported tower-measured and LLS-inferred currents.

2. Analysis

2.1. Lightning Currents and Associated Far Magnetic Fields

[6] Figure 1a illustrates the case of lightning strike to a tall grounded object, in which two lossless uniform transmission lines represent the lightning channel (whose characteristic impedance is Z_{ch}) and the tall strike object of height h (whose characteristic impedance is Z_{ob}). The current reflection coefficient at the bottom of the tall object and the current reflection coefficient at the top of the object for upward-propagating waves are given by $\rho_{bot} = (Z_{ob} - Z_{gr})/(Z_{ob} + Z_{gr})$ and $\rho_{top} = (Z_{ob} - Z_{ch})/(Z_{ob} + Z_{ch})$,

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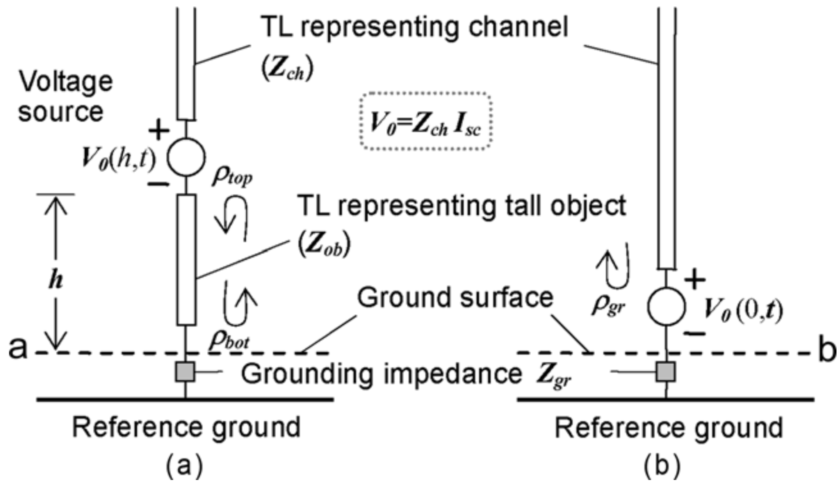

Figure 1. Transmission line representation of lightning strikes (a) to a tall grounded object of height h and (b) to flat ground. Adapted from *Baba and Rakov* [2005a].

respectively. Figure 1b illustrates the case of the same lightning strike to flat ground. The current reflection coefficient at the channel base is given by $\rho_{gr} = (Z_{ch} - Z_{gr})/(Z_{ch} + Z_{gr})$. In the above equations, Z_{gr} is the lumped grounding impedance. Expressions for current along a tall strike object and along the lightning return-stroke channel, to be used in Section 2.2.2 in deriving far-field-to-current conversion factors for lightning strikes to tall objects, are given by *Baba and Rakov* [2005a].

[7] Figure 2a shows waveforms of current at the top ($z' = h$) and bottom ($z' = 0$) of strike object of height h = 100 m, for $\rho_{top} = -0.5$, $\rho_{bot} = 1$. Figure 2b shows the waveform of current at the channel base ($z' = 0$) for the same lightning strike to flat ground, for $\rho_{gr} = 1$. In these calculations, current waveform proposed by *Nucci et al.* [1990] was used to specify the lightning short-circuit current, $I_{sc}(h, t)$ and $I_{sc}(0, t)$. This waveform is characterized by peak of $I_{sc.peak}$ = 11 kA and 10-to-90% risetime of RT = 0.15 μs, which are thought to be typical for subsequent strokes. In Figure 2a, the initial peak of current at the object top is $I_{top.ini.peak}$ = 8.2 kA, the largest peak current at the object top is $I_{top.peak}$ = 12 kA, and the peak current at the object bottom is $I_{bot.peak}$ = 16 kA. In Figure 2b, the peak current at the channel base is $I_{bas.peak} = I_{sc.peak}$ = 11 kA.

[8] Figure 3 shows waveforms of azimuthal magnetic field, calculated using the expression for the radiation component of magnetic field due to an infinitesimal dipole [*Uman et al.*, 1975] that was integrated over the radiating sections of the channel and the strike object, for a lightning strike to a tall grounded object of height h = 100 m on perfectly conducting ground at horizontal distance d = 50 km. Also shown in Figure 3 is the azimuthal magnetic field waveform for the same lightning strike to flat ground. In these field calculations, the return-stroke wavefront speed was set to $v = 0.5\ c$.

