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Effect of vertically extended strike object on the distribution of current along the lightning channel

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[1] On the basis of a distributed-source representation of the lightning channel, the mathematical formulations of the so-called engineering lightning return stroke models are generalized to take into account the presence of a vertically extended strike object. The strike object is modeled as a lossless uniform transmission line, and the reflection coefficients are all assumed to be constant. The distribution of current along the lightning channel for each model is expressed in terms of the "undisturbed" current, object height, and current reflection coefficients at the top and the bottom of the object. The undisturbed current is defined as the current that would flow in the channel if the current reflection coefficients at the extremities of the strike object were equal to zero, that is, the characteristic impedances of the lightning channel and the strike object were equal to each other and equal to the grounding impedance of the strike object. The distributed-source representation of the lightning channel adopted in this study allows for a more general and straightforward formulations of the generalized return-stroke models than the traditional representations implying a lumped current source at the bottom of the channel, including a self-consistent treatment of the impedance discontinuity at the tower top. *INDEX TERMS:* 3304 Meteorology and Atmospheric Dynamics: Atmospheric electricity; 3324 Meteorology and Atmospheric Dynamics: Lightning; 3367 Meteorology and Atmospheric Dynamics: Theoretical modeling; *KEYWORDS:* Lightning, tall tower, current distribution, reflections, lightning modeling

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1. Introduction

[2] The interaction of lightning with tall strike objects has recently attracted considerable attention among lightning researchers [e.g., *Beierl*, 1992; *Montandon and Beyeler*, 1994; *Guerrieri et al.*, 1998; *Janischewskyj et al.*, 1996; *Fuchs*, 1998; *Shostak et al.*, 1999b; *Baba and Ishii*, 2001; *Rakov*, 2001]. For this reason, some of the return stroke models, initially developed for the case of return strokes initiated at ground level, have been extended to take into account the presence of a vertically extended strike object [e.g., *Diendorfer and Uman*, 1990; *Rachidi et al.*, 1992; *Zundl*, 1994a; *Guerrieri et al.*, 1994, 1996, 1998, 2000; *Rusan et al.*, 1996; *Motoyama et al.*, 1996; *Rachidi et al.*, 1998, 2001; *Janischewskyj et al.*, 1998, 1999a; *Shostak et al.*, 1999a, 1999b, 2000; *Goshima et al.*, 2000; *Kordi et al.*, 2000]. In some of these models, it is assumed that a current pulse $i_o(t)$ associated with the return-stroke process is injected at the lightning attachment point both into the strike object and into the lightning channel [e.g., *Guerrieri et al.*, 1994, 1996, 1998; *Rusan et al.*, 1996; *Motoyama et al.*, 1996; *Rachidi et al.*, 1998, 2001; *Janischewskyj et al.*, 1998, 1999a; *Shostak et al.*, 1999a; *Goshima et al.*, 2000]. The upward-moving wave propagates along the channel at the return-stroke speed v as specified by the return-stroke model. The downward-moving wave propagates at the speed of light along the strike object, assumed to be a lossless uniform transmission line characterized by constant non-zero reflection coefficients at its top and its bottom. As noted by *Guerrieri et al.* [2000], the assumption of two identical current waves injected into the lightning channel and into the strike object implies that their characteristic impedances are equal to each other, which means that, to a certain extent, such models are not self-consistent in that (1) there is no impedance discontinuity at the tower top at the time of lightning attachment to the tower, but (2) there is one when the reflections from ground arrive at the tower top.

[3] In this paper, we present a generalization of several return stroke models, Bruce-Golde (BG) model, transmission line (TL) model, traveling current source (TCS) model, modified transmission line model with linear current decay with height (MTLL), and modified transmission line model with exponential current decay with height (MTLE), taking into account the presence of a vertically extended strike object, which does not employ the assumption that identical current pulses are launched both upward and downward from the object top. The extension for the TL, MTLL and MTLE models is based on a distributed-source representa-

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tion of the return-stroke channel [*Rachidi and Nucci*, 1990; *Cooray*, 2002], which allows more general and straightforward formulations of these models than the traditional representations implying a lumped current source at the bottom of the channel. The TCS model inherently assumes a distributed-source channel, while the BG model can be viewed as a special case of the TCS model [e.g., *Rakov and Uman*, 1998]. We first consider in detail the MTLE model and then extend the results to the BG, TL, TCS, and MTLL models.

