On the use of lumped sources in lightning return stroke models

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[1]   We consider the use of lumped voltage and current sources in engineering lightning return stroke models with emphasis on those including a tall strike object. If the model is to be used for computing remote electric and magnetic fields, we suggest a representation of the lightning channel as a transmission line energized by a lumped voltage source, with the voltage magnitude being expressed in terms of the lightning short-circuit current and equivalent impedance of the lightning channel. Such a representation assures appropriate boundary conditions (reflection and transmission coefficients) at the channel attachment point and is equivalent to a distributed-shunt-current-source representation of the lightning channel. This is in contrast with the use of series ideal current source which presents infinitely large impedance to current waves reflected from the ground and/or from discontinuities in the lightning channel, such as the moving return stroke front or branches, and therefore is inadequate when such reflections are involved. If the model is to be used only for injecting lightning current into a grounded object or system, a Norton equivalent circuit (an ideal current source in parallel with the equivalent impedance of the lightning channel) is sufficient to represent the lightning discharge.

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1.   Introduction

[2]   This paper is concerned primarily with the transmission line (TL) model of the lightning return stroke [*Uman and McLain*, 1969] generalized to include a tall strike object, although the results can be readily extended to the modified TL model with linear current decay with height (MTLL) [*Rakov and Dulzon*, 1987] and the modified TL model with exponential current decay with height (MTLE) [*Nucci et al.*, 1988]. All these models belong to the class of "engineering" models [*Rakov and Uman*, 1998]. Additionally considered here is the use of a Norton equivalent circuit in simulating lightning interaction with grounded objects and systems. The TL model has been widely used in various lightning electromagnetic pulse (LEMP) calculations [e.g., *Zeddam and Degauque*, 1990; *Krider*, 1994]. This model assumes that a current wave, injected at the lightning channel base, propagates upward along the channel without attenuation or distortion. The resultant longitudinal channel current $I(z', t)$ at any height z' and any time t is related to current $I(0, t)$ at the channel base by a simple equation: $I(z', t) = I(0, t - z'/v)$, where v is the return stroke wave front speed. The TL model is often visualized [e.g., *Rakov and Uman*, 1998; *Gomes and Cooray*, 2000; *Thottappillil and Uman*, 2002] as incorporating a lumped current source at the channel base, which injects a specified current into the channel. The primary reason for the assumption of a lumped current source (not a lumped voltage source) at the channel base is a desire to use directly the channel-base current, known from measurements for both natural and triggered lightning, as an input parameter of the model.

[3]   "Engineering" models were originally developed for lightning strikes to flat ground and did not consider any reflections (for example, from a channel branching point) of an upward propagating current wave. Recently, considerable attention has been attracted to the interaction of lightning with tall grounded strike objects [e.g., *Motoyama et al.*, 1996; *Guerrieri et al.*, 1998; *Shostak et al.*, 1999; *Goshima et al.*, 2000; *Rachidi et al.*, 2001, 2002; *Bermudez et al.*, 2003; *Pavanello et al.*, 2004]. When a model is generalized to include a tall grounded object, it is necessary to consider reflections, which requires specification of appropriate reflection coefficients at the top and bottom of the strike object. In this case, the use of an ideal current source at the channel attachment point, inserted in series between the channel and strike object [e.g., *Motoyama et al.*, 1996; *Guerrieri et al.*, 1998; *Goshima et al.*, 2000], does not allow one to obtain a self-consistent solution. This is because the ideal current source has an infinitely large impedance, and hence the lightning channel is electrically isolated from the strike object (transient processes in the lightning channel and in the tall object are totally decoupled). In general, a lumped source connected in series at the junction between two transmission lines representing the channel and strike object is required to

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(1) inject the same current in both the lines and (2) be "seen" as a zero impedance by reflected waves returning to the junction point. A series current source satisfies the first requirement, but not the second one. Thus a series current source is not suitable for modeling of lightning return strokes when reflections are involved. Note that a shunt current source at the junction point satisfies the second requirement, but not the first one, unless the characteristic impedances of the channel and strike object are assumed to be the same. In order to avoid the problem with lumped current sources, *Rachidi et al.* [2002] have proposed a distributed-shunt-current-source representation of the lightning channel. An alternative approach would be to use a lumped series voltage source at the junction point that can satisfy both the requirements listed above.

[4] It is important to distinguish between two applications of lightning models: (1) computation of electric and magnetic fields needed, for example, in studying lightning-induced effects in electrical circuits, and (2) injection of lightning current into a grounded object or system in studying direct lightning strike effects. Indeed, in the first application, the distribution of current along the lightning channel as well as along the tall strike object, if present, should be specified, while in the second application the induced effects are usually neglected and the distribution of current only in the grounded object or system subjected to lightning is important (the actual current in the lightning channel is immaterial). In this paper, we propose the use of a lumped voltage source in the TL model generalized to include a tall strike object in the first application and show that a Norton equivalent circuit [*Carlson*, 1996] is sufficient in the second application. The Norton equivalent circuit consists of an ideal current source producing a lightning short-circuit current that is connected in parallel with the equivalent impedance of the lightning channel (often assumed to be equal to the characteristic impedance of the lightning channel). This equivalent circuit is to be connected in parallel with a "load", which can be either lumped or distributed circuit representing the grounded object or system subjected to lightning.

[5] The structure of this paper is as follows. In section 2, on the basis of the representation of a lightning strike to grounded object by a lossless transmission line excited at its attachment point to the object by a lumped voltage source, we derive expressions for current along the tall strike object and along the lightning channel, as a function of the lightning short-circuit current. We compare our current expressions with those derived by *Rachidi et al.* [2002], who used the distributed-shunt-current-source representation of the lightning channel. We also consider the special case of a strike object of negligible height, represented by its grounding impedance. In section 3, we show that the Norton equivalent circuit representation of lightning is sufficient in engineering studies of lightning currents injected into a grounded object or system. In Appendix A, we examine the properties of Norton (current source) equivalent circuit in comparison with those of Thevenin (voltage source) equivalent circuit.

[6] Throughout the analysis presented in this paper, we assume that the equivalent impedance of lightning channel is constant (although in reality it varies with lightning current) and that the transmission lines representing vertical conductors, lightning channel or strike object, are uniform (although in reality they are nonuniform). We also assume that the lightning channel extends from its termination point on ground or on grounded object to infinity. These assumptions are traditionally used in engineering studies of lightning interaction with various objects and systems [e.g., *Shostak et al.*, 1999; *Goshima et al.*, 2000; *Rakov*, 2001; *Rachidi et al.*, 2002; *Bermudez et al.*, 2003].