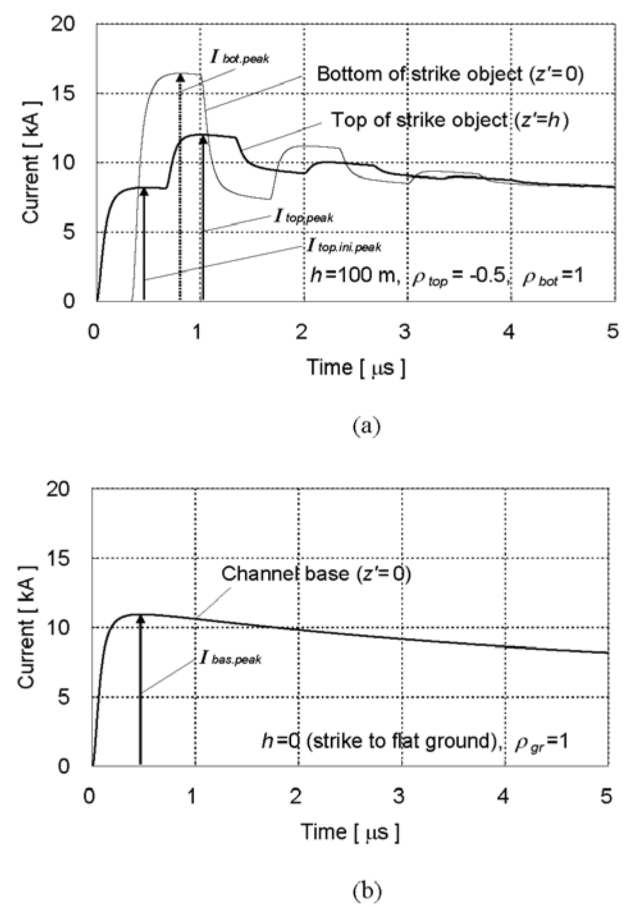

Figure 2. (a) Waveforms of current for a lightning strike to a 100-m object at the top (z' = 100 m) and bottom (z' = 0) of the object, and (b) waveform of current at the channel base (z' = 0) for the same lightning strike to flat ground. *Nucci et al.*'s [1990] current waveform was used to specify the lightning short-circuit current.

2.2. Far-Field-to-Current Conversion Factors

2.2.1. Lightning Strikes to Flat Ground

[9] The peak of azimuthal magnetic field (radiation-component) $H_{flat.peak}$ on perfectly conducting ground plane

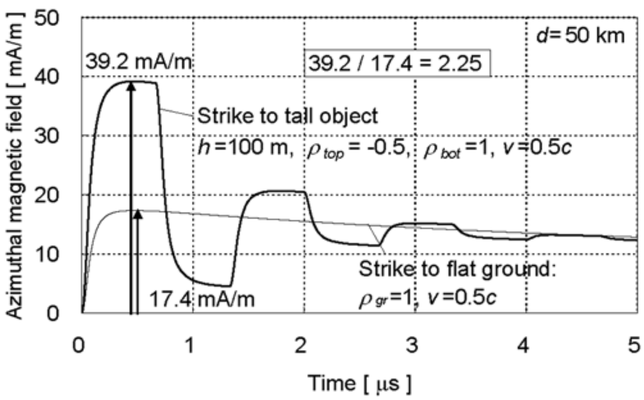

Figure 3. Azimuthal magnetic field waveforms calculated on perfectly conducting ground at distance d = 50 km for a lightning strike to a grounded object of height h = 100 m and for the same strike to flat ground. Note that the largest magnetic field peak corresponds to the initial (not the largest) current peak at the object top (see Figure 2a).

at far distance d from the lightning channel terminating on ground, according to the transmission line (TL) model [*Uman et al.*, 1975], is given by

$$H_{flat.peak} = \frac{1}{2\pi c d}\, v\, I_{bas.peak} = \frac{1}{2\pi c d}\, v\, \frac{1+\rho_{gr}}{2} I_{sc.peak} \qquad (1)$$

so that the far-(magnetic-)field-to-current conversion factor is given by

$$F_{H\_flat} = \frac{I_{bas.peak}}{H_{flat.peak}} = \frac{2\pi c d}{v} \qquad (2)$$

where $I_{bas.peak}$ is the peak current at the channel base ($z' = 0$), and $I_{sc.peak}$ is the peak of the lightning short-circuit current.

[10] Note that the far-(magnetic)-field-to-current conversion factor employed in modern lightning detection systems (0.185 in the NALDN and 0.23 in the ALDIS and EUCLID, where the magnetic field is expressed in so-called LLP units) has been determined empirically comparing system's response to triggered-lightning strokes terminating on small-height grounded objects and directly measured channel-base currents for these same strokes. This empirical conversion factor implies a constant value of return-stroke speed.