2. MTLE Model for a Return Stroke Initiated at Ground Level

[4] Consider first the case of a return stroke initiated at ground level. The spatial-temporal distribution of the current along the vertical channel according to the MTLE model is defined by *Nucci et al.* [1988] and *Nucci and Rachidi* [1989] as:

$$i(z,t) = e^{-z/\lambda} i(0, t - z/v) u(t - z/v) \tag{1}$$

where z is the height above ground, λ is the attenuation height, $i(0,t)$ is the current at the channel base, and v is the return-stroke speed assumed to be constant. u is the unit-step function which, for sake of simplicity, will be omitted in the following equations of sections 2 and 3. This model implies a specified current source connected at the bottom of the channel.

[5] As shown by *Rachidi and Nucci* [1990], the MTLE model can also be expressed in terms of current sources distributed along the channel, these sources representing the effect of the charge initially stored in the corona sheath surrounding the leader channel core. Each elemental source is turned on when the upward-moving return stroke front reaches its altitude, as illustrated in Figure 1, with the resultant current contribution propagating downward at the speed of light. Figure 1 applies to all the engineering models considered in this paper, although for the BG model c (speed of light) should be replaced with infinity.

[6] The general expression for the current source located at height z' is given by [*Rachidi and Nucci*, 1990]

$$\begin{array}{ll} di_s(z', t = 0) & t < z'/v \\ di_s(z', t) = f(t - z'/v) e^{-z'/\lambda} dz' & t \geq z'/v \end{array} \tag{2}$$

where $f(t)$ is an arbitrary function.

[7] The general expression for the current distribution along the channel can be written as

$$i(z,t) = \int_z^H di_s\left(z', t - \frac{z' - z}{c}\right) = \int_z^H f\left(t - \frac{z'}{v} - \frac{z' - z}{c}\right) e^{-z/\lambda} dz' \tag{3a}$$

where c is the speed of light, and H is the return stroke wave front height as seen by the observer at height z, which is given by $H = H(z,t) = (t + z/c)/(1/v + 1/c)$. If the current contributions from the distributed current sources propagated downward at an infinitely large speed, as is the case in the BG model, the expression for H would reduce to $H = vt$.

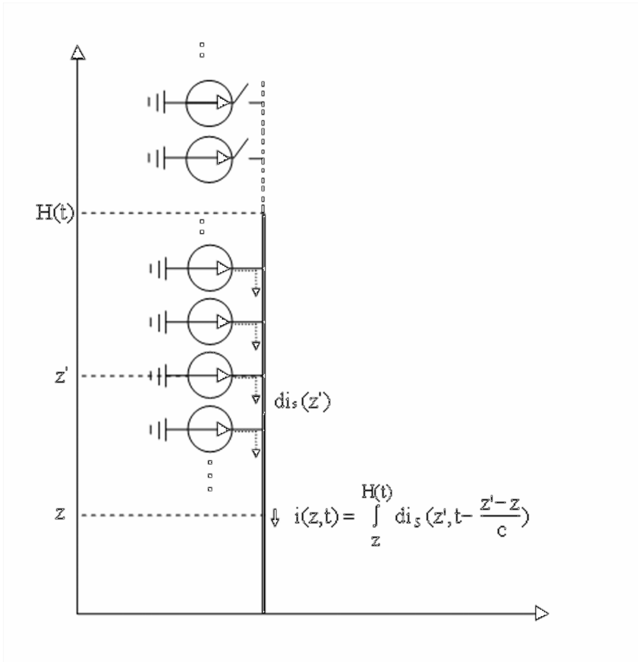

Figure 1. Distributed-source representation of the lightning channel in engineering return-stroke models for the case of no strike object and no reflections at ground.