2. Models for Computing Lightning Electric and Magnetic Fields

[7] Before considering a more general case of lightning striking a tall object, we examine the case of lightning striking an object of negligible height, represented by its grounding impedance (a lumped-circuit grounded object such as an electrically-short grounding electrode). We will refer to this case as lightning strike to flat ground. The transmission line is assumed to support wave propagation at a speed lower than the speed of light, which can be achieved, for example, by adjusting the per-unit-length inductance or/and capacitance of the line.

2.1. Lightning Strike to Flat Ground

[8] In this section, we derive an expression for current along the lightning channel using a representation of lightning strike to flat ground comprising (see Figure 1a) a lossless uniform transmission line representing the lightning channel (whose characteristic impedance is Z_{ch}), a lumped grounding impedance (Z_{gr}), and a lumped voltage source generating an arbitrary voltage waveform $V_0(0, t)$.

[9] From Figure 1a, current, $I(z', t)$, at an arbitrary height z' along the channel and at an arbitrary time t, is given by

$$I(z', t) = \frac{1}{Z_{ch} + Z_{gr}} V_0\left(0, t - \frac{z'}{v}\right) = \frac{Z_{ch}}{Z_{ch} + Z_{gr}} I_{sc}\left(0, t - \frac{z'}{v}\right), \quad (1)$$

where v is the return stroke wave front speed, and $I_{sc}(0, t)$ is the lightning short-circuit current at $z' = 0$ given by

$$I_{sc}(0, t) = \frac{V_0(0, t)}{Z_{ch}}. \quad (2)$$

The lightning short-circuit current is defined here as the lightning current that would be measured at an ideally grounded object ($Z_{gr} = 0$ or $Z_{gr} \ll Z_{ch}$) of negligible height. Most currents measured in triggered-lightning experiments [e.g., *Depasse*, 1994; *Rakov et al.*, 1998] and natural-lightning currents measured on tall objects [e.g., *Berger et al.*, 1975; *Visacro et al.*, 2004a, 2004b] can be viewed as short-circuit currents. The lightning short-circuit current represents the lightning discharge regardless of the impedance "seen" by this discharge at its termination point and therefore can be used for comparison of lightning strikes under a variety of conditions, including strikes to flat ground and to electrically-long strike objects. The latter case is considered in sections 2.2 and 2.3.2 [see also *Rakov*, 2001].

[10] Although no downward propagating wave would be present in the uniform transmission line representing the lightning channel shown in Figure 1a, the current reflection
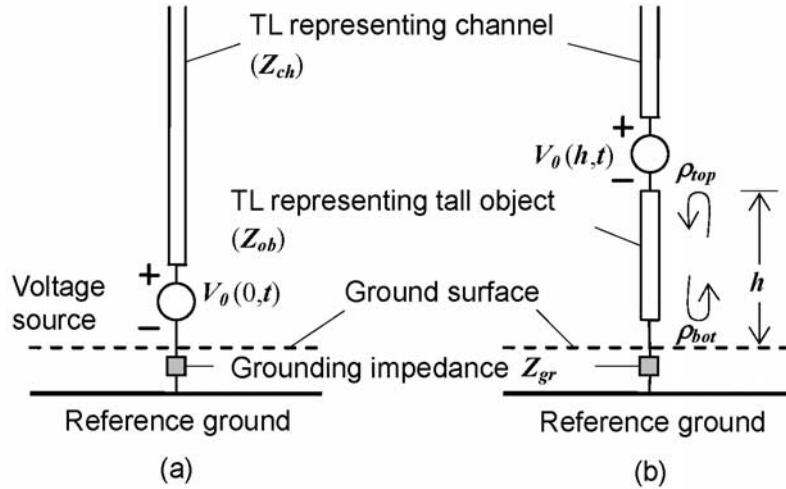
Figure 1. Lightning strikes (a) to flat ground and (b) to a tall grounded object of height h, represented by lossless transmission lines connected in series with a lumped voltage source generating an arbitrary voltage waveform, $V_0(0, t)$ or $V_0(h, t)$, and a lumped grounding impedance (Z_{gr}). Z_{ch} is the characteristic impedance of the transmission line representing the lightning channel, and Z_{ob} is that representing the tall strike object. Here ρ_{top} is the current reflection coefficient at the top of the strike object for upward propagating waves, and ρ_{bot} is the current reflection coefficient at the bottom of the strike object.

coefficient at ground for downward propagating current waves can be expressed as

$$\rho_{gr} = \frac{Z_{ch} - Z_{gr}}{Z_{ch} + Z_{gr}}. \tag{3}$$

We will show later that the configuration shown in Figure 1a is equivalent to a distributed-shunt-current-source representation of the lightning channel (with no lumped source at the channel base), for which the current reflection coefficient at ground is required. Using equation (3), we can formally rewrite equation (1) as

$$I(z', t) = \frac{Z_{ch}}{Z_{ch} + Z_{gr}} I_{sc}\left(0, t - \frac{z'}{v}\right) = \frac{1 + \rho_{gr}}{2} I_{sc}\left(0, t - \frac{z'}{v}\right)$$
$$= \frac{1}{2}\left[I_{sc}\left(0, t - \frac{z'}{v}\right) + \rho_{gr} I_{sc}\left(0, t - \frac{z'}{v}\right)\right]. \tag{4a}$$

Note that the difference between equation (4a) and its counterpart, equation (11), for the distributed-shunt-current-source representation of the lightning channel proposed by *Rachidi et al.* [2002] (see section 2.3.1) is the speed in the second term: v in equation (4a) and c in equation (11). If $\rho_{gr} = 1$, probably the most practical case, since Z_{gr} is typically much smaller than Z_{ch} which is expected to be of the order of hundreds of ohms to some kiloohms [*Gorin et al.*, 1977; *Gorin and Shkilev*, 1984], equation (4a) reduces to

$$I(z', t) = I_{sc}\left(0, t - \frac{z'}{v}\right). \tag{4b}$$

Note that equation (4b) postulates that the channel-base current, $I(0, t)$, in the TL model is the same as the lightning short-circuit current. This is consistent with the fact that $I(0, t)$ used as an input to the TL model is normally based on measurements for which one can assume $\rho_{gr} = 1$. Indeed, *Rakov et al.* [1998] showed that triggered-lightning peak currents measured under a variety of grounding conditions

ranging from nearly ideal to poor are similar, suggesting that for those measurements Z_{ch} was much larger than Z_{gr}. Further, most of natural-lightning currents measured at the top of tall objects (usually well grounded) or at the bottom of objects of relatively small height (60 m or less) are not much influenced by the transient process excited in the object [*Rakov*, 2001, 2003; *Miyazaki and Ishii*, 2004; *Visacro et al.*, 2004a, 2004b] and therefore can be viewed as short-circuit currents as well.