2.2.2. Lightning Strikes to Tall Objects

[11] Far-field-to-current conversion factors for lightning strikes to tall objects are defined here as the product of (a) the far-field-to-current conversion factor for lightning strikes to flat ground, F_{H_flat} given by (2), and (b) an appropriate correction factor to account for the transient process in the strike object.

[12] The peak of azimuthal magnetic field at far distance d on perfectly conducting ground [*Uman et al.*, 1975] due to two current waves of the same magnitude, $(1 - \rho_{top})I_{sc}(h, t)/2$, propagating upward and downward at speeds v and c, respectively, from the series voltage source placed at the junction point between the strike object and vertical lightning channel is given by

$$\begin{aligned} H_{tall.peak} &= \frac{1}{2\pi c d}\,(v+c)\, I_{top.ini.peak} \\ &= \frac{1}{2\pi c d}\,(v+c)\,\frac{1-\rho_{top}}{2} I_{sc.peak} \end{aligned} \qquad (3)$$

Note that (3) is valid only for $0 \le RT \le 2h/c$ when $\rho_{bot} = 1$ and for $0 \le RT \le h/c$ when $0 < \rho_{bot} < 1$ [*Baba and Rakov*, 2005b]. Using (2) and (3), one can express the far-(magnetic-)field-to-current conversion factor for lightning strikes to tall objects for the initial peak current at the object top as

$$\begin{aligned} F_{H\_tall\_ini.top} &= \frac{I_{top.ini.peak}}{H_{tall.peak}} = \frac{2\pi c d}{v+c} = \frac{v}{v+c} F_{H\_flat} \\ &= f_{tall\_ini.top}\, F_{H\_flat} \end{aligned} \qquad (4)$$

In (4), $f_{tall\_ini.top}$ is the correction factor to account for the transient process in the strike object, which also applies to the case of far vertical electric field. Clearly, $F_{H\_tall\_ini.top}$ is smaller than F_{H_flat} due to the presence of two waves moving in opposite directions from the top of the strike object. Both $F_{H\_tall\_ini.top}$ and $f_{tall\_ini.top}$ increase with increasing v. The maximum value of $f_{tall\_ini.top}$ corresponds to $v = c$ and is equal to 0.5. When $v = 0.5c$, $f_{tall\_ini.top} = 0.33$ (see Table 1). Note that $f_{tall\_ini.top} = (1 - \rho_{top})/(1 + \rho_{gr})/k_{tall}$, where k_{tall} is the far-field enhancement factor due to the presence of tall strike object, introduced by *Baba and Rakov* [2005b, equation B5].

[13] When the risetime, RT, of $I_{sc}(h, t)$ is less than $2h/c$, and its overall duration is much longer than $2h/c$, the largest peak of current at the object top ($z' = h$), $I_{top.peak}$, occurs between $t = 2h/c$ and $4h/c$ and can be approximated by $I_{top.peak} \approx (1 - \rho_{top})/2 \times [1 + \rho_{bot}\,(1 + \rho_{top})]\, I_{sc.peak} = [1 + \rho_{bot}\,(1 + \rho_{top})]\, I_{top.ini.peak}$. The peak current at the object bottom ($z' = 0$), $I_{bot.peak}$, occurs between $t = h/c$ and $3h/c$ and can be approximated by $I_{bot.peak} \approx (1 - \rho_{top})/2 \times (1 + \rho_{bot})I_{sc.peak} = (1 + \rho_{bot})\, I_{top.ini.peak}$. Therefore, far-field-to-current conversion factors for the largest peak current at the

Table 1. Correction Factors to be Used in Far-Field-to-Current Conversion Equations to Account for Transient Process in the Strike Object[a]

Peak Current of Interest	Correction Factor Equation	Expected Value[b]
Peak current at the channel base for strikes to flat ground, $I_{bas.peak}$	$f_{flat} = 1$	1
Initial peak current at the object top, $I_{top.ini.peak}$	$f_{tall\_ini.top} = \dfrac{v}{v+c}$	0.33
Largest peak current at the object top, $I_{top.peak}$	$f_{tall\_top} = [1 + \rho_{bot}\,(1 + \rho_{top})]\,\dfrac{v}{v+c}$	0.5
Peak current at the object bottom, $I_{bot.peak}$	$f_{tall\_bot} = (1 + \rho_{bot})\,\dfrac{v}{v+c}$	0.67

[a]Far-field-to-current conversion equations including correction factors, f, are given by, $I = f(2\pi c d/v)H$ and $I = f(2\pi\varepsilon_0 c^2 d/v)E$; f_{tall_top}, $f_{tall\_ini.top}$, and f_{tall_bot} are valid only for $RT \le 2\ h/c$ when $\rho_{bot} = 1$ and for $RT \le h/c$ when $0 < \rho_{bot} < 1$.