[8] In particular, the current at the channel base can be expressed as

$$i(0,t) = \int_0^H f\left(t - \frac{z'}{v} - \frac{z'}{c}\right) e^{-z/\lambda} dz' \tag{4}$$

It is important to note that in the above formulation the reflections at ground of the downward propagating contributions from the current sources distributed along the channel have been implicitly disregarded, that is, the equivalent impedance at the strike point has been assumed to be equal to the characteristic impedance of the channel. If this is not the case, that is, the reflections at ground of the downward propagating contributions from the current sources distributed along the channel are to be taken into account, equation (3) will assume a different form. We shall further consider this point at the end of section 3.1.

3. MTLE Model in the Presence of a Vertically Extended Strike Object

[9] The geometry of the problem is shown in Figure 2, which also applies to all other engineering models discussed in this paper. We will consider the strike object (tower) as a lossless uniform transmission line of length h. We will assume that the propagation speed along the strike object is equal to the speed of light, and that the current reflection coefficients at its extremities (the top and the bottom) are constants. We will also disregard any upward connecting leader and any reflections at the return stroke wave front. The physics involved in the process of possible reflections of the upward-propagating current pulses at the return stroke wavefront is rather complicated. These reflections could, in principle, influence the front propagation speed,

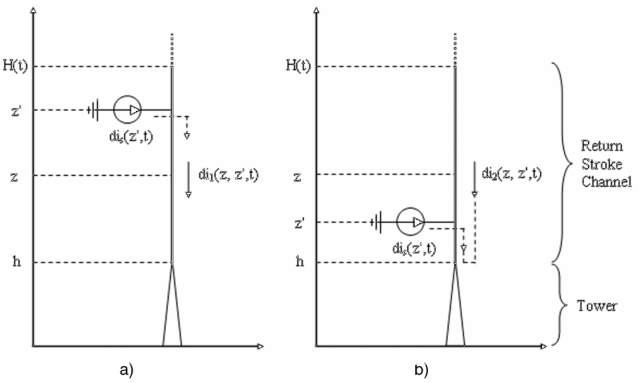

Figure 2. Same as Figure 1, but generalized to include a tall strike object (tower): (a) $z'>z>h$ (only the initial incident wave is shown, di_1 also includes reflections from the top and the bottom of the object); (b) $h<z'<z$ (only reflection from the object top is shown; di_2 also includes reflections from the bottom of the object). The total current $i(z,t)$ is obtained by integrating di_1 and di_2 within appropriate limits and summing the two resultant current contributions.

and they are poorly understood. However, it is possible to include such reflections in the calculations [see, for example, *Heidler and Hopf*, 1994; *Shostak et al.*, 1999a]. *Shostak et al.* [1999a] have shown that some fine structure of the radiated field could be attributed to these reflections.

[10] The reflection coefficient (for the current) at the bottom of the object can be expressed in terms of the characteristic impedance of the strike object Z_t and the equivalent impedance of the grounding system Z_g. If the grounding system is "electrically long", Z_g can be viewed, at least at early times, as the characteristic impedance of the grounding system.

$$\rho_g = \frac{Z_t - Z_g}{Z_t + Z_g} \tag{5}$$

In a similar way, we can define two reflection coefficients at the top of the strike object for the upward, ρ_t^+, and downward, ρ_t^-, propagating current waves,

$$\rho_t^+ = \frac{Z_t - Z_{ch}}{Z_t + Z_{ch}} \qquad \rho_t^- = \frac{Z_{ch} - Z_t}{Z_{ch} + Z_t} = -\rho_t^+ \tag{6}$$

To simplify the notations, we define

$$\rho_t^+ = \rho_t \qquad \rho_t^- = -\rho_t \tag{7}$$

3.1. Distribution of Current Along the Lightning Channel

[11] Consider the current $di_1(z,z',t)$ due to an elemental current source $di_s(z',t)$ located at height $z' > z$ (see Figure 2a). If we assume that both reflection coefficients ρ_t and ρ_g are equal to zero, we can write

$$di_1(z,z',t) = di_s\left(z', t - \frac{z'-z}{c}\right) = e^{-(z'-h)/\lambda} f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c}\right)dz' \tag{8}$$