2.2. Lightning Strike to a Tall Grounded Object

[11] In this section, we will generalize the model developed in section 2.1 to include a tall strike object of height h. We will derive expressions for current along the tall strike object and along the lightning channel using a configuration comprising (see Figure 1b) two lossless uniform transmission lines representing the lightning channel (whose characteristic impedance is Z_{ch}) and tall grounded object (whose characteristic impedance is Z_{ob}), a lumped grounding impedance (Z_{gr}), and a lumped voltage source generating an arbitrary voltage waveform $V_0(h, t)$. We assume that the current propagation speed along the tall grounded object is equal to the speed of light c. The current reflection coefficient at the bottom of the tall object for downward propagating waves (ρ_{bot}) and the current reflection coefficient at the top of the tall object for upward propagating waves (ρ_{top}) are given by

$$\rho_{bot} = \frac{Z_{ob} - Z_{gr}}{Z_{ob} + Z_{gr}}, \qquad \rho_{top} = \frac{Z_{ob} - Z_{ch}}{Z_{ob} + Z_{ch}}. \tag{5}$$

The configuration considered in this section (Figure 1b) differs from that discussed in section 2.1 (Figure 1a) by the presence of a distributed circuit (transmission line representing the tall strike object) between the lumped voltage source and the lumped grounding impedance. As a result, current equations involve the summations of multiple

waves bouncing between the bottom and top of the object and ground-reflected waves being transmitted into the lightning channel. Note that the current initially injected into the tall object and into the channel is given by $I(h, t) = V_0(h, t)/(Z_{ch} + Z_{ob})$.

[12] From Figure 1b, currents along the tall object ($0 \leq z' \leq h$) and along the lightning channel ($z' \geq h$) are given by the following two equations, which become identical when $z' = h$:

$$
\begin{aligned}
I(z', t) &= \frac{1}{Z_{ch} + Z_{ob}} V_0\left(h, t - \frac{h - z'}{c}\right) \\
&+ \rho_{bot} \frac{1}{Z_{ch} + Z_{ob}} V_0\left(h, t - \frac{h + z'}{c}\right) \\
&+ \rho_{top}\rho_{bot} \frac{1}{Z_{ch} + Z_{ob}} V_0\left(h, t - \frac{h - z'}{c} - \frac{2h}{c}\right) \\
&+ \rho_{bot}\rho_{top}\rho_{bot} \frac{1}{Z_{ch} + Z_{ob}} V_0\left(h, t - \frac{h + z'}{c} - \frac{2h}{c}\right) \\
&\cdots \\
&= \sum_{n=0}^{\infty} \left[\begin{array}{l} \rho_{bot}^n\rho_{top}^n \frac{1}{Z_{ch} + Z_{ob}} V_0\left(h, t - \frac{h - z'}{c} - \frac{2nh}{c}\right) \\ +\rho_{bot}^{n+1}\rho_{top}^n \frac{1}{Z_{ch} + Z_{ob}} V_0\left(h, t - \frac{h + z'}{c} - \frac{2nh}{c}\right) \end{array} \right] \\
&= \frac{1 - \rho_{top}}{2} \sum_{n=0}^{\infty} \left[\begin{array}{l} \rho_{bot}^n\rho_{top}^n I_{sc}\left(h, t - \frac{h - z'}{c} - \frac{2nh}{c}\right) \\ +\rho_{bot}^{n+1}\rho_{top}^n I_{sc}\left(h, t - \frac{h + z'}{c} - \frac{2nh}{c}\right) \end{array} \right]
\end{aligned}
$$

for $0 \leq z' \leq h$ (along the strike object)   (6a)

$$
\begin{aligned}
I(z', t) &= \frac{1}{Z_{ch} + Z_{ob}} V_0\left(h, t - \frac{z' - h}{v}\right) \\
&+ \left[\begin{array}{l} (1 + \rho_{top})\rho_{bot} \frac{1}{Z_{ch} + Z_{ob}} V_0\left(h, t - \frac{z' - h}{v_{ref}} - \frac{2h}{c}\right) \\ +(1 + \rho_{top})\rho_{bot}\rho_{top}\rho_{bot} \frac{1}{Z_{ch} + Z_{ob}} V_0\left(h, t - \frac{z' - h}{v_{ref}} - \frac{4h}{c}\right) \\ +(1 + \rho_{top})\rho_{bot}\rho_{top}\rho_{bot}\rho_{top}\rho_{bot} \frac{1}{Z_{ch} + Z_{ob}} V_0\left(h, t - \frac{z' - h}{v_{ref}} - \frac{6h}{c}\right) \\ \cdots \end{array} \right] \\
&\quad \times u\left(t - \frac{z' - h}{v}\right) \\
&= \frac{1}{Z_{ch} + Z_{ob}} V_0\left(h, t - \frac{z' - h}{v}\right) \\
&+ \sum_{n=1}^{\infty} \rho_{bot}^n\rho_{top}^{n-1}(1 + \rho_{top}) \frac{1}{Z_{ch} + Z_{ob}} V_0\left(h, t - \frac{z' - h}{v_{ref}} - \frac{2nh}{c}\right) \times u\left(t - \frac{z' - h}{v}\right) \\
&= \frac{1 - \rho_{top}}{2} \left[\begin{array}{l} I_{sc}\left(h, t - \frac{z' - h}{v}\right) \\ + \sum_{n=1}^{\infty} \rho_{bot}^n\rho_{top}^{n-1}(1 + \rho_{top}) I_{sc}\left(h, t - \frac{z' - h}{v_{ref}} - \frac{2nh}{c}\right) \end{array} \right] u\left(t - \frac{z' - h}{v}\right) \\
&= \frac{1}{2} \left[\begin{array}{l} I_{sc}\left(h, t - \frac{z' - h}{v}\right) - \rho_{top} I_{sc}\left(h, t - \frac{z' - h}{v}\right) \\ +(1 - \rho_{top})(1 + \rho_{top}) \sum_{n=1}^{\infty} \rho_{bot}^n\rho_{top}^{n-1} I_{sc}\left(h, t - \frac{z' - h}{v_{ref}} - \frac{2nh}{c}\right) \end{array} \right] \\
&\quad \times u\left(t - \frac{z' - h}{v}\right)
\end{aligned}
$$

for $z' \geq h$ (along the lightning channel)   (6b)

where n is an index representing the successive multiple reflections occurring at the two ends of the strike object, v is the return stroke wave front speed, and v_{ref} is the speed of upward propagating current waves reflected from ground and then transmitted into the lightning channel. Although for the linear transmission line representing lightning channel in Figure 1b v_{ref} should be the same as v, $v_{ref} = v$, we use different symbols for these two speeds (v in the second term and v_{ref} in the third term of equation (6b).) in order to facilitate comparison of equation (6b) with its counterpart, equation (13b), for the distributed-shunt-current-source representation of the lightning channel proposed by *Rachidi et al.* [2002] (see section 2.3.2). In this latter representation, it is assumed that $v_{ref} = c$. Note that setting $v_{ref} = v$ allows one to avoid the situation when faster moving ground reflections catch up with the slower moving front and the necessity to deal with reflections at that front. Note also that in deriving equations (6a) and (6b) we used the definition of ρ_{top}, from which $Z_{ch}/(Z_{ch} + Z_{ob}) = (1 - \rho_{top})/2$ (see equation (5)) and the definition of the lightning short-circuit current given by

