Comment on "Radio frequency radiation beam pattern of lightning return strokes: A revisit to theoretical analysis" by Xuan-Min Shao, Abram R. Jacobson, and T. Joseph Fitzgerald

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1. Introduction

[1] It is shown that the expression of *Shao et al.* [2004] for the radiation component of electric field from lightning return stroke does not give the correct analytical expression for the radiation beam pattern for the traveling current source (TCS) model, even though their expression for differential radiation electric field is general for a moving current element. It is also shown that their expression does not give the correct radiation field pattern for a traveling step function wave whose magnitude is exponentially decaying with height (the modified transmission line (MTLE) model).

[2] In their paper, *Shao et al.* [2004] assume a moving differential current element, idz', and derive an equation for the resultant electric field, dE, containing the so-called F factor, $(1 - (v/c)\cos\theta)^{-1}$, using a simple differential transformation between the retarded time and stationary time/space. The same F factor was previously obtained, using a different approach, by *Rubinstein and Uman* [1990] for fields from the traveling current discontinuity and was obtained and examined for the transmission line (TL) model by a number of researchers [e.g., *Le Vine and Willett*, 1992; *Thottappillil et al.*, 1998]. *Shao et al.* [2004, paragraph 1] (hereinafter referred to as SJF) state that the F factor is "fundamental and is explicitly associated with the radiation beam pattern but is not limited only to the lossless TL return stroke model." In order to illustrate this point they integrate their equation for dE over the entire lightning channel length to obtain the F factor for the TCS model. These comments are prompted by the fact that some of SJF's results and inferences appear to be in conflict with our own published work on the subject.

2. Discussion

[3] We agree that equation (7) of SJF for dE from a moving current element is general (applicable to a variety of lightning return stroke models), but we think that their equation (10) for E is not. We show that SJF's equation (10) is applicable only to models in which both the return stroke front and the current distribution (wave) behind the front move forward at the same speed v. Further, we show that equation (10) does not give correct analytical expression for the radiation beam pattern for the TCS model and that expression (12) given by SJF for the TCS model is incomplete. It is also shown that their equation (10) does not give the correct radiation field pattern for a traveling step function wave whose magnitude is exponentially decaying with height (the MTLE model). Part of the problem with equation (10) is its failure to take into account properly the contribution to the field from current discontinuity at the wave front, which is an intrinsic feature of the TCS model. Even if the contribution due to current discontinuity at the wave front were added to equation (10), it would only be applicable to return stroke current distributions that could be expressed as the sum of propagating waves not to an arbitrarily specified current distribution behind the return stroke wave front.

[4] SJF derive the radiation field from a moving differential current element in a coordinate system that moves with the pulse. Therefore they get their equation (7), slightly modified and reproduced as equation (1) below, for the differential radiation electric field involving the explicit F factor,

$$d\overline{E} = \frac{1}{4\pi\varepsilon_0 c^2}\frac{1}{r}\frac{\sin\theta}{(1-(v/c)\cos\theta)}\frac{\partial i(z',t')}{\partial t'}dz'\hat{a}_\theta, \tag{1}$$

where $i(z', t')$ is the current at position z' and retarded time $t' = t - r(z')/c$ and r is the distance from dz' at position z' to the field point. SJF state that (1) is a general result for a traveling current pulse and that the F factors associated with the TL and TCS models can be derived as special cases from (1) by integrating dE over the entire channel length, assuming that the physical length of the active channel at t' is much smaller than distance r. SJF obtained an equation (see their equation (10)) for the far

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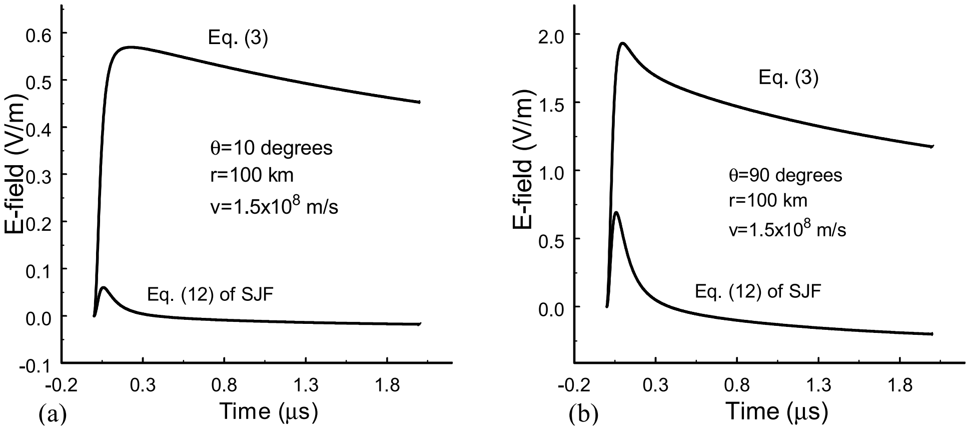