[b]Expected values are calculated for $\rho_{top} = -0.5$ and $\rho_{bot} = 1$, and $v = 0.5c$ [*Rakov*, 2001, 2007].

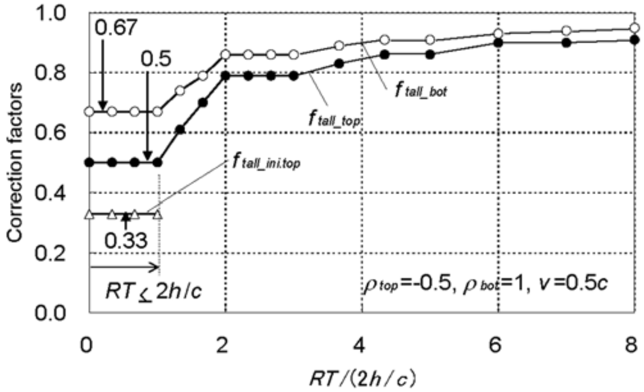

Figure 4. Correction factors, f, in far-field-to-current conversion equations, $I = f(2\pi cd/v)H$ or $I = f(2\pi\varepsilon_0 cd/v)E$, to account for transient process in the strike object as a function of $RT/(2h/c)$. Values of f were calculated from peak currents at the top and bottom of a 100-m-high strike object and associated azimuthal magnetic field peaks at distance d = 50 km. A ramp-like current waveform was used to specify the lightning short-circuit current. When $RT > 2h/c$, the initial peak coincides with the largest peak, so that $f_{tall\_ini.top} = f_{tall\_top}$.

object top and for the peak current at the object bottom can be expressed as

$$F_{H\_tall\_top} = \frac{I_{top.peak}}{H_{tall.peak}} = \frac{\left[1+\rho_{bot}\left(1+\rho_{top}\right)\right] I_{top.ini.peak}}{H_{tall.peak}} = \left[1+\rho_{bot}\left(1+\rho_{top}\right)\right]\frac{v}{v+c}F_{H\_flat} = f_{tall\_top}F_{H\_flat} \quad (5)$$

$$F_{H\_tall\_bot} = \frac{I_{bot.peak}}{H_{tall.peak}} = \frac{(1+\rho_{bot})I_{top.ini.peak}}{H_{tall.peak}} = (1+\rho_{bot})\frac{v}{v+c}F_{H\_flat} = f_{tall\_bot}F_{H\_flat} \quad (6)$$

Expressions for all the correction factors considered here are summarized in Table 1. It is worth noting that (4), (5), and (6) are valid only for $0 \le RT \le 2h/c$ when ρ_{bot} = 1 and for $0 \le RT \le h/c$ when $0 \le \rho_{bot} < 1$ (see also Figure 4).

3. Discussion

[14] *Lafkovici et al.* [2006] reported that the peak currents, I_{CNT}, measured directly near the top of the 553-m CN Tower for 21 strokes in 7 presumably upward flashes initiated from the tower in 2005 were a factor of 0.38, on average, smaller than the corresponding NALDN-reported peak currents, I_{NALDN}. This ratio of I_{CNT} and I_{NALDN} was estimated as the inverse of the slope of linear regression equation, $I_{NALDN} = 2.61\ I_{CNT} - 1.83$, reported by *Lafkovici et al.*, with the intercept (−1.83) being neglected. I_{CNT} corresponds to the largest peak current at the object top, $I_{top.peak}$, and the corresponding correction factor for ρ_{top} = −0.5, ρ_{bot} = 1, and $v = 0.5c$ is equal to 0.5 (see Table 1; it would be 0.45 if ρ_{bot} were assumed to be 0.7), which is not much different from the observed ratio I_{CNT}/I_{NALDN} = 0.38. Note that for the 553-m CN Tower, $2h/c$ = 3.7 μs, which is larger than current risetimes (0.18 to 2.0 μs; A. Hussein, personal communication, 2007) in *Lafkovici et al.*'s study. It is worth mentioning that *Lafkovici et al.* expressed caution regarding their directly measured currents, stating that the CN Tower current measuring system is in need of more accurate calibration.