$$ I_{sc}(h, t) = \frac{V_0(h, t)}{Z_{ch}}. \tag{7} $$

As stated in section 2.1, the lightning short-circuit current is the lightning current that would be measured at an ideally grounded object ($Z_{gr} = 0$ or $Z_{gr} \ll Z_{ch}$) of negligible height ($h \approx 0$), which represents the lightning discharge regardless of the impedance "seen" by this discharge at its termination point and therefore can be used for comparison of lightning strikes to flat ground and to electrically-long strike objects. This means that for such comparison $I_{sc}(h, t)$ and $V_0(h, t)$ in equation (7) should be the same as $I_{sc}(0, t)$ and $V_0(0, t)$, respectively, in equation (2), in which case the same charge is transferred to ground regardless of the presence of the tall object (we checked this by confirming that integration of equation (4a) at $z' = 0$ from $t = 0$ to ∞ was equal to that of equation (6a)). On the other hand, current initially injected into the tall strike object is different from the current that the same lightning would inject directly into the ground, as discussed next.

[13] When $\rho_{bot} = 1$ and $\rho_{top} = -0.5$ (for example, $Z_{ch} = 900\ \Omega$, $Z_{ob} = 300\ \Omega$, and $Z_{gr} = 0\ \Omega$), the magnitude of current waves initially injected into the tall object and into the channel is $(1 - \rho_{top})I_{sc}(h, t)/2 = 0.75\ I_{sc}(h, t)$ from equations (6a) and (6b) (for $z' = h$, $t < 2h/c$, and $n = 0$). When $\rho_{top} = 0$, that is, when $Z_{ob} = Z_{ch}$ (matched conditions at the channel attachment point), the initially injected current is equal to $I_{sc}(h, t)/2$. When $\rho_{gr} = 1$ ($Z_{gr} = 0$), the magnitude of current injected into the channel at its termination on flat ground ($z' = 0$) is $(1 + \rho_{gr})I_{sc}(0, t)/2 = I_{sc}(0, t)$, from equation (4). The difference between the current in configuration of Figure 1a (equation (4a)) and the initial current in configuration of Figure 1b (equations (6a) and (6b)) is due to a transient process excited in the tall strike object. If the current waveshape were a step function, channel currents in configurations of Figures 1a and 1b would become identical as the time approached infinity, that is, when the transient process ended.

[14] Equations (6a) and (6b) readily reduce to equation (4a) if h approaches zero (if the tall strike object in configuration of Figure 1b is removed, which results in configuration of

Figure 1a). In this case, terms in equations (6a) and (6b) become

$$I_{sc}\left(h, t - \frac{h - z'}{c} - \frac{2nh}{c}\right) \cong I_{sc}\left(0, t + \frac{z'}{c}\right),$$
$$I_{sc}\left(h, t - \frac{h + z'}{c} - \frac{2nh}{c}\right) \cong I_{sc}\left(0, t - \frac{z'}{c}\right),$$
$$I_{sc}\left(h, t - \frac{z' - h}{v}\right) \cong I_{sc}\left(0, t - \frac{z'}{v}\right), \quad (8)$$
$$I_{sc}\left(h, t - \frac{z' - h}{v_{\text{ref}}} - \frac{2nh}{c}\right) \cong I_{sc}\left(0, t - \frac{z'}{v_{\text{ref}}}\right).$$

Since $|\rho_{\text{bot}}\rho_{\text{top}}|$ is less than 1 (unless $Z_{ch} = 0$, which is physically unreasonable), the geometrical series in equations (6a) and (6b) can be reduced as [e.g., *Spiegel and Liu*, 1998]

$$\sum_{n=0}^{\infty} \rho_{\text{bot}}^n \rho_{\text{top}}^n = \sum_{n=1}^{\infty} \rho_{\text{bot}}^{n-1} \rho_{\text{top}}^{n-1} \cong \frac{1}{1 - \rho_{\text{bot}}\rho_{\text{top}}}. \quad (9)$$

We set $v_{\text{ref}} = v$ in equation (6b), as required for a linear transmission line representation of the lightning channel considered here. Additionally, in the case of h approaching zero, upward current waves, reflected from ground and then transmitted into the channel propagate with no or little delay relative to the upward propagating return-stroke wave front. As a result, v_{ref} in this case (h approaches zero) should not be much different from v even without the assumption that the transmission line representing the lightning channel is linear. Hence equations (6a) for $z' = 0$ and (6b) for $z' \geq 0$ become

$$I(z', t) = \frac{1 - \rho_{\text{top}}}{2} \sum_{n=0}^{\infty} \left[\begin{array}{l} \rho_{\text{bot}}^n \rho_{\text{top}}^n I_{sc}\left(h, t - \frac{h - z'}{c} - \frac{2nh}{c}\right) \\ + \rho_{\text{bot}}^{n+1} \rho_{\text{top}}^n I_{sc}\left(h, t - \frac{h + z'}{c} - \frac{2nh}{c}\right) \end{array} \right]$$

$$= \frac{1 - \rho_{\text{top}}}{2}(1 + \rho_{\text{bot}}) \sum_{n=0}^{\infty} \rho_{\text{bot}}^n \rho_{\text{top}}^n I_{sc}(0, t)$$

$$\cong \frac{1 - \rho_{\text{top}} + \rho_{\text{bot}} - \rho_{\text{top}}\rho_{\text{bot}}}{2\left(1 - \rho_{\text{bot}}\rho_{\text{top}}\right)} I_{sc}(0, t)$$

$$= \frac{1 + \rho_{gr}}{2} I_{sc}(0, t)$$

$$= \frac{1}{2}\left[I_{sc}(0, t) + \rho_{gr} I_{sc}(0, t) \right]$$

$$\text{for } z' = 0 \quad \text{(at the channel base)} \quad (10a)$$

$$I(z', t) = \frac{1}{2}\left[\begin{array}{l} I_{sc}\left(h, t - \frac{z' - h}{v}\right) - \rho_{\text{top}} I_{sc}\left(h, t - \frac{z' - h}{v}\right) \\ + \left(1 - \rho_{\text{top}}\right)\left(1 + \rho_{\text{top}}\right) \sum_{n=1}^{\infty} \rho_{\text{bot}}^n \rho_{\text{top}}^{n-1} I_{sc}\left(h, t - \frac{z' - h}{v_{\text{ref}}} - \frac{2nh}{c}\right) \end{array} \right]$$