Figure 1. Comparison of electric fields at $r = 100$ km for the TCS return stroke model using incomplete equation (12) of SJF and equation (3) of this comment for return stroke speed 1.5×10^8 m/s. Effects of ground plane on the fields are not included. Angle with respect to vertical, θ, is (a) 10° and (b) 90° (ground surface). Figure 1 illustrates the importance of the second term in equation (3) that accounts for the current discontinuity at the return stroke front and is missing in equation (12) of SJF.

radiation field from a return stroke, which is slightly modified and reproduced below; two versions of which are labeled (2a) and (2b) here in order to facilitate discussion,

$$\overline{E} = \frac{1}{4\pi\varepsilon_0 c^2 r} \frac{\sin\theta \hat{a}_\theta}{(1-(v/c)\cos\theta)} \int_0^{L'} \frac{\partial i(z',t')}{\partial t'} dz' \qquad (2a)$$

$$\overline{E} = \frac{1}{4\pi\varepsilon_0 c^2 r} \frac{v\,\sin\theta \hat{a}_\theta}{(1-(v/c)\cos\theta)} (i(L',t') - i(0,t')). \qquad (2b)$$

In (2a) and (2b), $L' = vt'$ is the length of the active channel at t'.

[5] From equation (2b), SJF obtained their expressions (11) and (12) for the TL and TCS models, respectively. Equation (11) of SJF for the TL model is identical to equation (47) of *Thottappillil et al.* [1998] (hereinafter referred to as TUR), except for the sign (it should be plus; there is a sign error in equation (11) of SJF, which has its origin in the final form of their equation (10), similar to equation (2b) above) if both equations are expressed in the same notation and assuming that there is no current discontinuity at the return stroke wave front. However, equation (12) for the TCS model given by SJF is not correct. The correct expression is given by equation (49) of TUR, reproduced in SJF's notation as equation (3) below:

$$\overline{E} = \frac{1}{4\pi\varepsilon_0}\frac{\sin\theta}{c^2 r}\left[\frac{c}{1+\cos\theta}\left(i\left(0, L'\left(\frac{1}{u}+\frac{1}{c}\right)\right) - i(0,t')\right) + \frac{u}{1-\frac{u}{c}\cos\theta} i\left(0, L'\left(\frac{1}{u}+\frac{1}{c}\right)\right)\right]\hat{a}_\theta. \qquad (3)$$

In equation (3), u is the speed of the upward extension of the return stroke channel and is the same as v in (1) and (2). The last term of equation (3) is missing in equation (12) of SJF, which shows that equation (2b) is not general. In the TCS model the return stroke wave front traveling upward at speed u injects the charges into the channel instantaneously, and as a result, current at the wave front is not equal to zero. The last term of (3) accounts for this current discontinuity that is intrinsic in the TCS model. In fact, as illustrated in Figure 1, the far radiation field predicted by the TCS model may be dominated by the last term of (3), missing in (12) of SJF.

[6] It appears that equation (2a) can only be applied to finding the far radiation field pattern for the TL model but not for any general current distribution in space and time, as the form of current $i(z', t')$ implies. Even in the case of models of TL type, equation (2b) has to be modified when there is a current discontinuity at the wave front since the first term of (2b) involving $i(L', t')$ does not really take care of possible current discontinuity at the wave front. TUR showed that the far turn-on field due to current discontinuity at the wave front for the TL model is given by an expression that is the same in magnitude as the first term of (2b) but has the opposite sign. Thus the total far field for the TL model, even with a current discontinuity at the wave front (for the TL model this implies that current at $z' = 0$ has an instantaneous rise at $t = 0$), is given by equation (11) of SJF, corrected for sign error as mentioned earlier.

[7] Now consider the case of a step function current wave that travels upward with speed v and whose magnitude decreases with height as $e^{-z'/\lambda}$, where λ is a decay height constant. This is in fact the well-known modified transmission line model with an exponential current decay with height (MTLE) [*Nucci et al.*, 1988]. The far radiation field (θ component) pattern from such a return stroke current distribution is discussed by *Wait* [1998] and by *Rakov and Tuni* [2003], and their expression, without including the effect of ground, is given by equation (4) below,

$$\overline{E} = +\frac{1}{4\pi\varepsilon_0}\frac{Fv\sin\theta}{c^2 r} I_0 e^{-\frac{L'(t)}{\lambda}}\hat{a}_\theta \approx +\frac{1}{4\pi\varepsilon_0}\frac{Fv\sin\theta}{c^2 r} I_0 e^{-\frac{Fv(t-r/c)}{\lambda}}\hat{a}_\theta. \qquad (4)$$