Now, for the general case when ρ_t and ρ_g are different from zero, multiple reflections from the top and bottom of the strike object are to be added to the right-hand side of equation (8). Taking into account multiple reflections, we obtain the expression for the elemental current at height z, for $z' > z$

$$\begin{aligned} di_1(z,z',t) = e^{(-z'-h)/\lambda}dz' \Bigg\{ & f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c}\right) \\ & -\rho_t f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c} - \frac{2(z-h)}{c}\right) \\ & +(1-\rho_t)\rho_g(1+\rho_t) f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c} - \frac{2z}{c}\right) \\ & +(1-\rho_t)\rho_g^2\rho_t(1+\rho_t) f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c} - \frac{2z}{c} - \frac{2h}{c}\right) \\ & +(1-\rho_t)\rho_g^3\rho_t^2(1+\rho_t) f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c} - \frac{2z}{c} - \frac{4h}{c}\right) \\ & + \cdots\cdots\cdots \Bigg\} \end{aligned} \tag{9}$$

Regrouping the terms, we get for $z' > z$

$$\begin{aligned} di_1(z,z',t) = e^{-(z'-h)/\lambda}dz' \Bigg\{ & f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c}\right) \\ & - \rho_t f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c} - \frac{2(z-h)}{c}\right) \\ & + (1-\rho_t)(1+\rho_t)\sum_{n=1}^{\infty}\rho_g^n\rho_t^{n-1} \\ & \cdot f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c} - \frac{2z}{c} - \frac{2(n-1)h}{c}\right) \Bigg\} \end{aligned} \tag{10a}$$

where n is an index representing the successive multiple reflections occurring at the two ends of the strike object.

[12] Due to reflections at the top and at the bottom of the object, elemental sources below the observation point at z also contribute to the current at that point. The current $di_2(z,z',t)$ due to an elemental current source $di_s(z',t)$ located at height $h < z' < z$ (see Figure 2b) is given by

$$\begin{aligned} di_2(z,z',t) = e^{-(z'-h)/\lambda}dz' \Bigg\{ & -\rho_t f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c} - \frac{2(z-h)}{c}\right) \\ & + (1-\rho_t)(1+\rho_t)\sum_{n=1}^{\infty}\rho_g^n\rho_t^{n-1} \\ & \cdot f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c} - \frac{2z}{c} - \frac{2(n-1)h}{c}\right) \Bigg\} \end{aligned} \tag{10b}$$

The total current at height z can be obtained by integrating equations (10a) and (10b) within appropriate limits and summing the two resultant current contributions

$$i(z,t) = \int_z^H di_1(z,z',t) + \int_h^z di_2(z,z',t) \tag{11}$$

Now, combining equations (1) and (3a) for the case of the return stroke initiated at ground yields

$$i\left(0, t - \frac{z}{v}\right)e^{-z/\lambda} = \int_z^H f\left(t - \frac{z'}{v} - \frac{z'-z}{c}\right)e^{-z'/\lambda}dz' \tag{12}$$

In order to be able to extend equation (12) to the geometry of Figure 2 (return stroke initiated at the top of a tall strike object), let us first define the 'undisturbed' current as the current that would be measured at the object top (lightning attachment point) if both reflection coefficients ρ_t and ρ_g were equal to zero. Note that under these ideal conditions the 'undisturbed' current waveform would also be measured at any point along the strike object and would be measured at ground level when h = 0. Applying the above definition of the undisturbed current to (12) we can write

$$i_o\left(h, t - \frac{z-h}{v}\right)e^{-(z-h)/\lambda} = \int_z^H f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c}\right)e^{-(z'-h)/\lambda}dz' \tag{13}$$