$$\times u\left(t - \frac{z' - h}{v}\right)$$

$$= \frac{1}{2}\left[1 - \rho_{top} + \left(1 - \rho_{\text{top}}\right)\left(1 + \rho_{\text{top}}\right) \sum_{n=1}^{\infty} \rho_{\text{bot}}^n \rho_{\text{top}}^{n-1} \right] I_{sc}\left(0, t - \frac{z'}{v}\right)$$

$$= \frac{1}{2}\left[1 - \rho_{top} + \left(1 - \rho_{\text{top}}\right)\left(1 + \rho_{\text{top}}\right)\rho_{\text{bot}} \frac{1}{1 - \rho_{\text{bot}}\rho_{\text{top}}} \right] I_{sc}\left(0, t - \frac{z'}{v}\right)$$

$$= \frac{1 - \rho_{\text{top}} + \rho_{\text{bot}} - \rho_{\text{bot}}\rho_{\text{top}}}{2\left(1 - \rho_{\text{bot}}\rho_{\text{top}}\right)} I_{sc}\left(0, t - \frac{z'}{v}\right) = \frac{1 + \rho_{gr}}{2} I_{sc}\left(0, t - \frac{z'}{v}\right)$$

$$= \frac{1}{2}\left[I_{sc}\left(0, t - \frac{z'}{v}\right) + \rho_{gr} I_{sc}\left(0, t - \frac{z'}{v}\right) \right]$$

$$\text{for } z' \geq 0 \quad \text{(along the lightning channel)}. \quad (10b)$$

As expected, equation (10b), which corresponds to the strike to tall object configuration of Figure 1b with $h \rightarrow 0$, is identical to equation (4a), which corresponds to the strike to flat ground configuration of Figure 1a. Equation (10a) is simply the special case of equation (10b) for $z' = 0$.

[15] It follows from the above that the transmission line representation of lightning strike to a tall object or to flat ground including a lumped voltage source at the channel attachment point allows a simple and self-consistent formulation of current both along the lightning channel and along the object. The voltage magnitude is expressed in terms of the lightning short-circuit current, and equivalent impedance of the lightning channel. As noted above, most currents measured in triggered-lightning experiments [e.g., *Depasse*, 1994; *Rakov et al.*, 1998] and natural-lightning currents measured on tall objects [e.g., *Berger et al.*, 1975; *Visacro et al.*, 2004a, 2004b] can be viewed as short-circuit currents. The equivalent impedance of the lightning channel is thought to be in the range from hundreds of ohms to several kiloohms [*Gorin et al.*, 1977; *Gorin and Shkilev*, 1984]. This impedance is usually much larger than the grounding impedance, so that ρ_{gr} in equation (4a) is usually equal to 1, in which case equation (4b) applies.

[16] By multiplying the first and second terms in equation (6b) by $[1 - (z' - h)/H]$, where H is the total length of the lightning channel, or $\exp[-(z' - h)/\lambda]$, where λ is the decay height constant, we can extend equation (6b) to the case of the MTLL model [*Rakov and Dulzon*, 1987] or the MTLE model [*Nucci et al.*, 1988], respectively. Note that the current attenuation functions, $[1 - (z' - h)/H]$ and $\exp[-(z' - h)/\lambda]$, do not influence equation (6a).

2.3. Comparison With *Rachidi et al.*'s [2002] Distributed-Current-Source Representation of the Lightning Channel

[17] In this section, we compare current expressions derived in sections 2.1 and 2.2 (see equations (4a), (6a), and (6b)) with those derived for the TL model by *Rachidi et al.* [2002], who used a distributed-shunt-current-source representation of the lightning channel.

2.3.1. Lightning Strike to Flat Ground

[18] The expression for current along the lightning channel, derived by *Rachidi et al.* [2002] for the case of lightning strike to flat ground (see their equation (3b)), is given by

$$I(z', t) = \left[I_{mc}\left(0, t - \frac{z'}{v}\right) + \rho_{gr} I_{mc}\left(0, t - \frac{z'}{c}\right) \right] u\left(t - \frac{z'}{v}\right), \quad (11)$$

where ρ_{gr} is the current reflection coefficient at ground for downward propagating current waves, given by equation (3), and $I_{mc}(0, t)$ is the lightning current that would be measured at ground in the case of matched conditions ($Z_{gr} = Z_{ch}$) at the channel attachment point. From equation (1) (see Figure 1a), for $z' = 0$ and $Z_{gr} = Z_{ch}$ we obtain

$$I_{mc}(0, t) = \frac{V_0(0, t)}{Z_{ch} + Z_{gr}} = \frac{1}{2} \frac{V_0(0, t)}{Z_{ch}} = \frac{I_{sc}(0, t)}{2}. \quad (12)$$

It follows from equation (12) that one cannot arbitrarily replace $I_{sc}(0, t)$ in, for example, equation (4b) by $I_{mc}(0, t)$; an additional coefficient of 2 would be required in this case. The

matched-conditions current is a hypothetical current that cannot be measured because it requires that $Z_{gr} = Z_{ch}$, while in reality, typically, $Z_{gr} \ll Z_{ch}$ [*Rakov et al.*, 1998]. Note that equation (3b) of *Rachidi et al.* [2002], on which equation (11) is based, is for the MTLE model [*Nucci et al.*, 1988] and therefore includes the current attenuation function, $\exp(-z'/\lambda)$, which is dropped in equation (11) to reduce the Rachidi et al. equation to the case of the TL model. Also note that equation (11) disregards any reflections at the return stroke wave front that should occur in the case of $v < c$ [*Rachidi et al.*, 2002]. Such reflections were considered by *Heidler and Hopf* [1994] and *Shostak et al.* [1999].