where I_0 is the current magnitude at $z' = 0$, $F = (1 - (v/c)\cos\theta)^{-1}$, and $L'(t) \approx Fv(t - r/c)$ when $L'(t) \ll r$. The exact expression for $L'(t)$ is given by equation (25) of TUR. Equation (4) cannot be obtained directly from equation (2b), which can be shown as follows. Application of $i(z', t') = I_0 e^{-z'/\lambda}$ to (2b), excluding the current discontinuity at the wave front, gives (after correcting for the sign error)

$$\overline{E} = \frac{-1}{4\pi\varepsilon_0 c^2 r} \frac{v \sin\theta \hat{a}_\theta}{(1 - (v/c)\cos\theta)} \left(I_0 e^{-\frac{L'(t)}{\lambda}} - I_0 \right). \tag{5a}$$

The wave front discontinuity gives a field term that is equal in magnitude but opposite in sign to the first term of (5a) and that has to be added to (5a) in order to obtain the total far radiation field. The resulting expression contains only the second term of (5a) and can be written as

$$\overline{E} = +\frac{1}{4\pi\varepsilon_0} \frac{Fv \sin\theta}{c^2 r} I_0 \hat{a}_\theta. \tag{5b}$$

It is clear that (5b) is not equivalent to (4), confirming that equation (2b) (equation (10) of SJF) is not general.

[8] TUR clearly show that the general expressions for electromagnetic fields from arbitrarily specified current distributions that vary in time and space (expressions (7) and (8) of TUR) do not require any explicit correction involving F factor. F factor can arise when one carries out certain analytical simplifications of field expressions. The case of far electromagnetic fields from a traveling current discontinuity, first considered by *Rubinstein and Uman* [1990] and later discussed by *Rubinstein and Uman* [1991] and by TUR, is one example of that. Current distributions associated with certain return stroke models, e.g., the TL and TCS models, also may give rise to the F factor in the analytical simplification of far radiation field expressions. Other situations when the F factor can come out explicitly are (1) the length of a moving line as seen by a remote observer (apparent length) and (2) the speed of a traveling wave as seen by a remote observer (apparent speed). These two cases are also discussed by TUR. SJF introduced one more situation, the gradient of retarded time, which led them to a field expression containing the F factor for a differential current element. Only if one of the above situations is involved, may the analytic expression for the field contain the F factor. Therefore the F factor need not be an intrinsic, explicit entity associated with radiation field patterns.

[9] A truly general electric field equation that is valid for any model (including those with current discontinuity at the return stroke front) and at any distance from the lightning channel is equation (7) of *Thottappillil and Rakov* [2001b] (see also equation (5.21) of *Thottappillil* [2003]). This equation does not contain any explicit F factor unless there is a current discontinuity at the wave front, which is one of the situations when F factor arises as mentioned earlier. Thus the F factor is not a ''fundamental'' quantity, as SJF suggest; it is rather one formal way to account for the retardation effects, which are indeed fundamental in computing fields due to moving sources.

[10] In the traditional formulation of electromagnetic fields using the current dipole technique, F factor appears only in the radiation field term, associated with the radiation field of the current discontinuity at the traveling wave front and/or due to the wave behind the front. However, this may not be the case for other formulations, as explained below. *Thottappillil and Rakov* [2001a] (hereinafter referred to as TR) discuss three different but equivalent expressions for electric fields from propagating current distributions that vary in time and space. In two of the approaches in which the electric field expressions involve charge densities, the current/charge discontinuity at the return stroke wave front contributes to part of the intermediate (induction) electric field term also, and the F factor can appear in this term through the expression for the speed of the wave front as seen by the remote observer (notice dL'/dt in equations (6) and (9) of *TR*). The explicit expression for dL'/dt, involving the F factor, is equation (31) of TUR, equivalently written as equation (6) below:

$$\frac{dL'}{dt} = \frac{v}{1 - (v/c)\cos\theta(L')}. \tag{6}$$

In equation (6), $\theta(L')$ is the angle between the direction of propagation and the line connecting the retarded position of the wave front and the field point.

3. Concluding Remarks

[11] In summary, SJF present a new approach to deriving a radiation electric field equation for the TL model when there is no current discontinuity at the return stroke front. This approach (see their equation (11)) is identical, except for the SJF sign error, to that found in the literature (e.g., equation (47) of TUR). However, their electric field equation (10), asserted to be general, is incapable of handling models with current discontinuity (intrinsic in some models) at the return stroke front, and their electric field equation (12) for the TCS model is incomplete. The correct equation for the TCS model is given by TUR (see their equation (49)). A truly general electric field equation that is valid for any model and for any distance to the field point is equation (7) of *Thottappillil and Rakov* [2001b].

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