Substituting (10a) and (10b) into (11) and taking into account (13), we obtain the final expression for the current distribution along the channel for $h < z < H$

$$\begin{aligned} i(z,t) = {} & e^{-(z-h)/\lambda} i_o\left(h, t - \frac{z-h}{v}\right) - \rho_t i_o\left(h, t - \frac{z-h}{c}\right) \\ & + (1-\rho_t)(1+\rho_t)\sum_{n=1}^{\infty}\rho_g^n\rho_t^{n-1} i_o\left(h, t - \frac{z-h}{c} - \frac{2nh}{c}\right) \end{aligned} \tag{14}$$

Note that retaining only one term (n = 1) in the sum and setting ρ_t = 0 and h = 0 in equation (14) we can obtain a generalized form of equation (3a) mentioned at the end of section 2, in which the reflections at ground of the downward propagating contributions from the current sources distributed along the channel are taken into account,

$$i(z,t) = e^{-z/\lambda} i_o\left(0, t - \frac{z}{v}\right) + \rho_g i_o\left(0, t - \frac{z}{c}\right) \tag{3b}$$

where $\rho_g = \frac{Z_{ch}-Z_g}{Z_{ch}+Z_g}$, different from (5), and

$$i_o(0,t) = \int_0^H f\left(t - \frac{z'}{v} - \frac{z'}{c}\right)e^{-z'/\lambda}dz'$$

3.2. Distribution of Current Along the Strike Object

[13] Considering a point along the strike object, $0 < z < h$, and applying the same procedure as in section 3.1., we obtain the following expression for the current due to an elemental source located at z':

$$\begin{aligned} di(z,z',t) = e^{-(z'-h)/\lambda}dz' \Bigg\{ & (1-\rho_t) f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c}\right) \\ & + \rho_g(1-\rho_t) f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c} - \frac{2z}{c}\right) \\ & + \rho_g\rho_t(1-\rho_t) f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c} - \frac{2h}{c}\right) \\ & + \rho_g^2\rho_t(1-\rho_t) f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c} - \frac{2z}{c} - \frac{2h}{c}\right) \\ & + \rho_g^2\rho_t^2(1-\rho_t) f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c} - \frac{4h}{c}\right) \\ & + \rho_g^3\rho_t^2(1-\rho_t) f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c} - \frac{2z}{c} - \frac{4h}{c}\right) \\ & + \cdots\cdots\cdots \Bigg\} \end{aligned} \tag{15}$$

Regrouping terms, we get

$$\begin{aligned} di(z,z',t) = {} & (1-\rho_t)e^{-(z'-h)/\lambda}dz' \\ & \left\{ \sum_{n=0}^{\infty} \left[\rho_g^n\rho_t^n f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c} - \frac{2nh}{c}\right) \right.\right. \\ & \left.\left. + \rho_g^{n+1}\rho_t^n f\left(t - \frac{z'-h}{v} - \frac{z'-z}{c} - \frac{2z}{c} - \frac{2nh}{c}\right)\right]\right\} \end{aligned} \tag{16}$$

Then the total current at z, $0 < z < h$, is given by

$$i(z,t) = \int_h^H di(z,z',t) \tag{17}$$

Again, using the fact that the undisturbed current can be related to the current sources distributed along the channel (see equation (13)),

$$i_o(h,t) = \int_h^{H(t)} f\left(t - \frac{z'-h}{v} - \frac{z'-h}{c}\right)e^{-(z'-h)/\lambda}dz' \tag{18}$$

we obtain the current distribution along the strike object, $0 < z < h$

$$\begin{aligned} i(z,t) = (1-\rho_t)\sum_{n=0}^{\infty} \Bigg[& \rho_g^n\rho_t^n i_o\left(h, t - \frac{h-z}{c} - \frac{2nh}{c}\right) \\ & + \rho_g^{n+1}\rho_t^n i_o\left(h, t - \frac{h+z}{c} - \frac{2nh}{c}\right)\Bigg] \end{aligned} \tag{19}$$

Note that

$$1 - \rho_t = 1 - \frac{Z_t - Z_{ch}}{Z_t + Z_{ch}} = \frac{2Z_{ch}}{Z_t + Z_{ch}} \tag{20}$$

which is the transmission (refraction) coefficient at the junction point between the lightning channel and the strike object for downward-moving current waves. Note further that when only one pair of terms ($n = 0$) of the sum is retained, and $z = 0$, $\rho_g = 1$ and $\rho_t = 0$, equation (19) results in $i(0,t) = 2i_o(h,t - h/c)$, and for $h = 0$, $i(0,t) = 2i_o(0,t)$. The latter result can be also obtained from equation (3b) by setting ρ_g = 1 and z = 0.