[19] It is evident that the structure of equation (11) is the same as that of equation (4a). The difference between equations (4a) and (11) is the speed in the current argument in the second term: v in equation (4a) and c in equation (11). In our representation using a transmission line excited at its bottom by a lumped voltage source (see Figure 1a), a current wave whose magnitude is $(1 + \rho_{gr})I_{sc}/2$ propagates upward at speed v. In *Rachidi et al.*'s [2002] representation, shunt current sources distributed along the lightning channel are activated progressively when the return stroke wave front, propagating upward at speed v, arrives at their altitudes. The resultant partial current waves are assumed to propagate downward at the speed of light, c, and the upward waves reflected from ground are also assumed to propagate along the channel at the speed of light. The speed of both downward propagating current waves and upward propagating reflected current waves along the channel in Rachidi et al.'s model can be viewed as an adjustable parameter and therefore can be set at any reasonable value, with the structure of governing equation (11) remaining the same. One such reasonable speed value is v, the return-stroke front speed. In fact, equation (11) reduces at $\rho_{gr} = 1$ to equation (4b), the original TL model equation, when one replaces c with v. Thus, if one sets the speed of the current waves propagating along the lightning channel to v instead of c in equation (11), the latter becomes identical to equation (4a) proposed in this paper. This shows that our transmission line representation employing a lumped voltage source is equivalent to the distributed-current-source representation of lightning channel proposed by *Rachidi et al.* [2002], if the speed adjustment described above is made. It is worth noting that equations (4a) and (11) can be also made identical without the speed adjustment if one assumes that $\rho_{gr} = 0$, which requires that $Z_{gr} = Z_{ch}$, an unrealistic condition at the lightning attachment point, as noted above. The equivalence of equations (4a) and (11) at $\rho_{gr} = 0$ is apparently due to the implicit assumption of $\rho_{gr} = 0$ ($Z_{gr} = Z_{ch}$) used in developing the distributed-current-source representation of the lightning channel [*Rachidi and Nucci*, 1990; *Rachidi et al.*, 2002; *Cooray*, 2003]. Note that the unrealistic assumption of $\rho_{gr} = 0$ ($Z_{gr} = Z_{ch}$) is also implicitly used in the traveling current source [*Heidler*, 1985] and the *Diendorfer and Uman* [1990] models, as discussed by *Thottappillil et al.* [1997], *Rakov et al.* [2003], and *Schoene et al.* [2003]. Still another possibility to make equations (4a) and (11) identical is to assume that $v = c$, which, however, is inconsistent with optical observations [e.g., *Idone and Orville*, 1982; *Idone et al.*, 1984; *Mach and Rust*, 1989; *Olsen et al.*, 2004]. Typical values of v range from one third to two thirds of the speed of light.

[20] We now compare current distributions predicted by equations (4a) and (11). Figure 2 shows current waveforms at different heights, $z' = 0$, 300, and 600 m, along the lightning channel, calculated using equations (4a) and (11), in which we set $\rho_{gr} = 1$ ($Z_{gr} = 0\ \Omega$) and $v = 0.5c$. It is clear from Figure 2 that the current distribution calculated using equation (4a) is not much different from that calculated using equation (11) for the typical value of $v = 0.5c$. Note that the current waveform predicted by equation (4a) preserves its shape and amplitude as it propagates along the channel, consistent with the original TL model concept, while the current waveform predicted by equation (11) does not. As stated above, if ρ_{gr} is set to 0, or v set to c, or the speed of downward propagating current waves and upward propagating reflected current waves (along the channel) in *Rachidi et al.*'s [2002] model is set to v, the current waveforms at any height along the channel calculated using equation (11) become identical to the corresponding waveforms calculated using equation (4a). The larger the value of v, the closer current distributions predicted by equations (4a) and (11).

[21] In the next section, we compare in a similar manner current distributions predicted by equations (6a) and (6b), derived in this paper, and by their counterparts proposed by *Rachidi et al.* [2002] for the case of lightning strike to a tall object.

2.3.2. Lightning Strike to a Tall Grounded Object

[22] The expressions for current along a tall strike object and along the lightning channel, derived by *Rachidi et al.* [2002] (see their equations (25) and (24)), are given by

$$
I(z',t) = \left(1 - \rho_{top}\right) \sum_{n=0}^{\infty} \left[\begin{array}{l} \rho_{bot}^n \rho_{top}^n I_{mc}\left(h, t - \dfrac{h - z'}{c} - \dfrac{2nh}{c}\right) \\ + \rho_{bot}^{n+1} \rho_{top}^n I_{mc}\left(h, t - \dfrac{h + z'}{c} - \dfrac{2nh}{c}\right) \end{array} \right]
$$
for $0 \leq z' \leq h$ (along the strike object); (13a)

$$
I(z',t) = \left[\begin{array}{l} I_{mc}\left(h, t - \dfrac{z' - h}{v}\right) - \rho_{top} I_{mc}\left(h, t - \dfrac{z' - h}{c}\right) \\ + \left(1 - \rho_{top}\right)\left(1 + \rho_{top}\right) \sum_{n=1}^{\infty} \rho_{bot}^n \rho_{top}^{n-1} I_{mc}\left(h, t - \dfrac{z' - h}{c} - \dfrac{2nh}{c}\right) \end{array} \right]
$$
$$
\times u\left(t - \dfrac{z' - h}{v}\right)
$$
for $z' \geq h$ (along the lightning channel). (13b)

[23] Equation (6a) derived here for current along the tall strike object (see Figure 1b) is the same as equation (13a). Also, the structure of equation (6b) derived here for current along the lightning channel is the same as that of equation (13b). The differences are v in the second term and v_{ref} in the third term of equation (6b) (recall that we set $v_{ref} = v$ in this paper), whereas the corresponding speeds in *Rachidi et al.*'s [2002] expression (13b) are equal to c. This difference is due to the different choices of speed for current waves propagating along the lightning channel, as explained in section 2.3.1 above. If we adjust the current-wave propagation speeds in equation (13b), replacing c in two places in equation (13): in the second term and $(z' - h)/c$ of the third
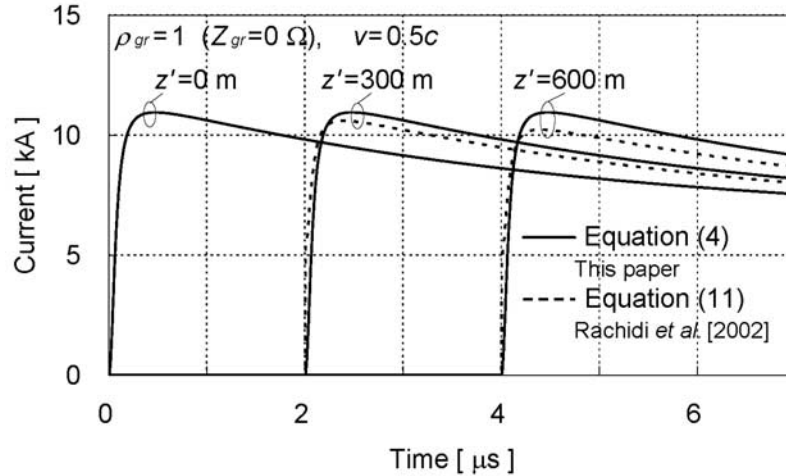
Figure 2. Current waveforms at different heights, $z' = 0$, 300, and 600 m, along the lightning channel for a lightning strike to flat ground, calculated using equations (4a) and (11). In these calculations, ρ_{gr} is set to 1 ($Z_{gr} = 0\ \Omega$) and v is set to 0.5c. The lightning short-circuit current $I_{sc}(0, t)$ is represented by a current pulse thought to be typical for subsequent lightning return strokes [*Nucci et al.*, 1990].

term, with v, this equation becomes identical to equation (6b), in which v_{ref} is set to v.