[15] In contrast to CN Tower observations, *Diendorfer et al.* [1998] reported that largest current peaks measured directly near the top of the 160-m Peissenberg tower in Germany for 44 return strokes and initial-stage pulses, all greater than about 3 kA, in 8 flashes that terminated on the tower in 1997–1998 were on average 0.88 of the corresponding current peaks estimated by the ALDIS. Further, *Diendorfer and Pichler* [2004] reported that the current peaks measured directly near the top of the 100-m Gaisberg tower in Austria for 334 strokes that terminated on the tower in 2000–2003 were, on average, 0.98 of the corresponding current peaks estimated by the EUCLID. We now discuss possible reasons why the flat-ground conversion factor appears to be applicable to Peissenberg and Gaisberg tower strikes. For strikes to a 160- or 100-m tower, if current risetimes are larger than 1.1 or 0.67 μs, respectively, the $RT < 2h/c$ condition (for ρ_{bot} = 1) assumed in deriving equations for far-field-to-current conversion factors is not satisfied. Figure 4 shows correction factors to account for transient process in the strike object as a function of $RT/(2h/c)$. Values of correction factors were calculated from peak currents at the top and bottom of a 100-m-high strike object and associated azimuthal magnetic field peaks at distance d = 50 km for ρ_{top} = −0.5, ρ_{bot} = 1, and $v = 0.5c$. A ramp-like current waveform was used to specify the lightning short-circuit current. Note that, since the initial peak current at the object top is identical to the largest peak current for $RT > 2h/c$, the correction factor for the initial peak current at the object top, $f_{tall\_ini.top}$ is shown only for $0 \le RT \le 2h/c$. In the case of $RT > 2h/c$, f_{tall_top} and f_{tall_bot} increase [*Baba and Rakov*, 2005b] relative to the values given in Table 1. For example, the correction factor for the largest peak at the object top is 0.79 for $RT = 4h/c$ and 0.90 and for $RT = 12h/c$, compared to 0.5 for $RT < 2h/c$. We estimated from Figure 4 the expected equivalent RTs for the 160-m Peissenberg tower and the 100-m Gaisberg tower (f_{tall_top} = 0.88 and 0.98, respectively) to be about $5(2h/c)$ = 5.5 μs and $>8(2h/c)$ = 5.3 μs, respectively.

[16] Interestingly, *Diendorfer and Pichler* [2004] found that essentially no correction factor was needed for either return strokes or initial-stage pulses. For the latter, the geometric mean 10-to-90% RT was estimated by *Miki et al.* [2005] to be 61 μs for the Peissenberg tower and 110 μs for the Gaisberg tower (both $\gg 2h/c$), although *Diendorfer and Pichler* and *Diendorfer et al.* considered only larger (>2 kA and >3 kA, respectively) pulses, as opposed to *Miki et al.* who included in their analysis all measurable pulses. For the case of $RT \gg 2h/c$, the tower acts as a lumped circuit and each of the correction factors is equal to unity.

[17] Another factor possibly contributing to the discrepancy between the CN-Tower data on the one hand and Peissenberg and Gaisberg tower data on the other hand is the propagation model used in the NALDN, but not in

the ALDIS or EUCLID. This model is to increase the measured field peak (normalized to 100 km) in order to compensate for its attenuation due to propagation over finitely-conducting ground. Uncompensated field propagation effects should lead to an overestimation of ratio of measured and inferred currents (inferred currents are smaller) as well as of the expected *RT*s (see above). When only sensors corresponding to relatively small propagation effects are considered, ratios of measured and inferred currents for the Peissenberg and Gaisberg towers become smaller, 0.71 [*Diendorfer et al.*, 1998, Figure 3] and 0.67 [*Schulz and Diendorfer*, 2004, Figure 3B], respectively, and the corresponding expected equivalent *RT*s become $1.7(2h/c) = 1.8$ μs and $1.5(2h/c) = 1.0$ μs. These latter values of *RT* appear to be reasonable.

4. Summary

[18] We have derived far-field-to-current conversion factors for lightning strikes to tall objects for (1) the initial peak current at the object top, (2) the largest peak current at the object top, and (3) the peak current at the object bottom. The correction factor derived for the 553-m CN Tower for case (2) is equal to 0.5, which is not much different from the observed ratio (0.38, on average) of the peak current measured directly near the tower top and corresponding LLS-reported peak current. In contrast, for the 160-m Peissenberg tower and 100-m Gaisberg tower, the observed ratios are close to 1 (0.88 and 0.98, respectively) due to the smaller round-trip time along the tower relative to the risetime of current waveforms and uncompensated field propagation effects in the ALDIS and EUCLID.

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