[14] Equation (19) can be represented by the equivalent circuit shown in Figure 3. Note that this circuit is similar to the one proposed by *Rakov* [2001, Figure 4a], although he used the short-circuit current, I, to define his current source (Norton equivalent circuit), while our current source in Figure 3 is given by $2i_o$, where i_o corresponds to matched conditions (Z_{ch} = Z_t = Z_g). As expected, the short-circuit current is twice the matched-conditions current, $I = 2i_o$.

4. Extension to Other Models

[15] Many of the so-called engineering models can be expressed using the following general expression [*Rakov and Uman*, 1998]:

$$i(z,t) = P(z)i(0, t - z/v^*)u(t - z/v) \tag{21}$$

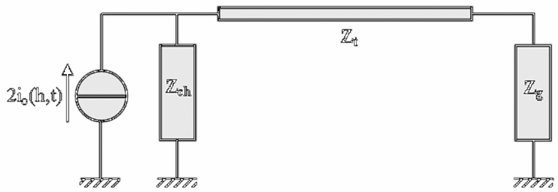

Figure 3. Equivalent circuit for the tower struck by lightning (Equation (19)). i_o is the 'undisturbed current'. Z_{ch} and Z_t are the characteristic impedances of the lightning channel and of the tall strike object respectively. Z_g is the equivalent grounding impedance.

where $P(z)$ is the current attenuation function, u is the unit-step function, v is the return stroke front speed, and v^* is the current-wave speed. The unit-step function needs to be shown explicitly in (21) in order to describe a possible current discontinuity (inherent in the BG and TCS models) at the return-stroke front. Table 1 summarizes the expressions for $P(z)$ and v^* for some of the most used return-stroke models. In Table 1, v is the return-stroke front speed, c is the speed of light, H_{tot} is the total channel height, and λ is the attenuation height.

[16] Recently, *Cooray* [2002] has shown that the current distribution $i(z,t)$ for any engineering model, not only for the MTLE model as previously shown by *Rachidi and Nucci* [1990], can be viewed as due to current sources distributed along the channel. The general expression for the distributed sources is given by

$$di_s(z',t) = \left[-\frac{\partial i(z',t)}{\partial z'} + \frac{1}{c}\frac{\partial i(z',t)}{\partial t}\right]dz' \qquad (22)$$

where c is the speed of light.

[17] Inserting (21) into (22), one gets the expression for distributed current sources as a function of channel-base current:

$$\begin{aligned} di_s(z',t) = \Bigg[& -\frac{dP(z')}{dz'} i(0, t - z'/v^*) u(t - z'/v) \\ & + P(z')\left(\frac{1}{c} + \frac{1}{v^*}\right)\frac{\partial i(0, t - z'/v^*)}{\partial t} u(t - z'/v) \\ & + P(z')\left(\frac{1}{c} + \frac{1}{v}\right) i(0, t - z'/v^*)\delta(t - z'/v)\Bigg] dz' \end{aligned} \qquad (23)$$

where δ is the Dirac distribution. The last term of (23) is non-zero only when there is a current discontinuity at the return stroke front.

Table 1. $P(z)$ and v^* for Different Return-Stroke Models

Model	P(z)	v^*
BG	1	∞
TL	1	v
TCS	1	$-c$
MTLL	$1-z/H_{tot}$	v
MTLE	$\exp(-z/\lambda)$	v

[18] Table 2 summarizes the resulting functions $di_s(z',t)$ for the five engineering return stroke models presented in Table 1. For the TL, MTLL, and MTLE models, it is assumed that there is no discontinuity at the return-stroke front, and for the BG model c in (23) is replaced with infinity.