[24] We now compare current distributions predicted by equations (6b) and (13b). Figure 3 shows current waveforms at different heights, $z' = 100$ m (at the top of the strike object), 400 m, and 700 m, along the lightning channel, calculated for a lightning strike to 100-m high object using equations (6b) and (13b). In these calculations, we set $\rho_{top} = -0.5$, $\rho_{bot} = 1$ (e.g., $Z_{ch} = 900\ \Omega$, $Z_{ob} = 300\ \Omega$, $Z_{gr} = 0\ \Omega$), and $v = 0.5c$. It is clear from Figure 3 that the overall current distribution calculated using equation (6b) with $v_{ref} = v$ is not much different from that calculated using equation (13b), although differences in detailed structure are appreciable. The differences disappear if one adjusts the speeds as discussed above. Note that the current waveform predicted by equation (6a) is the same at all heights above $z' = 100$ m, while that predicted by equation (13b) is not.

3. Models for Studying Lightning Currents Injected Into Grounded Object

[25] In this section, we examine the Norton equivalent circuit of lightning, comprising a lumped current source producing a short-circuit current $I_{sc}(t)$ connected in parallel with a lumped equivalent impedance of lightning channel Z_{ch} (usually assumed to be equal to the characteristic impedance of the channel). We will show that this engineering approach to study lightning currents injected into a grounded object or system is consistent with the models proposed in sections 2.1 and 2.2.

[26] Applications of Norton equivalent circuit to studying lightning currents injected into a lumped or distributed grounded object are illustrated in Figures 4a and 4b, respectively. General features of Norton equivalent circuit and Thevenin equivalent circuit are discussed in Appendix A. The Norton equivalent circuit was used by *Rakov*

et al. [1998] in examining the dependence of lightning peak current on grounding conditions at the strike point and by *Rakov* [2001, 2003] in finding the distribution of current along a tall strike object. The Norton equivalent circuit is also used in analyzing surges due to direct lightning strikes in power transmission lines with the Electromagnetic Transients Program (EMTP) [*Scott-Meyer*, 1982].

[27] The Norton equivalent circuit shown in Figure 4a injects current $I_{gr}(t)$ into the lumped grounding impedance Z_{gr}, given by

$$ I_{gr}(t) = \frac{Z_{ch}}{Z_{ch} + Z_{gr}} I_{sc}(t), \qquad \text{for}\quad z' = 0. \qquad (14) $$

The Norton equivalent circuit shown in Figure 4b initially injects current $I_{ob}(t)$ into the object (whose characteristic impedance is Z_{ob}), given by

$$ I_{ob}(t) = \frac{Z_{ch}}{Z_{ch} + Z_{ob}} I_{sc}(t), \qquad for\quad z' = h. \qquad (15) $$

Equation (15) is valid before the first reflection from Z_{gr} arrives at the source ($t < 2h/c$).

[28] The current injected into Z_{gr} in the circuit shown in Figure 4a, given by equation (14), is consistent with equation (1) derived for the lumped-voltage-source equivalent circuit shown in Figure 1a. Similarly, the current injected initially into Z_{ob} in the circuit shown in Figure 4b, given by equation (15), is consistent with equation (6a) (for $z' = h$, $t < 2h/c$, and $n = 0$) derived for the lumped-voltage-source circuit shown in Figure 1b. Also, the current reflection coefficient at the bottom of the strike object for downward propagating waves ρ_{bot} and that at
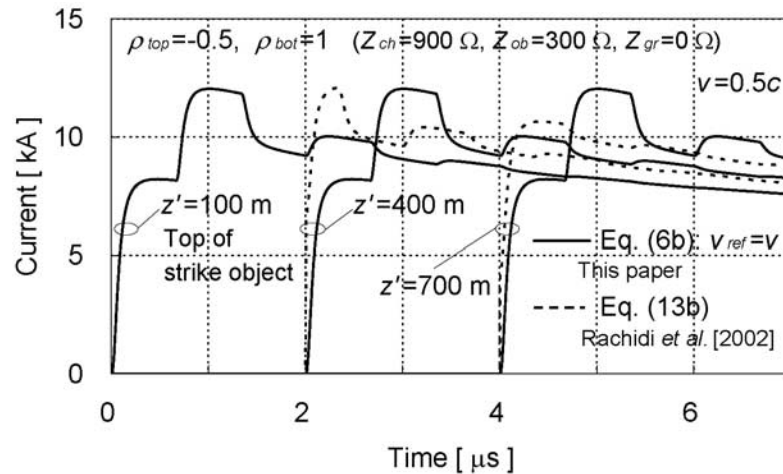
Figure 3. Current waveforms at different heights, $z' = 100$ m (at the top of the strike object), 400 m, and 700 m, along the lightning channel, calculated for a lightning strike to 100-m high object using equations (6b) and (13b). In these calculations, ρ_{top} is set to -0.5, ρ_{bot} is set to 1 ($Z_{ch} = 900$ Ω, $Z_{ob} = 300$ Ω, $Z_{gr} = 0$ Ω), and v is set to 0.5c. The lightning short-circuit current $I_{sc}(h, t)$ is represented by a current pulse thought to be typical for subsequent return strokes [*Nucci et al.*, 1990].

the top of the strike object for upward propagating waves ρ_{top} in the circuit shown in Figure 4b are the same as those in the lumped-voltage-source circuit shown in Figure 1b, given by equation (5). (Note that the circuit shown in Figure 4b is equivalent to the circuit proposed by *Rachidi et al.* [2002, Figure 3], which employs the matched-conditions current.) We will show in Appendix A that current flowing in Z_{ch} of the Norton equivalent circuit does not adequately represent the lightning-channel current. Thus the Norton equivalent circuit is adequate when only lightning currents injected into ground or into a tall grounded object [*Rakov*, 2001, 2003] are needed, that is, when current flowing in the lightning channel is of no concern.

4. Summary

[29] We proposed the use of a lumped voltage source in the TL model of the lightning return stroke generalized to include a tall strike object in computing remote electric and magnetic fields. The magnitude of voltage is expressed in terms of the lightning short-circuit current and equivalent impedance of the lightning channel. Such a model, as opposed to that incorporating a lumped current source, allows a simple and self-consistent formulation of current both along the lightning channel and along the strike object. The expression for current along the strike object (equation (6a)) is the same as its counterpart derived by *Rachidi et al.* [2002] (equation (13a)), who used the distributed-current-source representation of the lightning channel. Further, the structure of the channel current expression for the proposed model (equation (6b)) is the same as that of the corresponding equation derived by *Rachidi et al.* [2002] (equation (13b)). We show that the latter two equations become identical if we adjust the current-wave propagation speeds along the lightning channel in Rachidi et al.'s equation setting these speeds

equal to the return stroke front propagation speed. The proposed model for the case of lightning strike to a tall strike object readily reduces, as the object height decreases to zero, to the case of lightning strike to flat ground (equation (4a)). The latter version of the model includes the dependence of return-stroke current on grounding impedance at the strike point. Additionally, we showed that the Norton (current source) equivalent circuit of lightning discharge is consistent with our proposed voltage source
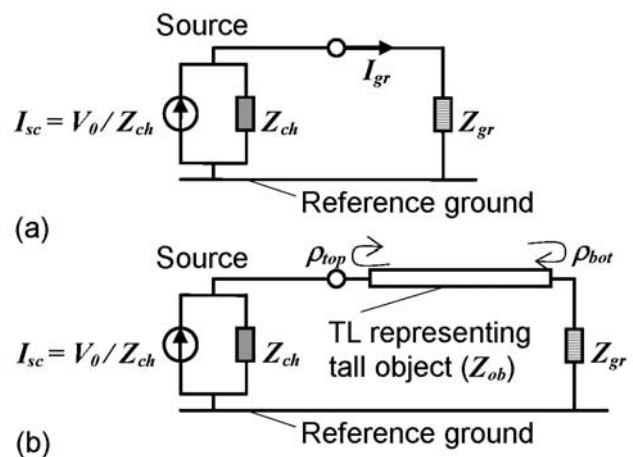

Figure 4. Engineering models of lightning strikes (a) to lumped grounding impedance and (b) to a tall grounded object, in which lightning is represented by the Norton equivalent circuit, labeled "source". The source output currents injected into the lumped grounding impedance Z_{gr} in Figure 4a and into an electrically-long object whose characteristic impedance is Z_{ob} in Figure 4b are consistent with the lumped-voltage-source models presented in Figures 1a and 1b, respectively.

model in terms of the current injected into grounded strike object or into flat ground.

Appendix A: Norton (Current Source) Equivalent Circuit Versus Thevenin (Voltage Source) Equivalent Circuit

[30] Thevenin and Norton equivalent circuits are generally used in reducing a linear circuit, containing impedances and sources, to an equivalent source and a passive "load" connected to this source. A Thevenin equivalent circuit comprises an ideal voltage source in series with an internal impedance, to be connected to a passive load of interest. The voltage source of the Thevenin equivalent circuit generates the voltage that would appear between two nodes of the original circuit when the two nodes are open circuited, that is, the open-circuit voltage. The internal impedance of the Thevenin equivalent circuit is the impedance of the original circuit seen from the two nodes of interest when the two nodes are open circuited. A Norton equivalent circuit comprises an ideal current source in parallel with an internal impedance, to be connected in parallel with a passive load of interest. The current source of the Norton equivalent circuit produces the current that would flow between two nodes of the original circuit when the two nodes are short circuited, that is, the short-circuit current. The internal impedance of the Norton equivalent circuit is defined in the same manner as that of the Thevenin equivalent circuit.

[31] In Appendix A we will show that while current injected into a load of a Norton equivalent circuit is equal to that of the corresponding Thevenin equivalent circuit, currents injected into the internal impedances of these two source circuits are different. Figure A1a shows a Thevenin equivalent circuit, comprising a lumped voltage source generating $V_{oc}(t)$, in series with lumped internal impedance Z_{in}^T, connected to a "load" impedance Z_L. Note that the short-circuit current of this circuit is $V_{oc}(t)/Z_{in}^T$. Figure A1b shows a Norton equivalent circuit, comprising a lumped current source producing $I_{sc}(t)$, in parallel with lumped internal impedance Z_{in}^N, connected to the same load impedance as in Figure A1a, Z_L. Note that the open-circuit voltage of this circuit is $Z_{in}^N I_{sc}(t)$. The currents injected into Z_L and Z_{in}^T of the circuit shown in Figure A1a are given by

$$I_L^T(t) = \frac{V_{oc}(t)}{Z_{in}^T + Z_L},\qquad\text{(A1a)}$$

$$I_{in}^T(t) = \frac{V_{oc}(t)}{Z_{in}^T + Z_L}.\qquad\text{(A1b)}$$

[32] The current injected into Z_L and Z_{in}^N of the circuit shown in Figure A1b are given by

$$I_L^N(t) = \frac{Z_{in}^N}{Z_{in}^N + Z_L} I_{sc}(t),\qquad\text{(A2a)}$$

$$I_{in}^N(t) = \frac{Z_L}{Z_{in}^N + Z_L} I_{sc}(t).\qquad\text{(A2b)}$$
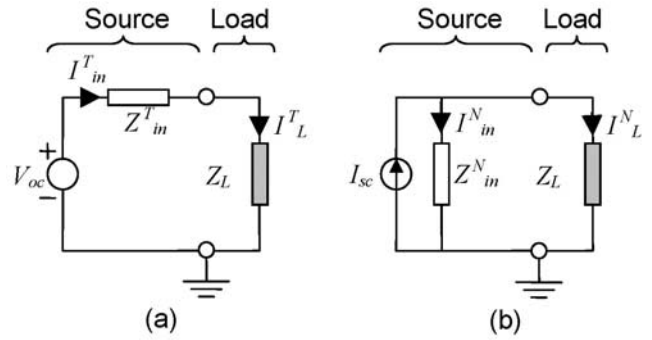

Figure A1. (a) A Thevenin equivalent circuit comprising a lumped voltage source generating $V_{oc}(t)$, in series with lumped internal impedance Z_{in}^T connected to a load impedance Z_L. (b) A Norton equivalent circuit comprising a lumped current source producing $I_{sc}(t)$, in parallel with lumped internal impedance Z_{in}^N, connected to the same load impedance as in Figure A1a, Z_L. The circuit shown in Figure A1b is equivalent to the circuit shown in Figure A1a in terms of the current injected into Z_L when $I_{sc}(t) = V_{oc}(t)/Z_{in}^T$ and $Z_{in}^T = Z_{in}^N$.

[33] It is clear from equations (A1a) and (A2a) that the Norton equivalent circuit shown in Figure A1b is equivalent to the Thevenin equivalent circuit shown in Figure A1a in terms of current injected into the load Z_L when $I_{sc}(t)$ and Z_{in}^N are given by

$$I_{sc}(t) = V_{oc}(t)/Z_{in}^T,\qquad\text{(A3)}$$

$$Z_{in}^N = Z_{in}^T.\qquad\text{(A4)}$$

Equation (A4) indicates that the impedance seen from the load terminals is the same in both circuits.

[34] We now compare currents injected into Z_{in}^T and Z_{in}^N. The current injected into the internal impedance Z_{in}^N of the Norton equivalent circuit (see equation (A2b)) is not equal to that injected into the internal impedance Z_{in}^T of the Thevenin circuit (see equation (A1b)) even when equations (A3) and (A4) are satisfied. Thus the Norton (current source) and Thevenin (voltage source) equivalent circuits are equivalent in terms of the load (strike object) current, but not in terms of internal impedance (lightning channel) current.

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