[19] Now, following a mathematical development similar to that in section 3, we obtain the general expression for the current distribution along the lightning channel, $h < z < H$,

$$\begin{aligned} i(z,t) = \Bigg[& P(z-h) i_o\left(h, t - \frac{z-h}{v^*}\right) - \rho_t i_o\left(h, t - \frac{z-h}{c}\right) \\ & + (1-\rho_t)(1+\rho_t)\sum_{n=1}^{\infty} \rho_g^n \rho_t^{n-1} i_o \\ & \cdot \left(h, t - \frac{z}{c} - \frac{(2n-1)h}{c}\right)\Bigg] u(t - z/v) \end{aligned} \qquad (24)$$

and for the current distribution along the strike object, $0 < z < h$,

$$\begin{aligned} i(z,t) = (1-\rho_t)\sum_{n=0}^{\infty} \Bigg[& \rho_g^n \rho_t^n i_o\left(h, t - \frac{h-z}{c} - \frac{2nh}{c}\right) \\ & + \rho_g^{n+1}\rho_t^n i_o\left(h, t - \frac{h+z}{c} - \frac{2nh}{c}\right)\Bigg] \end{aligned} \qquad (25)$$

Equations (24) and (25) apply to all engineering models that are described by equation (21), although for the BG model c should be replaced with infinity. Note that equation (25) is identical to equation (19); that is, the current distribution along the strike object is independent of the return-stroke model. This is in agreement with the fact that we have assumed the same undisturbed current for all models. For the MTLE model, $P(z-h) = e^{-(z-h)/\lambda}$, $v^* = v$, and equation (24) becomes identical to equation (14).

5. Summary

[20] Based on a distributed-source representation of lightning channel, five engineering lightning return stroke models (BG, TL, TCS, MTLL, and MTLE models) are extended to include a tall strike object. In the case of the TL, MTLL and MTLE models, the distributed-source representation of the lightning channel allows more general and straightforward formulations of these models, including a self-consistent treatment of the impedance discontinuity at the tower top, than does the traditional representation implying a

Table 2. Expressions for $di_s(z',t)$ as a Function of Channel-Base Current for Different Return Stroke Models

Model	$di_s(z',t)/\,dz'$
BG	$\frac{1}{v} i(0,t)\delta(t - z'/v)$
TL	$\left(\frac{1}{c}+\frac{1}{v}\right)\frac{\partial i(0,t-z'/v)}{\partial t} u(t - z'/v)$
TCS	$\left(\frac{1}{c}+\frac{1}{v}\right) i(0, t + z'/c)\delta(t - z'/v)$
MTLL	$\left[\left(\frac{1}{c}+\frac{1}{v}\right)\left(1 - \frac{z'}{H_{tot}}\right)\frac{\partial i(0,t-z'/v)}{\partial t} + \frac{i(0,t-z'/v)}{H_{tot}}\right] u(t - z'/v)$
MTLE	$\left[\left(\frac{1}{c}+\frac{1}{v}\right)\exp(-z'/\lambda)\frac{\partial i(0,t-z'/v)}{\partial t} + \frac{i(0,t-z'/v)}{\lambda}\exp(-z'/\lambda)\right] u(t - z'/v)$

lumped current source at the bottom of the channel. The object is represented by a lossless uniform transmission line, and current reflection coefficients at its extremities are assumed to be constant. The distribution of current along the lightning channel for each model is expressed in terms of the "undisturbed" current, object height, and current reflection coefficients at the top and bottom of the object. The undisturbed current corresponds to matched conditions, $Z_{ch} = Z_t = Z_g$, and is one-half of the short-circuit current of the equivalent lightning source (no strike object, $Z_g = 0$). The distribution of current along the strike object is clearly independent of the return-stroke model used, provided that the same undisturbed current is specified for each model.

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